MODELING OFDISTRIBUTION TRANSFORMERS BY A 3-D NUMERICAL TECHNIQUE

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Abstract: *Modeling of electromagnetic quantities (magnetic flux, leakage flux, eddy current loss, and electromagnetic force) in distributed power transformers remarkably plays an essential role for researchers and designers in the calculation and production of electrical equipment in general and transformer designs in particular. In the frame of this research, the distribution of magnetic flux densities along the tank wall and cover plate, the leakage flux in air regions, and the forces for both rated and short circuit modes in the windings of the transformer are computed/simulated via a 3-D numerical method. The method is herein developed with magnetic vector potential formulations and is applied to a practical application of the distributed transformer (630kVA-22/0,4kV).*

Keywords: Magnetic flux; Magnetic field; Leakageflux; Electromagnetic forces; Numerical method.

1. INTRODUCTION

Nowadays, the distribution transformer is one of the most vital and costliest electrical devices of the electrical system. Hence, themodeling of fields of leakage and fringing fluxes, eddy current losses, and electromagnetic forces in the distribution transformers is really a significant problem and have, in particular, become a matter of concern and topicality for researchers and designers. In addition, the hot spots in windings due to influences of magnetic fields and eddy current losses are also presented. High temperatures may accelerate aging and cause faults [1].

In order to simulate and solve the above electromagnetic quantities, many researchers have recently applied numerical methods (e.g., moment method, finite differential method, boundary finite element method, finite element method). Amongst these methods, the finite element method (FEM) has become the most popular technique for quasi-static magnetodynamic problems with low frequencies [1, 2].

In this development, the method is proposed for the magnetic vector potential formulations (***b***-conform formulations). The extension of the method is illustrated on a practical test of a distribution transformer of 630 kVA - 22/0,4 kV.

2. METHODOLOGY

A studied problem is defined in a domain Ω, with boundary $\partial\Omega = \Gamma = \Gamma_h \cup \Gamma_e$. The equations and material relations are written in Euclidean space $\mathbb{R}^3$[3]-[6]

$$\text{curl}\,\mathbf{H} = \mathbf{J}_s, \text{curl}\,\mathbf{E} = -j\omega\,\mathbf{B},\;\; \text{div}\,\mathbf{B} = 0, \qquad \text{(1a-b-c)}$$

with behavior relations of materials

$$\mathbf{B} = \mu\mathbf{H},\;\; \mathbf{J} = \sigma\mathbf{E}, \qquad \text{(2a-b)}$$

where j is called the imaginary unit, $\mathbf{B}$ is the magnetic induction (T), $\mathbf{E}$ is the electric field (V/m), $\mathbf{J}_s$ is the current density (A/m2), $\mathbf{H}$ is the magnetic field (A/m), μ is the relative permeability and σ is the electric conductivity (S/m).

The boundary conditions (BCs) for the normal or the tangential components of electromagnetic fields defined on Γ are defined as [6-9]:

$$\boldsymbol{n} \times \mathbf{H}|_{\Gamma_h} = 0, \boldsymbol{n} \times \mathbf{E}|_{\Gamma_e} = 0,\ \boldsymbol{n} \cdot \mathbf{B}|_{\Gamma_e} = 0, \qquad \text{(3a-b-c)}$$

where $\boldsymbol{n}$ is the unit normal exterior to Ω [2-5]. The full domain Ω is divided into two complementary parts Ω_c and Ω_c^C, where Ω_c is the conducting region take into account the eddy currents and Ω_c^C is the non-conducting region carrying the imposed electric current density $\mathbf{J}_s$. The fields presented in Maxwell's equations are presented in the frequency domain. The equations (1 a) and (1 b) are solved with the BCs taken the tangential component of $\mathbf{H}$ and $\mathbf{E}$ in (3 a-b) and the normal component of $\mathbf{B}$ in (3 c) into account.

From equation (1c), the magnetic induction $\mathbf{B}$ can be derived from a vector potential $\mathbf{A}$ such that [4]

$$\text{curl}\ \mathbf{B} = \mathbf{A}. \qquad (4)$$

By combining (4) with (1 a), one gets a curl $(\mathbf{E} + j\omega\mathbf{A}) = 0$, which leads to the definition of an electrical scalar potential ν such that

$$\mathbf{E} = -j\omega\mathbf{A} - \text{grad}\ \nu. \qquad (5)$$

For the magnetodynamic problem, the fields **H, B, E, J** are defined and validated to satisfy Tonti's diagram (Fig. 1) [4]. This means that $\mathbf{H} \in \boldsymbol{F}_h(\text{curl};\Omega)$, $\mathbf{J} \in \boldsymbol{F}_h(\text{div};\Omega)$, $\mathbf{E} \in \boldsymbol{F}_e(\text{curl};\Omega)$ và $\mathbf{B} \in \boldsymbol{F}_e(\text{div};\Omega)$, where $\boldsymbol{F}_h(\text{curl};\Omega)$ and $\boldsymbol{F}_e(\text{dive};\Omega)$ are function spaces containing BCs and the fields defined on Γ_h and Γ_e of studied domain Ω.

