

Synthesis of controllers based on linear matrix inequalities for a magnetic levitation system

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ABSTRACT

This paper presents the results of synthesizing controllers based on Linear Matrix Inequalities (LMIs) for a magnetic levitation system with a strongly nonlinear mathematical model: integral-state feedback control and sliding mode control. Comparative simulation results and evaluations of the controllers in MATLAB/Simulink software have demonstrated that the controllers perform well and are robust to changes in the mass of the object, with the object's position closely following the setpoint signal. However, in cases where system parameters change significantly, the control system with the integral-state feedback controller may operate inaccurately or even become unstable, whereas the sliding mode controller still ensures stability and robustness against changes in system parameters

Keywords: Magnetic levitation system; Linear Matrix Inequalities (LMIs); Integral-state feedback control; Sliding mode control; Nonlinear mathematical model.

1. INTRODUCTION

Nowadays, magnetic levitation systems have practical importance in various engineering applications such as high-speed maglev trains, frictionless bearings, and wind tunnel models [1, 2]. These systems are inherently unstable in open-loop with strongly nonlinear mathematical models, making stabilization and precise control challenging. Therefore, the synthesis of control systems for magnetic levitation is a very crucial and urgent task.

In [3], the authors used a linear PID controller to control the magnetic levitation system with linearized descriptions. In [4, 5], linearization feedback techniques were employed to design nonlinear control laws for magnetic levitation systems. The adaptive backstepping method was utilized in [6, 7] to control magnetic levitation systems with uncertain parameters and external disturbances. In [8], Lepetic applied the fuzzy predictive control method to the MagLev system. Jalili-Kharaajoo [9] proposed a sliding mode controller (SMC) for the magnetic levitation system. In [10], Hafiz and colleagues proposed three effective adaptive sliding mode controllers for the MagLev system: the adaptive terminal sliding mode controller (AT-SMC), the adaptive backstepping sliding mode controller (ABS-SMC), and the adaptive integral backstepping sliding mode controller (AIBS-SMC) under uncertain system parameters and external disturbances. In [11], the proportional-integral sliding mode controllers (PI-SMC) were developed in both continuous and discrete time domains, providing anti-chattering and superior robustness compared to conventional sliding mode control.

Over the years, the control synthesis method using Linear Matrix Inequalities (LMI) [12] has emerged as a powerful tool to address complex control problems that are difficult or impossible to solve analytically. Linear Matrix Inequalities are widely used and increasingly developed in optimal control, robust control, adaptive control, etc.

In this paper, the authors propose the synthesis of integral-state feedback controllers and sliding mode controllers based on Linear Matrix Inequalities for nonlinear magnetic levitation systems.

2. MATHEMATICAL MODEL OF THE SYSTEM AND SYNTHESIS OF CONTROLLERS BASED ON LINEAR MATRIX INEQUALITIES

2.1. Description of the magnetic levitation system

The diagram of the magnetic levitation system is shown in figure 1 [10], where R and L are the resistance and inductance of the coil, respectively. F_g is the gravitational force, F_e is the electromagnetic force, x is the position of the object relative to the coil, u is the supply voltage, m is the mass of the object, and i is the current.

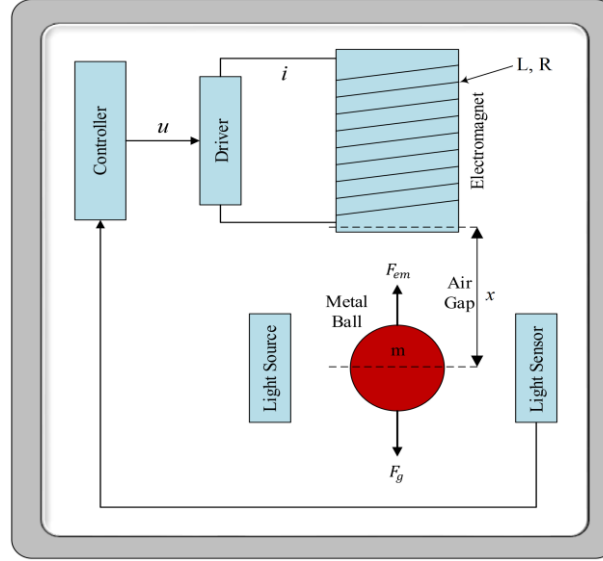


Figure 1. Diagram of the magnetic levitation system.

