Study the variation of the pitching frequency of sounding rockets

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ABSTRACT

Sounding rockets typically feature an axially symmetric design and are launched vertically to facilitate research and high-altitude atmospheric data collection. Manufacturing errors can cause axial asymmetry, leading to undesirable rocket trajectory dispersion. Sounding rockets are often designed to spin around their axis to mitigate these effects. However, axial spinning motion can resonate with short-period oscillations, creating large normal loads that may damage the rocket's structures. This paper focuses on analyzing the variations in the pitching frequency, which may help predict the roll resonance phenomenon. In this study, the authors constructed a six-degree-of-freedom dynamic model for a sounding rocket, considering all aerodynamic problems and the variation of inertial characteristics. To determine the pitching frequency, an impulse is applied to the rocket to generate short-period oscillation. The Fourier transform is then used to analyze and obtain the frequency of the rocket while oscillating in space. The results demonstrate agreement with the theoretical model, thereby substantiating the validity of the current method. The findings of this research provide valuable recommendations for the design and manufacturing process of sounding rockets, which may help mitigate the adverse effects of motion resonance during flight.

Keywords: Sounding rocket; Resonance; Short-period oscillations; Fourier transform.

1. INTRODUCTION

Sounding rockets are a type of small-scale rockets, typically operating at altitudes ranging from 40 to 200 km, though they can sometimes reach altitudes up to 2000 km [1]. Their primary purpose is to carry scientific research equipment for space exploration, atmospheric studies, Earth observation, and related fields [2]. Sounding rockets mainly function in the altitude range that lies between the observational limits of balloons and satellites. Another characteristic of sounding rockets is their high speed and load factor, especially those using solid fuel engines, which can speed up rockets to three to four times the speed of sound and a load factor over 30G [3]. These features make sounding rockets complex objects for design, manufacturing, and research.

Numerous studies have addressed issues related to the design and manufacturing of sounding rockets, covering topics such as aerodynamics [4, 5], structure [6], dynamics [7-9], propulsion, and more. Among these, the dynamics of sounding rocket flight is particularly crucial, directly affecting trajectory stability and dispersion. While studying sounding rocket flight dynamics, researchers have observed a complex phenomenon, which is the resonance between the rocket's roll motion and its pitching oscillation [10]. This phenomenon may potentially lead to a high load factor and even structural failures. This issue has been discussed in several studies, such as [10, 11], in which researchers have shown that frequency locking during resonance could cause launch safety hazards and structural destruction.

To investigate this resonance phenomenon, it is necessary to accurately determine the pitching frequency of sounding rockets. The sounding rockets' inertia characteristics, including their mass, moment of inertia, and center of gravity, may change considerably during flight. Additionally, they operate within a wide range of speeds and altitudes, affecting their aerodynamic properties. These factors result in continuous changes in the dynamic characteristics in general and the pitching frequency in particular. Therefore, determining the variation of the pitching frequency of a sounding rocket is relatively complex. There has not been any research in Vietnam addressing this issue.

This paper utilizes a theoretical model and a six degrees of freedom simulation model to determine the variation in the pitching frequency. The calculated results from both models are compared for verification. The sounding rocket model used in this paper is a simplified version of a rocket model that has been developed in Vietnam.

2. METHODOLOGY

2.1. Theoretical model to determine pitching frequency

The theoretical model used to investigate the dynamic characteristics of sounding rockets is based on [12]. This model relies on the linear equations of motion used to study the short-period longitudinal dynamics of aircraft. The undamped natural pitching frequency ω_n and the damping ratio ζ are determined as follows [12]:

$$\begin{cases} \omega_n = \left(M_q \dfrac{N_\alpha}{V} - M_\alpha \right)^{1/2} \\ \zeta = -\left(M_q + M_{\dot{\alpha}} + \dfrac{N_\alpha}{V} \right) \Big/ \left(2\omega_n \right) \end{cases} \tag{1}$$

