Investigation of factors affecting tire-road separation using a half-car model

Vu Ngoc Tuan*

Le Quy Don Technical University, 236 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam.
*Corresponding author: tuan.vungoc@lqdtu.edu.vn
Received 4 Apr. 2024; Revised 17 May 2024; Accepted 12 Jun. 2024; Published 25 Jun. 2024.
DOI: https://doi.org/10.54939/1859-1043.j.mst.96.2024.152-158

ABSTRACT

This paper conducts numerical simulations to delve into the phenomenon of tire-road separation using a half-car model. It scrutinizes scenarios where the tires lose contact with the surface, delving into the time response and phase portrait domains. This analysis captures vertical displacements and pitch dynamics in both in-contact and no-contact states. The research examines tire-road detachment for individual wheels and both wheels concurrently while also considering variations in road conditions and suspension structures. By comparing outcomes between scenarios where tires maintain contact and where they separate, the study offers valuable insights for vehicle designers, aiding them in selecting optimal suspension parameters to mitigate tire-road separation.

Keywords: Vehicle vibrations; Half-Car dynamics; Tire-Road separation; Half-Car vibration; Vehicle safety; Ride comfort.

1. INTRODUCTION

Research on vehicle vibrations typically operates under the assumption that tires remain in contact with the surface [1]. Moreover, Gillespie and Guiggiani indicate that many recent mathematical models overlook the separation condition to simplify numerical simulations and optimization processes [2, 3]. However, in reality, cars may lose contact with the road due to factors like high speeds or large bumps [4]; the interaction between the wheel and the surface, particularly when the wheel loses contact with the road, has rarely been studied [5]. Recent studies in separation dynamics have demonstrated that accounting for the possibility of separation between the wheel and the ground leads to a more realistic system [6-12]. While these studies often focus on tire-road separation in suspension systems with 2 degrees of freedom (2DOF), which is adequate for providing a basic understanding of vibration dynamics, a quarter-car model is limited in its ability to study longitudinal and lateral interconnections due to its focus on only one corner of the vehicle [13, 14]. Consequently, it primarily simulates body bounce while neglecting pitch and roll modes [15]. To explore these modes, researchers might apply the separation assumption in a more comprehensive vehicle model known as the full-car model. However, solving the governing equations related to the discontinuous state of the full-car model is more complex due to various wheel and suspension constraints. Therefore, this study examines how the progression of separation impacts the vertical displacements and pitch motions of a 4DOF half-car model and validates the system's ride comfort under specific conditions. Under the separation condition, contact is lost when the displacement of the unsprung mass exceeds the road excitation, and the contact force becomes zero due to the lack of interaction [16].

2. VEHICLE MODELING

In this section, the article introduces a half-car model along with its corresponding differential equations in the dimensional domain. Furthermore, the notion of separation time between the road and the tire will be presented.

2.1. Half-car model

A vehicle's half-car vibrating model, depicted in figure 1 and featuring four degrees of freedom, is employed for investigating vibration dynamics. This model encompasses vertical body bounce x, body pitch θ, wheels' hops x_1, x_2, and independent ground inputs x_{r1}, x_{r2}.