Let $x_1 = x$, $x_2 = \dot{x}$, $x_3 = i$, $\mathbf{x} = [x_1 \ x_2 \ x_3]^T$, The nonlinear mathematical model of the magnetic levitation system, formulated based on [5], is as follows:

$$\begin{cases} \dot{x}_1 = x_2; \dot{x}_2 = g - \frac{k}{m} \left(\frac{x_3}{x_1} \right)^2; \dot{x}_3 = -\frac{R}{L} x_3 + \frac{2k}{L} \frac{x_2 x_3}{x_1^2} + \frac{1}{L} u, \end{cases} \quad (1)$$

$$y = x_1, \quad (2)$$

where g is the gravitational acceleration, k is the electromagnetic force constant, and y is the output variable of the system. The control objective is to design a controller such that the system is stable and the position of the object (output variable) closely follows the desired signal x_d with the specified quality requirements.

To achieve the above objective, this paper proposes the synthesis of controllers based on Linear Matrix Inequalities. To verify the robustness of the controllers, the paper examines the system's performance when the mass of the object m changes compared to its nominal value (relating to reality, for example, the mass of a maglev train changes when it is fully loaded with passengers compared to when it is empty). Additionally, in (1), it is easy to see that the coefficient in front of the control signal u depends on the inductance L of the coil. Therefore, when using conventional controllers (e.g., PID), if the value of L is not accurately measured, it will affect the control quality, potentially causing system instability. To further verify the robustness of the proposed controllers, the paper also simulates cases where the parameter L deviates from its nominal value.

Based on the characteristics of model (1), the proposed controllers for the magnetic levitation system have the following notable advantages. First, compared to traditional integral-state

feedback controllers, which typically use pole placement methods to calculate state feedback gain parameters for asymptotic system stability, the modern integral-state feedback controllers based on Linear Matrix Inequalities have the advantage of calculating these parameters directly without relying on the system's pole coordinates while still ensuring system stability according to the Lyapunov standard, with exponential convergence of the state variables to the origin. Next, compared to traditional sliding mode controllers, which choose sliding surface parameters to ensure that the characteristic polynomial of the sliding surface is Hurwitz, thus ensuring the control error converges to zero as the sliding function approaches zero, the proposed sliding mode controllers in the paper use Linear Matrix Inequalities to design the sliding surface function. This approach increases the flexibility and robustness of the sliding mode controller, reduces high-frequency oscillations in the control signal, ensures that the sliding function converges to zero within a finite time, and guarantees system stability according to Lyapunov with exponential convergence of the state variables to the origin, even under conditions of system parameter uncertainties or external disturbances.

2.2. Design of integral-state feedback control law

Using the nonlinear coordinate transformation in [5], (1) is converted to canonical form:

$$z_1 = x_1 - x_d; z_2 = x_2 - \dot{x}_d; z_3 = g - \frac{k}{m} \left(\frac{x_3}{x_1} \right)^2 - \ddot{x}_d. \quad (3)$$

Therefore, the dynamic equations in the new coordinates are described as follows:

$$\begin{cases} \dot{z}_1 = z_2; \dot{z}_2 = z_3; \dot{z}_3 = f(\mathbf{x}, \ddot{x}_d) + g(\mathbf{x})u, \end{cases} \quad (4)$$

where $f(\mathbf{x}) = -\frac{4k^2}{mL} \frac{x_2 x_3^2}{x_1^4} + \frac{2k}{m} \frac{x_2 x_3^2}{x_1^3} + \frac{2kR}{mL} \frac{x_3^2}{x_1^2} - \ddot{x}_d$, $g(\mathbf{x}) = -\frac{2k}{mL} \frac{x_3}{x_1^2}$.