Here, q - Dynamic pressure, α - The angle of attack, and M_α, $M_{\dot{\alpha}}$, M_q and N_α are defined by the following formulas [12]:

$$\begin{cases} M_\alpha = C_{m_\alpha} \dfrac{qSl}{I_y}; \; M_{\dot{\alpha}} = C_{m_{\dot{\alpha}}} \dfrac{qSl}{I_y} \\ M_q = C_{m_q} \dfrac{qSl}{I_y}; \; N_\alpha = C_{N_\alpha} \dfrac{qS}{m} \end{cases}$$

Here S and l - Reference area and length, m and I_y – Mass and moment of inertia about the y-axis, C_{m_α}, $C_{m_{\dot{\alpha}}}$, and C_{m_q} - The derivatives of the coefficients of pitch moment with respect to α, $\dot{\alpha}$ and q, and C_{N_α} - The derivative of the coefficient of normal force with respect to α.

Next, the actual oscillation frequency ω and the resonance frequency ω_r are determined using the following formulas [12]:

$$\begin{cases} \omega = \omega_n \sqrt{1-\zeta^2} \\ \omega_r = \omega_n \sqrt{1-2\zeta^2} \end{cases} \tag{2}$$

It should be noted that the above equations were developed for aircraft; however, they are possibly applied to rocket dynamics [13].

2.2. Equation of motion and the flight dynamics simulation program

The equations of motion for a 6-degree-of-freedom (6-DOF) model, accounting for variations in the inertial properties of the rocket, can be expressed as follows [14]:

$$\boldsymbol{F}^b + \boldsymbol{F}_T^b + \Delta m \ddot{\boldsymbol{r}}_O^b + \ddot{m}\boldsymbol{r}_{Oe}^b + 2\dot{m}\boldsymbol{\omega}^b \times \boldsymbol{r}_{Oe}^b = m_0 \ddot{\boldsymbol{r}}_{c_0}^b \tag{3}$$

$$\boldsymbol{M}_O^b + \boldsymbol{M}_T^b + \Delta \boldsymbol{I}^b \frac{d\boldsymbol{\omega}^b}{dt} + \boldsymbol{\omega}^b \times \left(\Delta \boldsymbol{I}^b \boldsymbol{\omega}^b \right) = -m_0 \ddot{\boldsymbol{r}}_O^b \times \boldsymbol{r}_{Oc_0}^b + \boldsymbol{I}_0^b \frac{d\boldsymbol{\omega}^b}{dt} + \boldsymbol{\omega}^b \times \left(\boldsymbol{I}_0^b \boldsymbol{\omega}^b \right) \tag{4}$$

where superscript b refers to the body-fixed coordinate system, m_0 and $\boldsymbol{I}_0^b$ are the initial mass and moment of inertia; Δm and $\Delta \boldsymbol{I}^b$ are the mass and moment of inertia lost; $\boldsymbol{F}_T^b$ và $\boldsymbol{M}_T^b$ are the forces and moments generated by the engine thrust; and $\boldsymbol{F}^b$ and $\boldsymbol{M}_O^b$ are external forces and moments; $\boldsymbol{\omega}^b$ is the angular velocity vector; $\boldsymbol{r}$ is the spatial coordinate vector; the subscript O refers to the origin of the coordinate system, which is set to be at the mass center of the propellant grain; c_0 is the initial center of mass position of the rocket.

The system of equations is solved numerically using MSC Adams software, where the aerodynamic external force components are determined using a panel method. To obtain the oscillation frequency of the rocket through the simulation method, an impulse with a short duration of 0.2 seconds is applied in the normal direction. Under the influence of this impulse, the rocket oscillates, and the oscillation frequency can be identified by observing the changes in the angle of attack and sideslip angle over time. To accurately determine the pitching frequency of the rocket, the Fourier transform is employed to derive the spectral distribution function of the oscillation around the time the impulse is applied.

2.3. Panel method for determining aerodynamic coefficients

A panel method is used to determine the aerodynamic coefficients acting on the rocket during flight. In this study, we employed the panel method based on Woodward's theory with some improvements to enhance the accuracy of aerodynamic calculations for sounding rockets. According to this method, the rocket's surface is divided into panels on which sources, vortices, and dipoles can be placed. The non-penetration boundary condition is applied at the control points of the panels. The pressure coefficient on each panel is determined using Bernoulli's equation, and the aerodynamic coefficients of the rocket are calculated by integrating over the entire surface. Detailed information about the panel method can be found in the reference [15].