$$\begin{tikzcd}
H_h^1(\Omega) \arrow[r, "\text{grad}_h"] \arrow[d, leftrightarrow] & \boldsymbol{H}_h(\textbf{curl};\Omega) \arrow[r, "\text{curl}_h"] \arrow[d, leftrightarrow] & \boldsymbol{H}_h(\text{div};\Omega) \arrow[r, "\text{div}_h"] \arrow[d, leftrightarrow] & L^2(\Omega) \arrow[d, leftrightarrow] \\
L^2(\Omega) & \boldsymbol{H}_e(\text{div};\Omega) \arrow[l, "\text{div}_e"'] & \boldsymbol{H}_e(\textbf{curl};\Omega) \arrow[l, "\text{curl}_e"'] & H_e^1(\Omega) \arrow[l, "\text{grad}_e"']
\end{tikzcd}$$

Figure 1. *Tonti's diagram [4].*

It should be noted that this diagram is also beneficial for the validation of dual finite element formulations ($\boldsymbol{h}$-conformal formulations).

3. WEAK FORMULATIONS AND FIELD DISCRITIZATIONS

3.1. Magnetodynamic weak formulations

In this part, the weak formulations for themagnetic vector potential are performed based on the set of Maxwell's equations (1a-b-c) together with the constitutive laws (2a-b) shown in Section 2. In order to satisfy precisely Faraday's law (1 b) in a strong sense, it takes $\mathbf{B} \in \boldsymbol{F}_e(\text{div};\Omega)$, $\mathbf{E} \in \boldsymbol{F}_e(\text{curl};\Omega)$. This is equivalent to verify the lower part of Tonti's diagram (Fig. 1). In addition, verifying exactly the constitutive laws (2 a - b) implies choosing $\mathbf{H} \in \boldsymbol{F}_e(\text{div};\Omega)$ and $\mathbf{j} \in \boldsymbol{F}_e(\text{curl};\Omega)$.

By starting from the weak form of Ampere's law (1 b), one has [4]:

$$\int_\Omega \text{curl}\ \mathbf{H} \cdot \mathbf{A}' d\Omega = \int_\Omega \boldsymbol{j} \cdot \mathbf{A}' d\Omega_s\,, \mathbf{A}' \in \boldsymbol{F}_e^0(\text{curl};\Omega), \qquad (6)$$

where the $\mathbf{A}' \in \boldsymbol{F}_e^0(\text{curl};\Omega)$ is a field of test functions independent of time. By applying the Green formula [4], (6) becomes

$$\int_{\Omega} \mathbf{H} \cdot \text{curl}\mathbf{A}' d\Omega + \int_{\Gamma_e} (\boldsymbol{n} \times \mathbf{H}) \cdot \mathbf{A}' d\Gamma_h = \int_{\Omega_s} \mathbf{J_s} \cdot \mathbf{A}' d\Omega_s, \quad \mathbf{A}' \in \boldsymbol{F}_e^0(\text{curl};\Omega), \tag{7}$$

In order to satisfy the lower part of the Tonti diagram (Fig. 1), equation (7) should introduce the behavior laws (2 a – b), i.e.

$$\int_{\Omega} \mu\mathbf{B} \cdot \text{curl } \mathbf{A}' d\Omega - \int_{\Omega} \sigma\mathbf{E} \cdot \text{curl}\mathbf{A}' d\Omega + \int_{\Gamma_e} (\boldsymbol{n} \times \mathbf{H}) \cdot \mathbf{A}' d\Gamma_h = \int_{\Omega_s} \mathbf{J_s} \cdot \mathbf{A}' d\Omega_s,$$
$$\mathbf{A}' \in \boldsymbol{F}_e^0(\text{curl};\Omega). \tag{8}$$