Choose $u = \frac{v - f(\mathbf{x}, \ddot{x}_d)}{g(\mathbf{x})}$, where v is the auxiliary control law designed to achieve the control

objectives mentioned above. To design the integral-state feedback control law with the aim of eliminating steady-state error and the influence of disturbances in the system model, we introduce an auxiliary state variable z_4 such that $\dot{z}_4 = z_1$. Therefore, we obtain a new state equation for the controlled object, described in the following matrix equation:

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}v, \quad (5)$$

where $\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

We need to design the auxiliary control law v such that $\mathbf{z} \rightarrow 0$ when $t \rightarrow \infty$ to achieve the initial control objective $x \rightarrow x_d$. The integral-state feedback control law is as follows:

$$v_{ISFC} = \mathbf{k}_1 \mathbf{z}. \quad (6)$$

In equation (6), $\mathbf{k}_1 \in \mathcal{R}^{1 \times 4}$ is the state feedback gain, which can be determined using Linear Matrix Inequalities. From (5) and (6), we obtain

$$\dot{\mathbf{z}} = (\mathbf{A} + \mathbf{B}\mathbf{k}_1) \mathbf{z}. \quad (7)$$

Consider the Lyapunov function of the form: $V = \mathbf{z}^T \mathbf{P}\mathbf{z}$, (8)

where $\mathbf{P} \in \mathfrak{R}^{4 \times 4}$ is a positive definite symmetric matrix. Using (7) with $\alpha > 0$:

$$\begin{aligned} \alpha V + \dot{V} &= \alpha \mathbf{z}^T \mathbf{P} \mathbf{z} + \dot{\mathbf{z}}^T \mathbf{P} \mathbf{z} + \mathbf{z}^T \mathbf{P} \dot{\mathbf{z}} \\ &= \mathbf{z}^T \left(\alpha \mathbf{P} + \mathbf{A}^T \mathbf{P} + \mathbf{k}_1^T \mathbf{B}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{P} \mathbf{B} \mathbf{k}_1 \right) \mathbf{z}. \end{aligned} \quad (9)$$

From equation (9), to ensure $\alpha V + \dot{V} \leq 0$ it is necessary to

$$\alpha \mathbf{P} + \mathbf{A}^T \mathbf{P} + \mathbf{k}_1^T \mathbf{B}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{P} \mathbf{B} \mathbf{k}_1 < 0. \quad (10)$$

Since \mathbf{P} and \mathbf{k}_1 are unknown, (10) is nonlinear. Therefore, we need to linearize (10). By multiplying both sides of (10) by matrix $\mathbf{M} = \mathbf{P}^{-1}$, we obtain:

$$\alpha \mathbf{M} + \mathbf{A} \mathbf{M} + \mathbf{M} \mathbf{A}^T + \mathbf{B} \mathbf{k}_1 \mathbf{M} + \mathbf{M} \mathbf{k}_1^T \mathbf{B}^T < 0. \quad (11)$$

Let \mathbf{Y} be the matrix satisfying $\mathbf{Y} = \mathbf{k}_1 \mathbf{M}$. We obtain the inequality in terms of \mathbf{Y} and \mathbf{M} :

$$\alpha \mathbf{M} + \mathbf{A} \mathbf{M} + \mathbf{M} \mathbf{A}^T + \mathbf{B} \mathbf{Y} + \mathbf{Y}^T \mathbf{B}^T < 0, \quad (12)$$

we can solve for the solutions \mathbf{Y} and \mathbf{M} of (11) and find \mathbf{k}_1 using the following formula:

$$\mathbf{k}_1 = \mathbf{Y} \mathbf{M}^{-1}. \quad (13)$$

Since $\alpha V + \dot{V} \leq 0$, it follows that $V(t) \leq V(0) e^{-\alpha t}$. When $t \rightarrow \infty$, then $V(t) \rightarrow 0$, meaning $\mathbf{z} \rightarrow 0$, which satisfies the control objective. The convergence rate of \mathbf{z} depends on the choice of $\alpha > 0$.

2.3. Design of Sliding Mode Control Law (SMC)

Assuming there is parameter uncertainty in the state model of the magnetic levitation system, we can rewrite the system (5) in the following form: $\dot{\mathbf{z}} = \mathbf{A} \mathbf{z} + \mathbf{B}(v + \Delta)$, (14)

where $\Delta = \Delta(\mathbf{x}, u, t)$ – the uncertain component is bounded in the mathematical model of the magnetic levitation system, caused by parameter uncertainties (e.g., changes in the mass m , or inaccurate measurement of inductance L); $|\Delta(\mathbf{x}, u, t)| \leq D$.