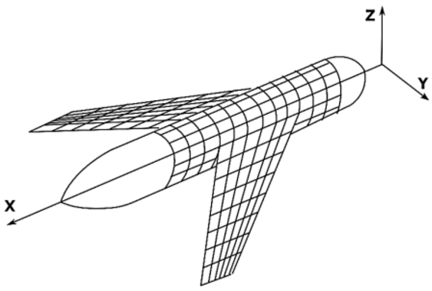

Figure 1. *Illustration of aircraft meshing using the panel method.*

To account for the effects of viscous drag and base drag, semi-empirical formulas are used [16] as follows:

$$C_{D_f} = 0.053 \times \frac{l}{d}\left(\frac{14.6M}{(ql)^{0.2}} + \frac{n \times 0.023}{(qc_{mac})^{0.2}} \times 2 \times \frac{S_f}{S_{ref}}\right) \tag{5}$$

$$\begin{cases} C_{D_b} = \left(0.12 + 0.13M^2\right) & if\ M \le 1 \\ C_{D_b} = \dfrac{0.25}{M} & if\ M > 1 \end{cases} \tag{6}$$

where C_{D_f} and C_{D_b} denote the skin-friction and the base drag coefficients, respectively; q is the dynamic pressure; l and d are the lengths and the diameter of the body; c_{mac} is the mean aerodynamic chord of the fins; n is the number of fins; S_f is the fin area; and S_{ref} is the reference area, which is set to be the cross-sectional area of the body.

Additionally, sounding rockets may suffer from strong oscillation during flight, it is crucial to investigate their unsteady aerodynamic components. In this paper, the authors used Munk's theory

[17] and the added mass effect [18] to determine these force components. The aerodynamic coefficients related to these components are related to the time derivatives of the angle of attack as follows:

$$\begin{cases} C_{N_{\dot{\alpha}}}^{body} = \dfrac{4}{S_{ref} l_{ref}} \displaystyle\int_0^l S(x)\,dx \\ C_{m_{\dot{\alpha}}}^{body} = \dfrac{4}{S_{ref} l_{ref}} \displaystyle\int_0^l S(x)(x - x_0)\,dx \end{cases} \tag{7}$$

$$\begin{cases} C_{N_{\dot{\alpha}}}^{fin} = \dfrac{n\pi c_{mac}{}^2 b}{S_{ref} l_{ref}} \\ C_{m_{\dot{\alpha}}}^{fin} = \dfrac{n\pi c_{mac}{}^2 b}{S_{ref} l_{ref}{}^2}\left(x_f - x_0\right) \end{cases} \tag{8}$$

where l_{ref} is the reference length; $S(x)$ is the function of the cross-sectional area of the body with respect to the *x* coordinate; x_f and x_0 are the positions of the fins and the reference point for the pitch moment calculation, respectively; *b* is the fin length; and *n* is the number of fins included in the calculation. The superscripts body and fin, respectively, denote the contributions of the body and the fins. The unsteady aerodynamic force and moment related to the body are calculated based on Munk's theory, while those of the fins are from the added mass effect. The subscripts *m* and *N* denote the normal force and pitch moment.

3. RESULTS AND DISCUSSION

3.1. Verification of aerodynamics

The method was initially applied to a simple rocket model, as depicted in Fig. 2.