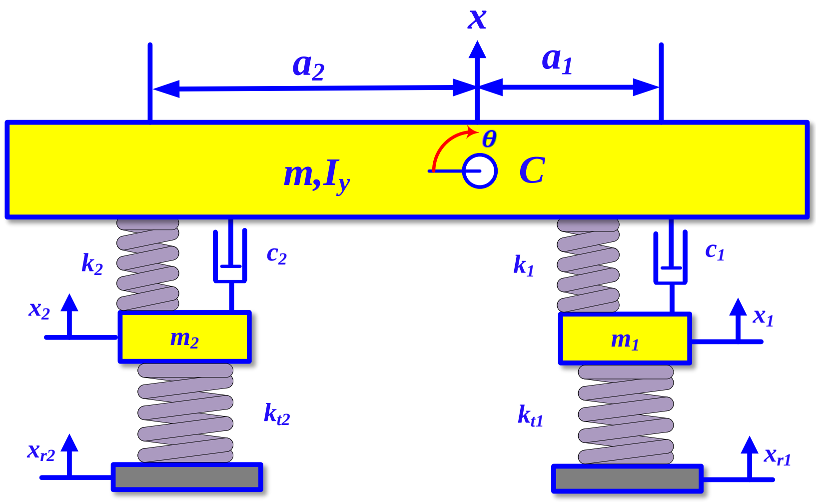

***Figure 1**. Half-car model for a vibrating investigation. Where: m is body mass; C is the mass centre; I_y is the lateral mass moment; a_1, a_2 are distances from the front and rear axles to the mass centre; c_1, c_2 are the damping coefficient of the dampers; k_1, k_2 are the stiffness of suspension springs; k_{t1}, k_{t2} are the stiffness of tires.*

2.2. Governing equations in the in-contact state

Under typical circumstances, the tire remains in constant contact with the surface. The differential equation system governing the model in this in-contact state is as follows [17]:

$$m\ddot{x} + c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) + k_1(x - x_1 - a_1\theta) + k_2(x - x_2 + a_2\theta) = 0 \quad (1)$$

$$I_y\ddot{\theta} - a_1c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + a_2c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) - a_1k_1(x - x_1 - a_1\theta) + a_2k_2(x - x_2 + a_2\theta) = 0 \quad (2)$$

$$m_1\ddot{x}_1 - c_1(\dot{x} - \dot{x}_1 - a_1\dot{\theta}) + k_{t1}(x_1 - x_{r1}) - k_1(x - x_1 - a_1\theta) = 0 \quad (3)$$

$$m_2\ddot{x}_2 - c_2(\dot{x} - \dot{x}_2 + a_2\dot{\theta}) + k_{t2}(x_2 - x_{r2}) - k_2(x - x_2 + a_2\theta) = 0 \quad (4)$$

2.3. Governing equations in the no-contact state

When the vehicle's suspension encounters periodic vertical inputs administered via the contact patch, it might lose contact with the ground, entering a state of separation. In this separation mode, the equation system governing the tire-road interaction will be as follows:

$$\ddot{x} = -\frac{k_1 + k_2}{m}y_1 + \frac{k_1a_1 - k_2a_2}{m}y_2 + \frac{k_1}{m}y_3 + \frac{k_2}{m}y_4 - \frac{c_1 + c_2}{m}y_5 + \frac{c_1a_1 - c_2a_2}{m}y_6 + \frac{c_1}{m}y_7 + \frac{c_2}{m}y_8 \quad (5)$$

$$\begin{aligned} \ddot{\theta} = {} & \frac{a_1k_1 - a_2k_2}{I_y}y_1 - \frac{a_1^2k_1 + a_2^2k_2}{I_y}y_2 - \frac{a_1k_1}{I_y}y_3 + \frac{a_2k_2}{I_y}y_4 \\ & + \frac{a_1c_1 - a_2c_2}{I_y}y_5 - \frac{a_1^2c_1 + a_2^2c_2}{I_y}y_6 - \frac{a_1c_1}{I_y}y_7 + \frac{a_2c_2}{I_y}y_8 \end{aligned} \quad (6)$$

$$\ddot{x}_1 = \frac{k_1}{m_1}y_1 - \frac{k_1a_1}{m_1}y_2 - \frac{k_1}{m_1}y_3 + \frac{c_1}{m_1}y_5 - \frac{c_1a_1}{m_1}y_6 - \frac{c_1}{m_1}y_7 - (1 + \frac{a_2}{m_1l}m)g \quad (7)$$

$$\ddot{x}_2 = \frac{k_2}{m_2}y_1 + \frac{k_2a_2}{m_2}y_2 - \frac{k_2}{m_2}y_4 + \frac{c_2}{m_2}y_5 + \frac{c_2a_2}{m_2}y_6 - \frac{c_2}{m_2}y_8 - (1 + \frac{a_1}{m_2l}m)g \quad (8)$$

The inclusion of pitch motion renders the equation system more complex compared to a quarter model with only two states. Consequently, the study employed an ODE solver in MATLAB to numerically solve the differential equations.