We need to design the sliding mode control law v such that $\mathbf{z} \rightarrow 0$ when $t \rightarrow \infty$ to achieve the initial control objective $x \rightarrow x_d$. Assume the sliding surface s has the form:

$$s = \mathbf{B}^T \mathbf{G} \mathbf{z}, \quad (15)$$

where $\mathbf{G} \in \mathfrak{R}^{4 \times 4}$ is an unknown positive definite symmetric matrix that needs to be designed using Linear Matrix Inequalities. The sliding mode control law v is designed as follows:

$$v_{SMC} = -(\mathbf{B}^T \mathbf{G} \mathbf{B})^{-1} \left[\mathbf{B}^T \mathbf{G} \mathbf{A} \mathbf{z} + (|\mathbf{B}^T \mathbf{G} \mathbf{B}| \delta + \varepsilon) \text{sign}(s) + ks \right]. \quad (16)$$

In (16), with the parameters $\delta \geq D$, $\varepsilon > 0$, $k > 0$, consider the Lyapunov function:

$$V_1 = s^2. \quad (17)$$

The time derivative of the Lyapunov function (17) is described as follows:

$$\begin{aligned} \dot{V}_1 &= 2s\dot{s} = 2s\mathbf{B}^T \mathbf{G} \dot{\mathbf{z}} = 2s\mathbf{B}^T \mathbf{G} [\mathbf{A} \mathbf{z} + \mathbf{B}(v_{SMC} + \Delta)] \\ &= 2 \left(s\mathbf{B}^T \mathbf{G} \mathbf{B} \Delta - |s| |\mathbf{B}^T \mathbf{G} \mathbf{B}| \delta \right) - 2\varepsilon |s| - 2ks^2, \\ \dot{V}_1 &\leq -2\varepsilon |s| - 2ks^2 = -2\varepsilon (V_1)^{1/2} - 2kV_1. \end{aligned} \quad (18)$$

Since $V_1(t) \rightarrow 0$ is reached in finite time when $t_0 = \frac{1}{k} \ln \left(\frac{k\sqrt{V_1(0)}}{\varepsilon} + 1 \right)$, thus for $\forall t > t_0$ we

have $s=0$, or $\mathbf{B}^T \mathbf{Gz} = \mathbf{z}^T \mathbf{GB} = 0$. Consider the following Lyapunov function:

$$V_2 = \mathbf{z}^T \mathbf{Gz}. \quad (19)$$

Transform the control function (16) into the following form:

$$v_{SMC} = \mathbf{k}_2 \mathbf{z} + \gamma(t), \quad (20)$$

where $\gamma(t) = -\mathbf{k}_2 \mathbf{z} - (\mathbf{B}^T \mathbf{GB})^{-1} [\mathbf{B}^T \mathbf{GAz} + (|\mathbf{B}^T \mathbf{GB}| \delta + \varepsilon) \text{sign}(s) + ks]$.

Then (14) is transformed into the following form:

$$\dot{\mathbf{z}} = (\mathbf{A} + \mathbf{Bk}_2) \mathbf{z} + \mathbf{B}(\gamma + \Delta). \quad (21)$$

Using (21), we find the time derivative of the Lyapunov function (19):

$$\begin{aligned} \dot{V}_2 &= \dot{\mathbf{z}}^T \mathbf{Gz} + \mathbf{z}^T \mathbf{G}\dot{\mathbf{z}} \\ &= \mathbf{z}^T (\mathbf{A}^T \mathbf{G} + \mathbf{k}_2^T \mathbf{B}^T \mathbf{G} + \mathbf{GA} + \mathbf{GBk}_2) \mathbf{z} + (\gamma + \Delta) (\mathbf{B}^T \mathbf{Gz} + \mathbf{z}^T \mathbf{GB}) \\ &= \mathbf{z}^T (\mathbf{A}^T \mathbf{G} + \mathbf{k}_2^T \mathbf{B}^T \mathbf{G} + \mathbf{GA} + \mathbf{GBk}_2) \mathbf{z}. \end{aligned} \quad (22)$$

From (19) and (22), we have the following matrix equation:

$$\alpha V_2 + \dot{V}_2 = \mathbf{z}^T (\alpha \mathbf{G} + \mathbf{A}^T \mathbf{G} + \mathbf{k}_2^T \mathbf{B}^T \mathbf{G} + \mathbf{GA} + \mathbf{GBk}_2) \mathbf{z}. \quad (23)$$