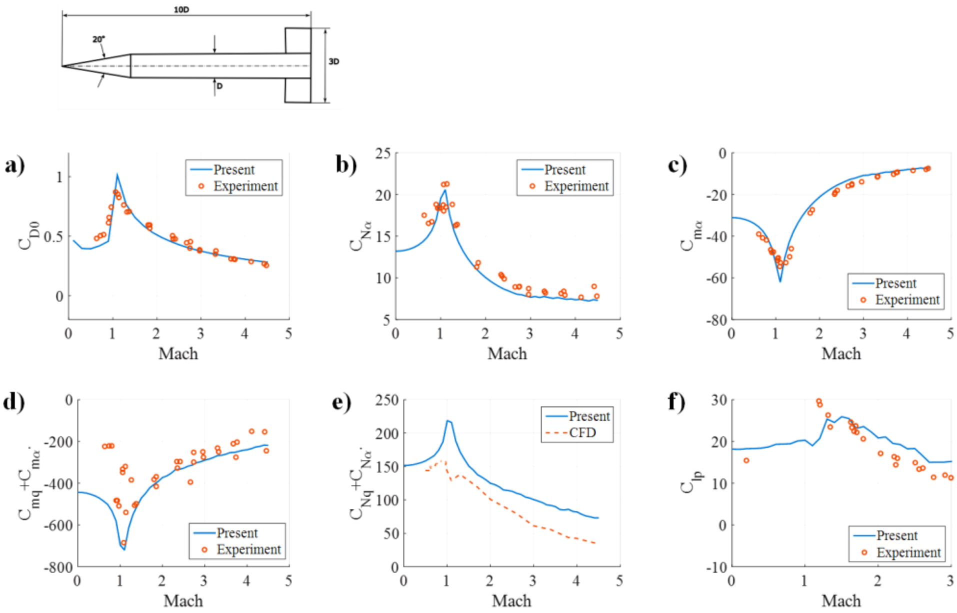

***Figure 2**. Validation of the present method for the simple rocket model:*
a) Frontal drag coefficient; b) and c) derivatives of the pitch moment and normal force coefficients with respect to the angle of attack; d) and e) dynamic derivatives of the pitch moment and normal force coefficients; f) Roll damping coefficient.

The resulting aerodynamic coefficients were then compared with wind-tunnel experimental data and CFD simulations [19-21]. Here, C_{N_α} and C_{m_α} are the derivatives of the normal force and pitch moment coefficients with respect to the angle of attack while C_{N_q} and C_{m_q} denote those with respect to the pitch rate. C_{l_p} is the roll damping coefficient. The reference area and the reference length are respectively equal to the cross-sectional area and the diameter of the rocket body.

3.2. Calculation results for sounding rocket dynamics

The aerodynamic calculation program was then applied to a sounding rocket model with parameters shown in Fig. 3 and table 1.

Figure 3. *Sounding rocket and its dimensions.*

Table 1. *Parameters of the sounding rocket.*

Mass	27.3 kg	Engine thrust	5.925 N
Body diameter	0.12 m	Launch angle	83°
Length	2.018 m	Propellant burn time	4.25 s
Propellant mass	16.7 kg		

The aerodynamic coefficients of the sounding rocket model were calculated using the panel method described in section 2.3, and are shown in Fig. 4. Here, the aerodynamic coefficients are calculated with a Mach number varying from 0 to 4.0. It can be observed that the Mach number significantly affects the aerodynamic coefficients, especially around a value of 1.0.