3. RESULTS AND DISCUSSION

Here, The analysis of the time response of the bicycle-car model begins with the assumption of harmonic excitations. Specifically, the input excitations for this study are sinusoidal functions that represent wavy roads in the following form: $x_{r1} = y_0 sin\omega t$; $x_{r2} = y_0 sin\omega t$. Consider a vehicle featuring independent suspension in the front, the practical half model of this car is detailed in table 1 [9].

Table 1. *Parameters of a half-car model.*

Parameter	Value [Unit]	Parameter	Value [Unit]
m	420 kg	I_y	1100 kgm^2
m_1	53 kg	m_2	76 kg
a_1	1.4 m	a_2	1.47 m
c_1	1000 Ns/m	c_2	1000 Ns/m
k_1	10000 N/m	k_2	13000 N/m
k_{t2}	200000 N/m	k_{t1}	200000 N/m

In practice, high speeds and large bumps impact the tire-road contact. Therefore, the simulation is conducted with varying road amplitude y_0 and input frequency ω. Initially, ride comfort is examined over time by solving the aforementioned equation system with two periodic vertical inputs from the road. To identify transitions between in-contact and no-contact states in the time response, the program incorporates two indicator variables, denoted as I_1 and I_2 which determine the state of the wheels at each evaluation point. A value of 0 signifies the in-contact state, while 0.1 indicates the contact-free state for the front wheel, and -0.1 denotes the absence of contact for the rear wheel.

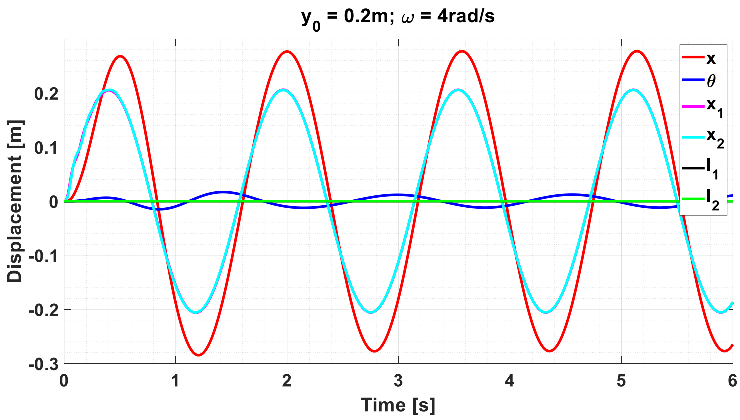

Figure 2. *Time response in the in-contact state.*

In the time response within the in-contact zone, as indicated in figure 2, the system remains in a state of no-separation, consistently maintaining contact with the surface. Despite a high road amplitude of $y_0 = 0.2\,m$, the tires remain inseparable and do not deviate from the contact condition. This could be attributed to a low input frequency $\omega = 4\,rad/s$, causing both indicators to align with the x −axis. It is noteworthy that the time response of the front wheel x_1 closely resembles that of the rear wheel x_2, except for the initial period when transient response is observed in the rear wheel.

At higher excitation frequencies, the tire may lose contact with the road. This separation phenomenon is illustrated in figure 3, particularly evident at the rear wheel, as the input frequency is increased to $\omega = 10rad/s$. The indicator I_2 tracks the states of the rear wheel at any given moment. For instance, at the beginning, the rear wheel maintains contact with the surface, separation occurs around $0.8s$ into the cycle, and contact is reestablished at $1.1s$. Consequently, the rear wheel remains out of contact with the surface for a certain duration during each cycle. Meanwhile, separation does not occur in the front tire, thus, I_1 consistently remains at zero.