From equation (23), to ensure $\alpha V_2 + \dot{V}_2 \leq 0$, it is necessary to

$$\alpha \mathbf{G} + \mathbf{A}^T \mathbf{G} + \mathbf{k}_2^T \mathbf{B}^T \mathbf{G} + \mathbf{GA} + \mathbf{GBk}_2 < 0. \quad (24)$$

Multiplying both sides of (24) by matrix $\mathbf{N} = \mathbf{G}^{-1}$, we obtain:

$$\alpha \mathbf{N} + \mathbf{AN} + \mathbf{NA}^T + \mathbf{Bk}_2 \mathbf{N} + \mathbf{Nk}_2^T \mathbf{B}^T < 0. \quad (25)$$

Let $\mathbf{H} = \mathbf{k}_2 \mathbf{N}$. We obtain the matrix inequality in terms of \mathbf{H} and \mathbf{N} :

$$\alpha \mathbf{N} + \mathbf{AN} + \mathbf{NA}^T + \mathbf{BH} + \mathbf{H}^T \mathbf{B}^T < 0. \quad (26)$$

From (26), we can solve for \mathbf{H} and \mathbf{N} and find \mathbf{k}_2 and \mathbf{G} using the following formula:

$$\mathbf{k}_2 = \mathbf{HN}^{-1}; \mathbf{G} = \mathbf{N}^{-1}. \quad (27)$$

Since $\alpha V_2 + \dot{V}_2 \leq 0, \forall t > t_0$, it follows that $V_2(t) \leq V_2(t_0) e^{-\alpha(t-t_0)}$. Therefore, when $t \rightarrow \infty$, meaning $\mathbf{z} \rightarrow 0$, which satisfies the control objective.

3. SIMULATION RESULTS AND ANALYSIS

The simulation to verify the control system with the synthesized controllers is conducted using MATLAB/Simulink with the nominal parameters of the system as follows [5]: $m = 0.025 \text{ kg}; g = 9.81 \text{ m/s}^2; k = 2.395 \text{ Nm}^2/\text{A}^2; R = 4.2 \Omega; L = 0.02 \text{ H}$. The parameters of the controllers are designed using the LMI Control Toolbox in MATLAB:

$$\mathbf{G} = \begin{bmatrix} 7.1995 & 0.4814 & 0.0108 & 36.1553 \\ 0.4814 & 0.0326 & 0.0007 & 2.3997 \\ 0.0108 & 0.0007 & 0.0000 & 0.0535 \\ 36.1553 & 2.3997 & 0.0535 & 182.9664 \end{bmatrix}; \left(\begin{array}{l} \mathbf{k}_1 = 10^4 \cdot [-1.2152 \quad -0.1131 \quad -0.0043 \quad -4.7006]; \\ \mathbf{k}_2 = 10^5 \cdot [-1.0142 \quad -0.0701 \quad -0.0017 \quad -5.0190]; \\ \alpha = 15; \delta = \varepsilon = k = 50. \end{array} \right)$$

The simulation results of position tracking control are presented in figures 2-5, where "Desired" represents the desired signal, "ISFC" represents the integral-state feedback control, and "SMC"

represents the sliding mode control.

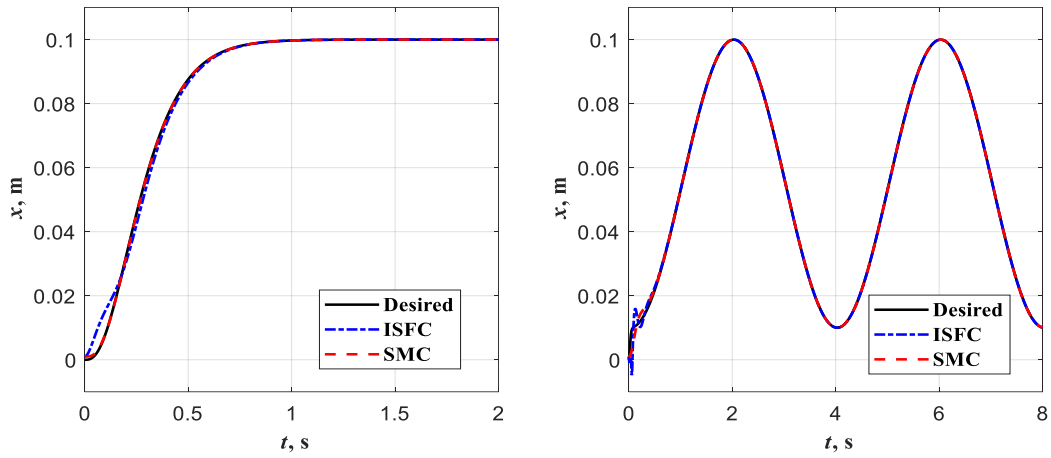


Figure 2. Position response of the object with constant and sinusoidal desired signals.