By introducing the magnetic vector potential **A** in (4) and magnetic field **E** in (5) into (8), one has

$$\int_{\Omega} \mu^{-1}\text{curl } \mathbf{A} \cdot \text{curl } \mathbf{A}' d\Omega + \int_{\Omega_c} \sigma j\omega \cdot \text{curl}\mathbf{A}' d\Omega_c + \int_{\Omega_c} \sigma\text{grad } v \cdot \mathbf{A}' d\Omega_c$$
$$+ \int_{\Gamma_e} (\boldsymbol{n} \times \mathbf{H}) \cdot \mathbf{A}' d\Gamma_h = \int_{\Omega_s} \mathbf{J_s} \cdot \mathbf{A}' d\Omega_s, \quad \mathbf{A}' \in \boldsymbol{F}_e^0(\text{curl};\Omega). \tag{9}$$

where $\boldsymbol{F}_e^0(\text{curl};\Omega)$ is a function space defined in Ω containing the basis functions for **A** as well as for the test function $\mathbf{A}'$. It should be noted that a gauge condition has to be imposed everywhere in Ω to get uniquely the magnetic vector potential and the electric scalar potential v in the conducting regions Ω_c.

The weak formulation (9) shows, by taking $\mathbf{A}' = \text{grad } v'$as a test function, that

$$\int_{\Omega_c} \sigma j\omega \cdot \text{grad } \mathbf{A}' d\Omega_c + \int_{\Omega_c} \sigma\text{grad } v \cdot \text{curl } v' d\Omega_c = \int_{\Gamma_g} (\boldsymbol{n} \cdot \mathbf{J_s}) v' d\Gamma_g,$$
$$v' \in \boldsymbol{F}_e^0(\text{curl};\Omega), \tag{10}$$

where Γ_g is the part of the boundary of Ω_c imposing electric currents. The tangential component of the magnetic field $\boldsymbol{n} \times \mathbf{H}$ in (9) is defined as a natural BC on the boundary Γ_g of the studied domain Ω. For that, the BC can be taken (2a) into account.

3.2. Discretization of fields

The field **A** in (9) is discretized by finite edge elements (FEs), generating the function space $\boldsymbol{F}_h^0(\text{curl};\Omega)$ defined in a mesh of Ω [4]. Thus, the discretization of **A** is expressed as:

$$\mathbf{A} = \sum_{i \in E(\Omega)} A_i s_{e,i}, \tag{11}$$

where $E(\Omega)$ is the set of edges of Ω, $s_{e,i}$ is basic edge function associated with edge i, and A_i is the circulation of **A** along the edge i of the studied domain. In the same way, a gauge condition has to be imposed to get a unique solution. For that, the magnetic vector potential **A** can be written as

$$\mathbf{A} = \sum_{i \in E(\Omega_c)} A_{c,i} s_{e,i} + \sum_{i \in E(\Omega_c^C)} A_{c,i} s_{e,i} \tag{12}$$

where $E(\Omega_c)$ is the set of edges of Ω_c, $E(\Omega_c^C)$ is the set of edges of a co-tree established in Ω_c [4].

The electromotive force (EMF)$\boldsymbol{F}_{total}$can be computed via the cross product of the magnetic leakage flux in the air gap (between the core and inductors) and the electric current density. This can be done by post-processing, i.e.,

$$\boldsymbol{F}_{total} = \frac{1}{2} \int_{\Omega} \mathrm{Re}|\mathbf{B} \times \boldsymbol{J}| \cdot d\Omega \tag{13}$$

4. NUMERICAL APPLICATION

The practical example is considered with a three-phase distribution transformer of 630kVA-22/0,4kV, where other parameters are given in the standard catalog of THIBIDI Company. The transformer is tested with the case ofrated and short-circuit modes. Figure 2 presents the model of the transformer with the tank wall (*left*) and the distribution of magnetic field flux density due to a three-phase current (*right*).