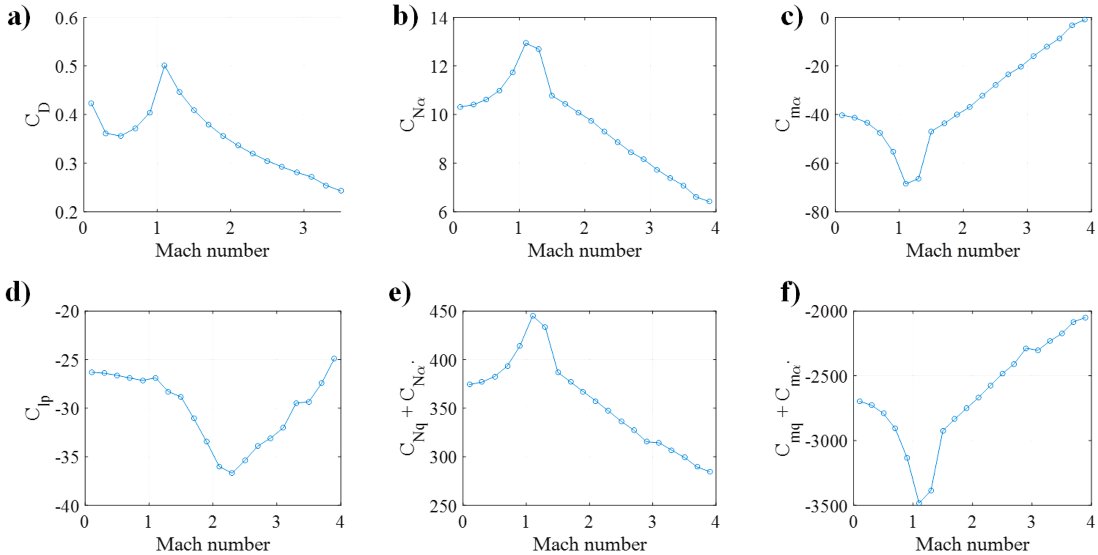

Figure 4. *The aerodynamic coefficients of the sounding rocket:*
a) Frontal drag coefficient; b) and c) derivatives of the normal force and pitch moment coefficients with respect to the angle of attack; d) roll damping coefficient; e) and f) dynamic derivatives of the normal force and pitch moment coefficients.

The aerodynamic coefficients presented in Fig. 4 are utilized as input data to determine the external forces acting on the rocket during flight. Alongside the six-degree-of-freedom equations (Eqs. 3 and 4), the point-mass trajectory equations [22] are also employed to compute the rocket's flight path. Figure 5 provides a comparison of the trajectory parameters derived from the theoretical model (the point-mass model), simulation model (the six-degree-of-freedom model), and data obtained from an IMU installed on board. The closely aligned trajectory data from the computational models and the launch data demonstrate the reliability of the research method employed in this study.

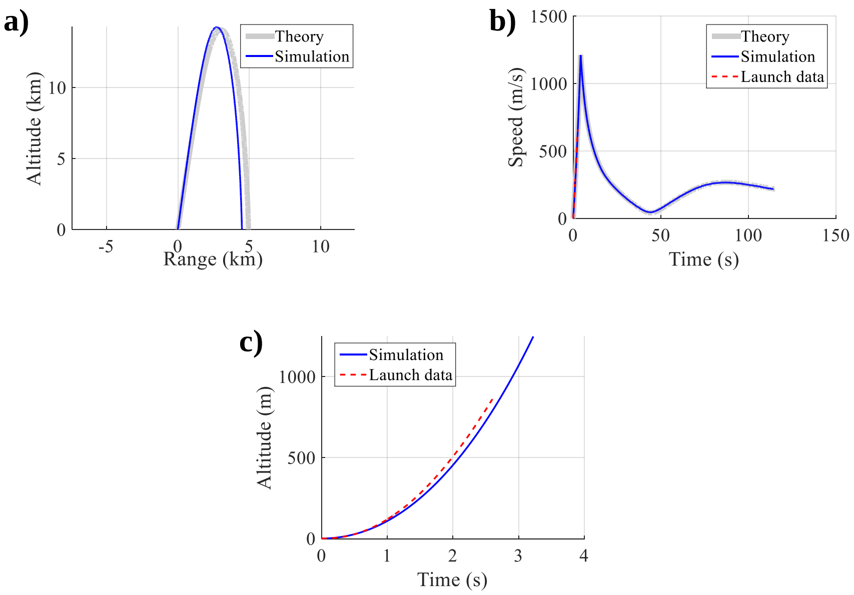

Figure 5. *Flight trajectory parameters of the rocket:*
a) Altitude vs range; b) Speed vs flight time; c) Altitude vs flight time.

Based on the theoretical trajectory shown in figure 5, the frequencies of the rocket are computed using Eqs. (1) and (2). The flow chart of the calculation process and the result are shown in figure 6. The results include the undamped natural frequency ω_n, the resonance frequency ω_r, and the actual pitching frequency ω, which are almost the same. To elucidate this, figure 7 presents the variation of the damping ratio ζ and the ratio values of the actual frequency and the resonance frequency to the undamped natural frequency. The computation results reveal minimal differences between the frequencies, with the largest deviation of approximately 15% occurring just after 4 seconds. Throughout the remaining flight duration, differences are consistently below 3%. Due to the similarity between these three quantities, it is necessary to determine only one of them. For practical applications, the actual frequency ω can be easily obtained from flight data. This frequency is important to characterize the rocket's dynamic behavior during flight and predict roll resonance.