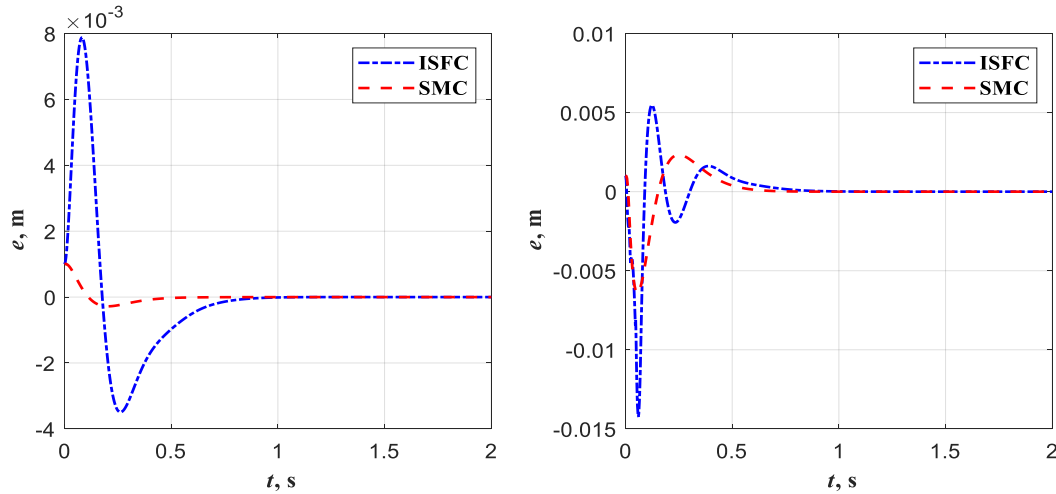


Figure 3. Position error of the object with constant and sinusoidal desired signals.

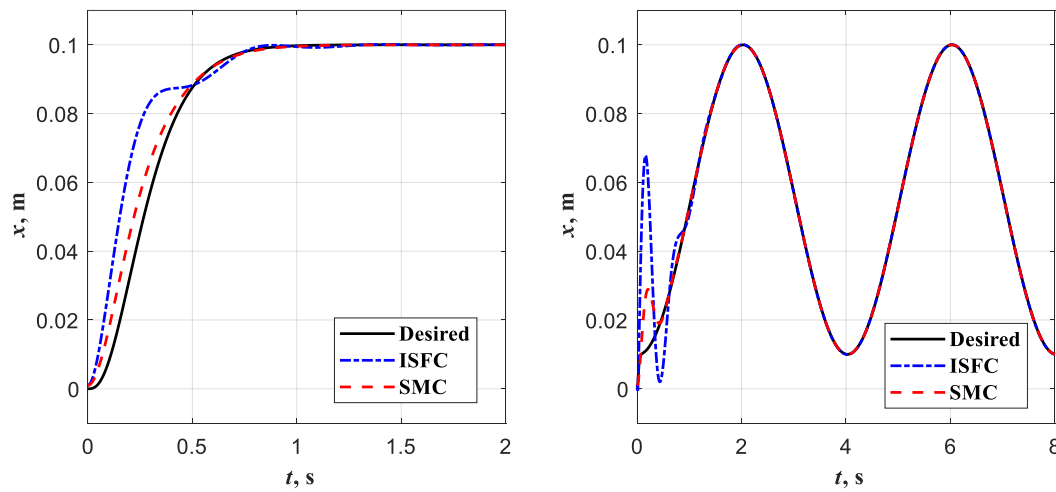


Figure 4. Position response of the object when $m' = 2m$.