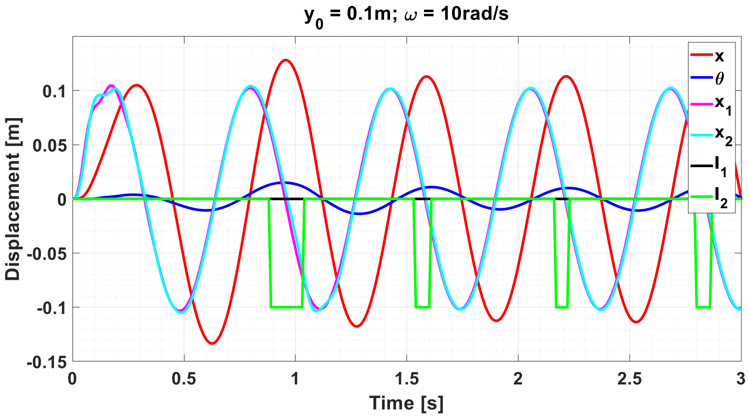

***Figure 3**. Time response with no-contact at the rear wheel only.*

Generally, figure 4 illustrates the intriguing separation phenomenon occurring in both tires, where they lose contact due to being unable to follow the road at a high input frequency of $\omega = 22rad/s$. Notably, the rear wheel experiences separation from the ground more frequently than the front wheel, as evidenced by the indicators I_1 and I_2. Consequently, the displacements of both wheels exhibit significant disparities, indicating that they do not remain coincident.

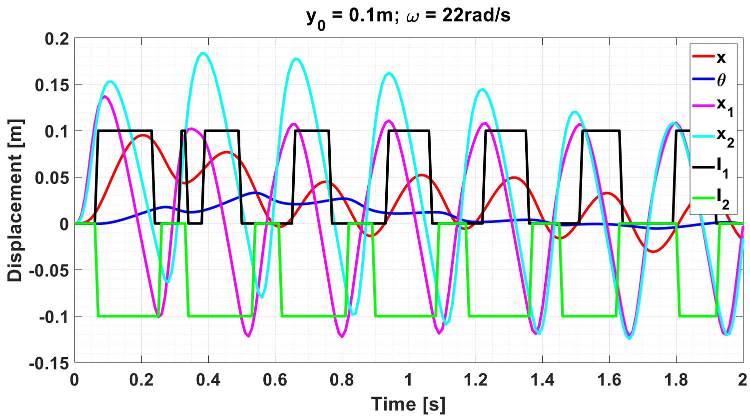

***Figure 4**. Time response with both wheels separated.*

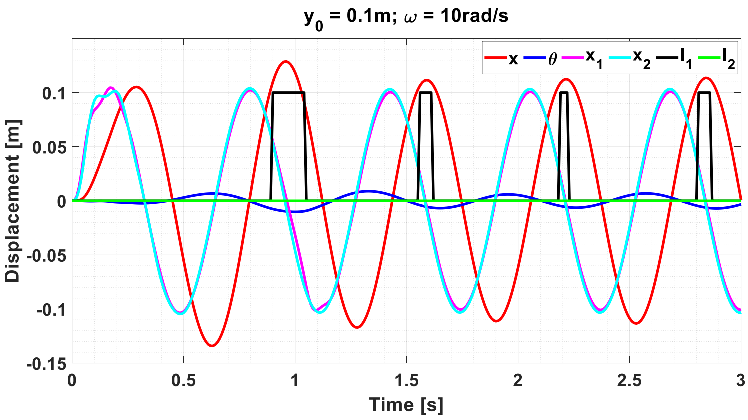

***Figure 5.** Time response with no-contact at the front wheel only.*

The front wheel could be separated only from the ground when the structure of suspension system changed, as can be seen in figure 5. To examine how the suspension structure affects tire-road separation, the paper adjusts the coefficient of the main stiffness. Here, the stiffness of springs in the front suspension is greater than the rear one, such as $k_1 = 13000$ N/m, $k_2 = 10000$ N/m while other inputs remained. Even though the dynamic responses of both wheels coincide, the rear tire consistently maintains contact with the road profile.