To determine the actual frequency of the sounding rocket, the oscillation of the side-slip angle after applying a normal impulse force was analyzed. In this study, the side-slip angle was selected for this analysis since it is not affected by other motions, such as the oscillation of the rocket after leaving the launch rail. The frequency corresponds to the location of the peak of the Fourier transform amplitude, as shown in Fig. 8.

Figure 9 presents the values of the rocket's pitching frequency estimated by the theoretical and simulation models. The figure shows that there is consistency between the theoretical and simulation results. This consistency can confirm the reliability of both models in determining the oscillation frequencies of sounding rockets. While comparing with studies by Crabill [13] for a shot put vehicle and by Busse and Kraft [23] sounding rocket Aerobee 150, a similarity in the variation trend of the frequency can be observed.

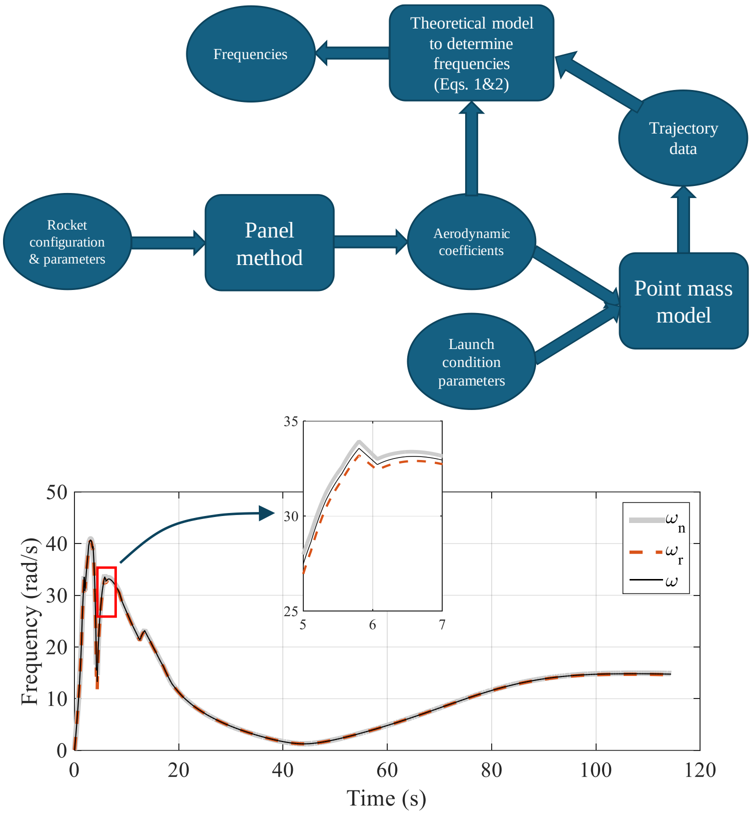

Figure 6. *The flow chart of the calculation process and the frequencies of the sounding rocket during flight.*