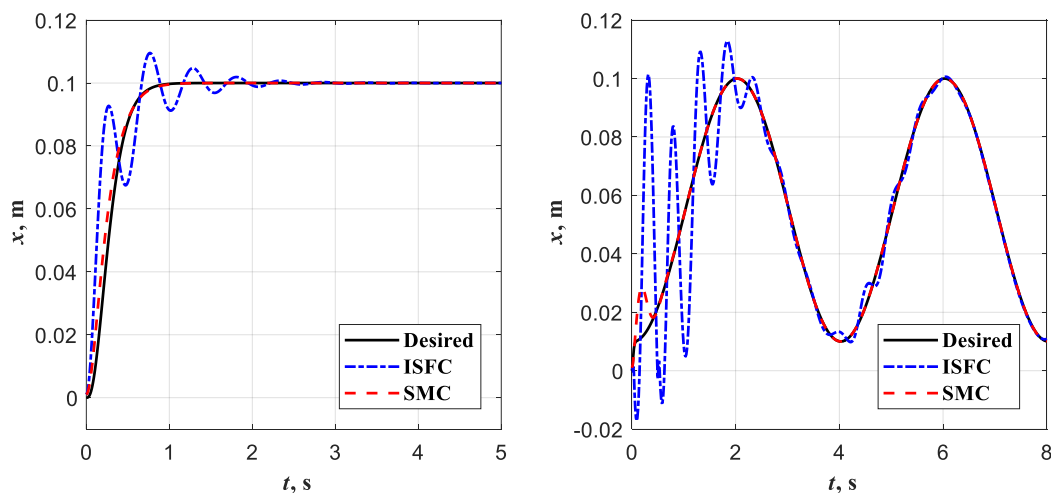


Figure 5. Position response of the object when $m' = 2m$, $L' = 1.5L$.

Observations:

- From the comparative simulation results, we see that both proposed controllers perform well at the nominal values of the system parameters, with the sliding mode controller exhibiting higher control quality compared to the integral-state feedback controller.

- When the mass of the object is doubled, both proposed controllers still perform well, but the sliding mode controller maintains higher control quality compared to the integral-state feedback controller.

- When the system parameters significantly deviate from the nominal values (e.g., the object's mass is doubled, inaccurate measurement of the coil's inductance), the integral-state feedback controller exhibits poorer control quality and may lead to system instability. In contrast, the sliding mode controller continues to perform very well, ensuring stability and robustness despite changes in system parameters.

4. CONCLUSIONS

Based on Linear Matrix Inequalities, the authors have proposed the synthesis of integral-state feedback controllers and sliding mode controllers for magnetic levitation systems. The stability of the controllers has been analyzed and demonstrated using Lyapunov stability theory. Comparative simulation results in MATLAB/Simulink have visually shown the performance of the controllers. For the integral-state feedback controller, the control system performs well at the nominal values of the system parameters. Additionally, the simulation results have proven the effectiveness of the sliding mode controller, showing robustness to system parameter changes, maintaining good dynamic and static characteristics, and ensuring stability and accurate position tracking.

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TÓM TẮT

Tổng hợp các bộ điều khiển trên cơ sở bất đẳng thức ma trận tuyến tính cho hệ thống nâng vật trong từ trường

Bài báo này trình bày kết quả tổng hợp các bộ điều khiển trên cơ sở bất đẳng thức ma trận tuyến tính cho hệ thống nâng vật trong từ trường có mô hình toán học phi tuyến mạnh: điều khiển tích phân-phản hồi trạng thái và điều khiển trượt. Các kết quả mô phỏng so sánh và đánh giá các bộ điều khiển trên phần mềm MATLAB/Simulink đã cho thấy các bộ điều khiển hoạt động tốt và bền vững với sự thay đổi của khối lượng vật, vị trí của vật bám sát theo tín hiệu đặt. Tuy nhiên, trong các trường hợp các tham số của hệ thống thay đổi mạnh, hệ thống điều khiển với bộ điều khiển tích phân-phản hồi trạng thái có thể hoạt động không chính xác, thậm chí mất ổn định, còn bộ điều khiển trượt vẫn đảm bảo tính ổn định và bền vững với sự thay đổi của tham số hệ thống.

Từ khóa: Hệ thống nâng vật trong từ trường; Bất đẳng thức ma trận tuyến tính; Điều khiển tích phân-phản hồi trạng thái; Điều khiển trượt.