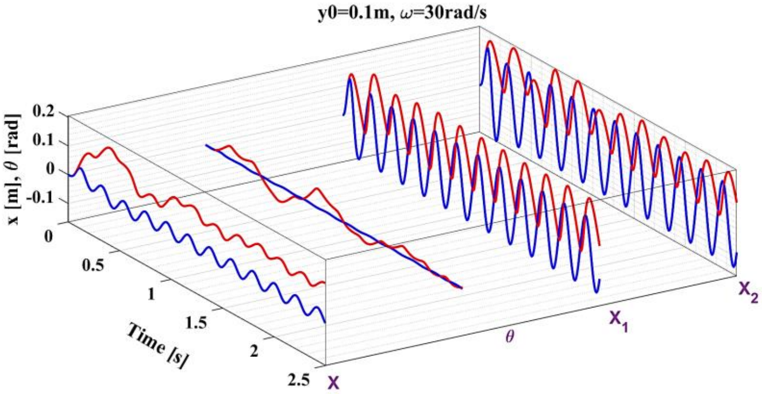

Figure 6. *Series of time response with no-contact assumption.*

To observe how the dynamic response changes in the absence of contact, a sequence of system time responses is illustrated in figure 6. These responses depict multiple displacements simultaneously in reaction to a high-frequency excitation with $\omega = 30\ rad/s$. The bouncing and pitching of the vehicle, along with the vertical displacements of both tires, are displayed in four planes represented by $x, \theta, x_1,$ and x_2, respectively. When the tires are in contact with the road, the body and two wheels appear to have three vertical responses, which are harmonic waves. However, this pattern does not hold true when separation is considered, leading to significant reductions in unexpected effects on vertical ride comfort and pitch motion due to high amplitudes.

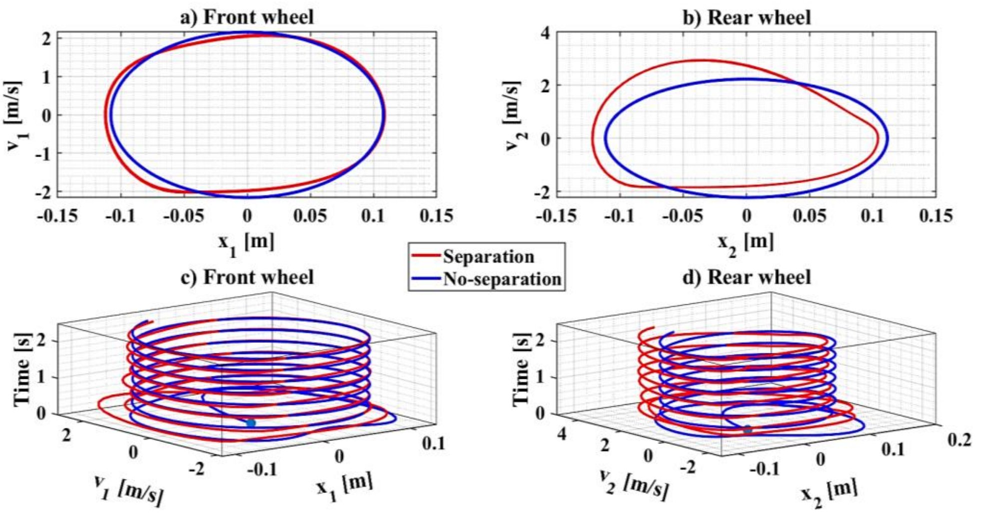

Figure 7. *Phase portraits of two wheels.*

Phase plane analysis is a widely used and valuable technique for studying the behaviors of nonlinear systems. It involves plotting trajectories in a two-dimensional graph within the configuration space of the dynamic system, representing the system's evolution based on two systemic state variables under various initial conditions. Tire-road separation introduces nonlinearity, causing dynamic response trajectories to diverge from those of a linear system without separation. Figure 7 illustrates phase portrait graphs in a dimensional field using input parameters from figure 6, excluding a transient period.