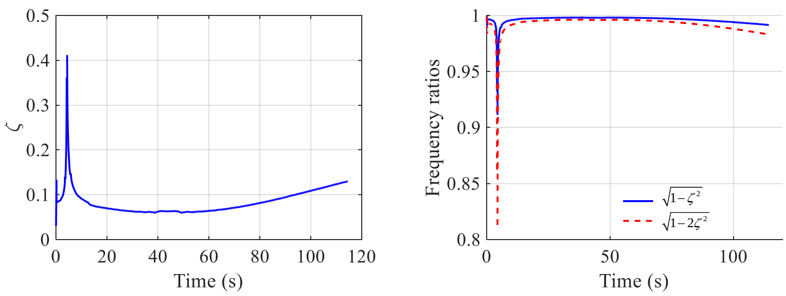

Figure 7. *Variation of damping and frequency ratios.*

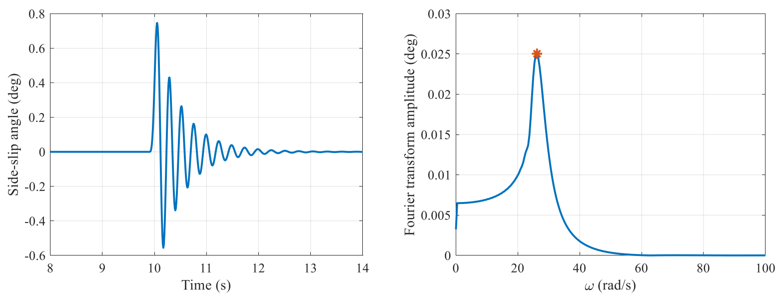

Figure 8. *The side-slip angle oscillation and its Fourier transform.*

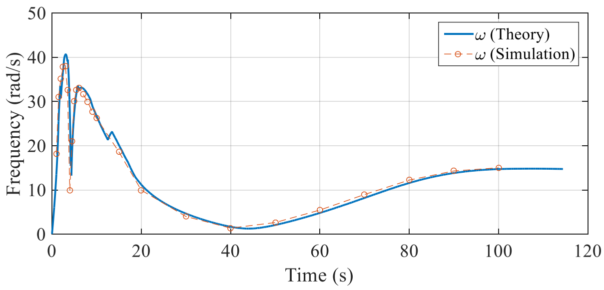

Figure 9. *The pitching frequencies of the sounding rocket during flight are obtained by different models.*

4. CONCLUSIONS

This paper has presented methods and models for determining the pitching frequencies of sounding rockets. The computed aerodynamic and flight dynamic parameters were compared and validated to ensure their reliability. During the flight, the rocket exhibits pitch/yaw oscillations with frequencies close to the undamped natural and resonance frequencies. The results have shown significant variation in the pitching frequency of the sounding rocket during flight. Therefore, such investigations are essential for predicting the roll resonance phenomenon of sounding rockets, which is important in the design stage.

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TÓM TẮT

Nghiên cứu sự thay đổi tần số dao động chúc ngóc của tên lửa thăm dò

Tên lửa thăm dò thường có thiết kế đối xứng trục và được phóng thẳng đứng phục vụ nghiên cứu, thu thập dữ liệu khí quyển tầng cao. Các sai số trong quá trình chế tạo gây ra sự bất đối xứng khiến quỹ đạo tên lửa bị tản mát không mong muốn. Để khắc phục vấn đề này, tên lửa thăm dò thường được thiết kế quay quanh trục nhằm trung bình hóa các sai số do chế tạo gây ra. Tuy nhiên, chuyển động quay quanh trục có khả năng cộng hưởng với dao động chúc ngóc chu kỳ ngắn tạo ra các quá tải cạnh lớn gây phá hủy kết cấu tên lửa. Bài báo tập trung vào việc phân tích sự thay đổi của tần số dao động chúc ngóc nhằm đưa ra dự đoán hiện tượng cộng hưởng đối với tên lửa thăm dò. Trong nghiên cứu này, các tác giả đã xây dựng mô hình động lực học 6 bậc tự do cho tên lửa thăm dò tính đến đầy đủ các vấn đề khí động lực học, sự thay đổi các đặc tính quán tính khi bay. Để xác định tần số chúc ngóc xung lực được tạo ra và tác động lên tên lửa gây ra dao động chu kỳ ngắn. Phép biến đổi Fourier được sử dụng để phân tích và xác định tần số dao động của tên lửa. Kết quả cho thấy sự tương đồng với mô hinh lý thuyết, qua đó độ tin cậy của phương pháp được khẳng định. Kết quả của nghiên cứu này giúp đưa ra những khuyến cáo trong quá trình thiết kế, chế tạo tên lửa thăm dò nhằm mục đích hạn chế các tác động tiêu cực gây ra bởi sự cộng hưởng giữa các kênh chuyển động trong quá trình bay.

Từ khoá: Tên lửa thăm dò; Cộng hưởng; Dao động chu kỳ ngắn; Phép biến đổi Fourier.