These graphs depict the dimensional phase portraits of both wheels' displacements corresponding to their velocities, as shown in figures 7a and 7b. In the case of the no-contact assumption, figures 7c, 7d show three directions of v, x, and investigation time, a 3-D helix of geometric pitch is formed, corresponding to an ellipse in the two-dimensional plane depicted in figures 7a, 7b. However, there is a noticeable deviation in the separation response from this elliptical pattern. It's worth noting that figures 7a and 7b omit the first half of the investigation time, while the other two graphs start from the beginning of the investigation time.

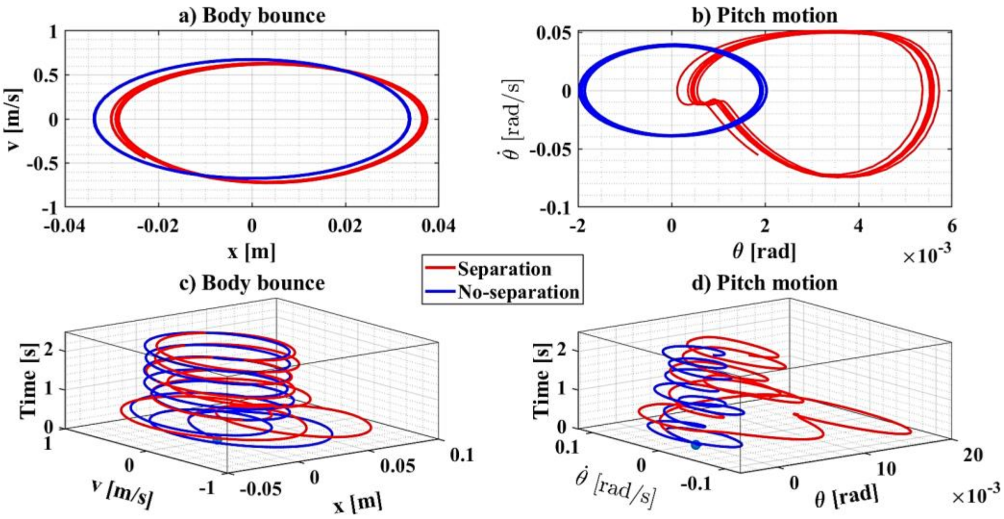

Figure 8. *Phase portraits of body bounce and pitch motion.*

Figure 8 displays the relationship between displacement and velocity for both the vehicle's body and pitch dynamics. The dynamic responses for both exhibit harmonic functions, forming an ellipse when projected onto a plane, as illustrated in figures 8a and 8b. However, when separation occurs, the body bounce and pitch motion become more chaotic, as depicted in figures 8c and 8d. This has a significant impact on passenger ride comfort.

4. CONCLUSIONS

The study concentrates on simulating the tire-road separation phenomenon using a half-car model in the nondimensional field. It examines the system's time response in both contact and no-contact states, indicated by two indicators. Furthermore, it investigates factors influencing the separation phenomenon, including road conditions and suspension structures, through two- and three-dimensional phase portraits. By comparing responses in these states, the paper underscores the differences resulting from tire-road separation, which ultimately diminishes passenger ride comfort. The novel findings, including the time response of separation dynamics for the 4DOF half-car model, could be valuable for future research and designers.

Acknowledgement: *The author extends appreciation to Professor Reza Jazar, alongside their colleague, and the College of Science, Technology, Engineering, and Mathematics at RMIT University, Australia, for their significant assistance and support in advancing the development of this research field.*

REFERENCES

[1]. Nguyen, Q. D. *"Tire-road separation duration in vibration dynamics".* Transport and Communications Science Journal, Vol. 14, Issue. 15, (2023).

[2]. Gillespie, T.D. *"Fundamentals of vehicle dynamics"* (Vol. 400). Warrendale, PA: Society of automotive engineers, (1992).
[3]. Guiggiani, M. *"The science of vehicle dynamics".* Pisa, Italy: Springer Netherlands, p.15, (2014).
[4]. Nguyen, Q. D. *"Limitation of the tire-road contact".* Journal of science & technology Vol.59-No.6B P-ISSN 1859-3585 E-ISSN 2615-9619, (2023).
[5]. Lukoshevichiene, O & Sokolovskij, E, *"Movement of a vehicle after loosing the touch of the wheels with road surface",* Transport Engineering, vol., pp. 29-34, (2001).
[6]. Nguyen, Q. D., Milani, S., Vo, D., Marzbani, H., & Jazar, R. N. *"Vibration analysis of the bicycle-car model considering tire-road separation".* SAE International Journal of Commercial Vehicles, (2021).
[7]. Nguyen, Q. D., Milani, S., Marzbani, H., & Jazar, R. N. *"Tire-road separation time reduction by an adaptive pid controller utilizing particle swarm optimization algorithm".* SAE International Journal of Commercial Vehicles, 14, (2021).
[8]. Nguyen, Q. D., Milani, S., Marzbani, H., & Jazar, R. N. *"Vehicle vibrations analysis of the quarter-car model considering tire-road separation".* In Dai l., Jazar r. (eds) nonlinear approaches in engineering applications: Design engineering problems. Springer, New York, (2021).
[9]. Nguyen, Q. D. *"Tire-road separation. Problem and solving method".* Journal of Water Resources & Environmental Engineering - No. 87, (2023).
[10]. Nguyen, Q. D., Milani, S., Marzbani, H., & Jazar, R. N. *"Vehicle ride analysis considering tire-road separation".* Journal of Sound and Vibration, 521, p.116674, (2022).
[11]. Khazaie, A., Hussaini, N., Marzbani, H. and Jazar, R.N. *"Quarter Car Suspension Model with Provision for Loss of Contact with the Road".* In Nonlinear Approaches in Engineering Applications (pp. 167-208). Springer, (2018).
[12]. N. Jazar, R. *"Advanced vibration: A modern approach".* Springer Science & Business Media, (2003).
[13]. Wong, JY, *"Theory of ground vehicles",* John Wiley & Sons, (2008).
[14]. Sadiku, S., & Leipholz, H. *"On the dynamics of elastic systems with moving concentrated masses".* Ingenieur-archiv, 57 (3), 223–242, (1987).
[15]. Stancioiu, D., Ouyang, H., & Mottershead, J. *"Dynamics of a beam and a moving two-axle system with separation".* Proceedings of the Institution of Mechanical, (2008).
[16]. Jazar, R.N. *"Vehicle vibrations".* In Vehicle Dynamics (pp. 819-881). Springer, (2017).

TÓM TẮT

Khảo sát ảnh hưởng của một số thông số đến hiện tượng tách bánh xe khỏi mặt đường thông qua việc sử dụng mô hình ½ động lực học dao động ô tô

Nội dung bài báo tập trung trình bày kết quả nghiên cứu hiện tượng tách bánh bằng phương pháp mô phỏng số thông qua việc sử dụng mô hình ½ động lực học dao động ô tô. Nội dung chính là xem xét kỹ lưỡng các tình huống trong đó tập trung vào hiện tượng khi bánh xe mất tiếp xúc với mặt đường. Các kết quả tính toán cả trong miền thời gian và miền tần số giúp phân tích chuyển vị thẳng đứng và góc lắc dọc của thân xe trong cả hai trường hợp bánh xe có tiếp xúc và mất tiếp xúc với đường. Đồng thời, cho phép chỉ ra ảnh hưởng của kết cấu của hệ thống treo và điều kiện đường đến hiện tượng tách bánh của từng bánh xe và của đồng thời cả hai bánh xe. Thông qua việc so sánh, đánh giá các kết quả trong cả hai trưởng hợp nêu trên sẽ cung cấp những thông tin có giá trị cho các nhà thiết kế nhằm lựa chọn các thông số tối ưu của hệ thống treo giúp giảm thiểu hiện tượng tách bánh xe khỏi mặt đường.

Từ khóa: Dao động ô tô; Mô hình động lực học 1/2 ; Hiện tượng tách bánh; Mô hình dao động ½; An toàn chuyển động; Độ êm dịu chuyển động.