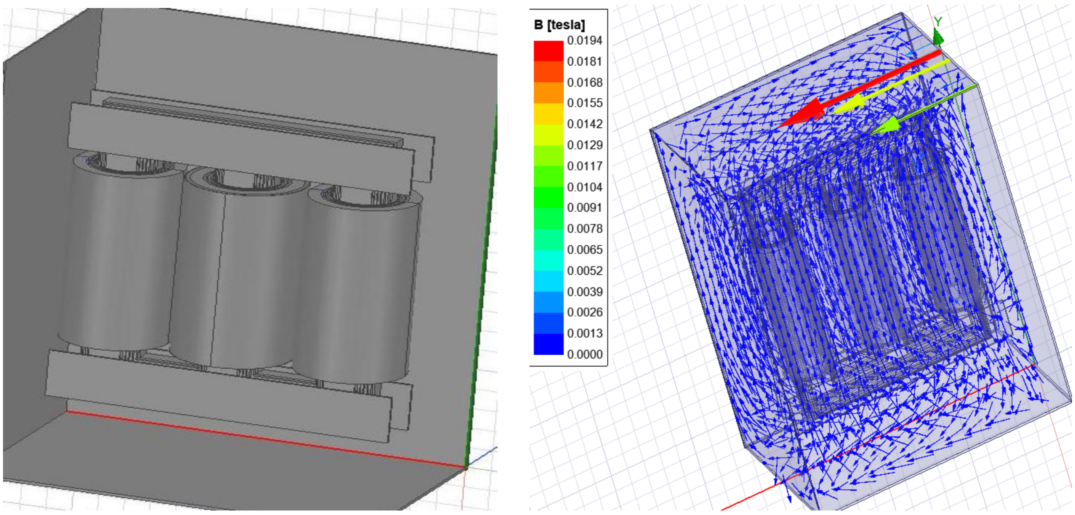

Figure 2. *Model of the transformer with the tank wall (left) and the distribution of magnetic field flux density due to a three-phase current (right).*

Figure 3 shows the distribution of the electromagnetic force (EMF) in the secondary windings for the rated mode (*left*) and the thermal color map of the secondary windings (*right*). In this situation, it can be seen that the hot spot appearing in the winding. This result serves as proof for increasing the electrical insulation for the windings. In addition, the value of radius EMFs with two different cases (rated and short-circuit modes) are indicated in Figure 4. For the rated mode (Fig. 4, *left*), the EMF of phase C is bigger than phase A and B and can touch 390N, corresponding to 105 degrees in comparison with the direction of the windings. For the short-circuit mode (Fig. 4, *right*), in the same way, the biggest EMF also appears in phase C and obtains the maximum value of 250 KN. This means that the distribution of leakage flux in phase C is the largest.

Figure 5 points out the distribution of magnetic flux density in the air along the tank wall (*left*) and along with the cover plate (*right*).

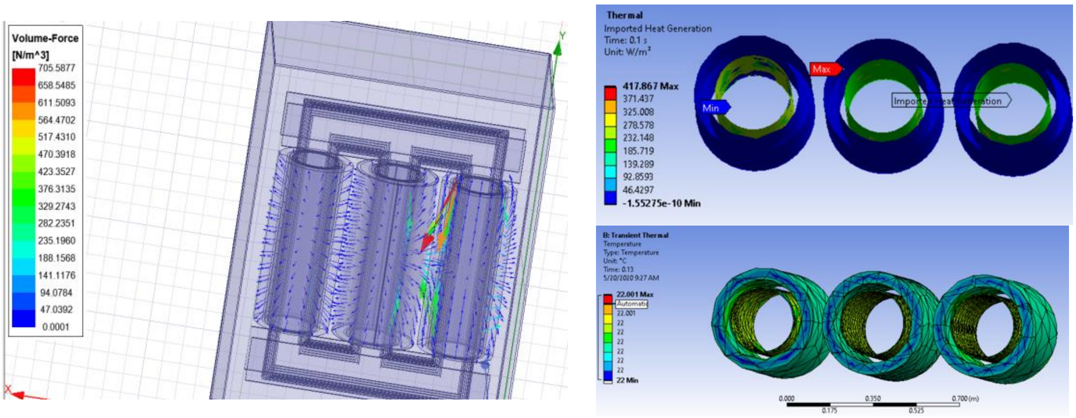

Figure 3. *Radius electromagnetic force (left) and the thermal color map of the secondary windings.*

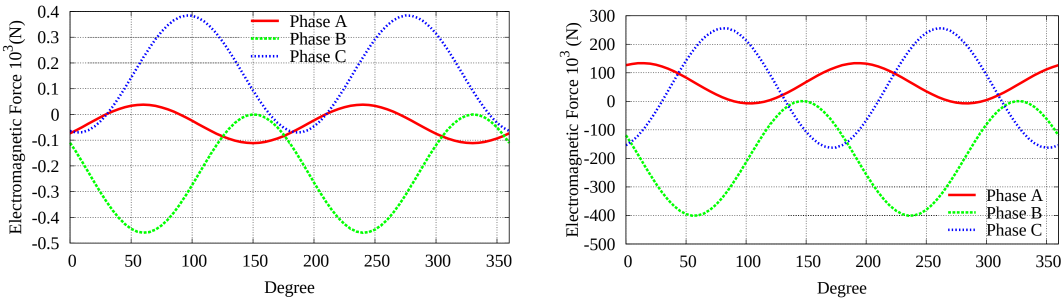

Figure 4. *Electromagnetic forces for rated currents (left) and short -circuit currents (right).*

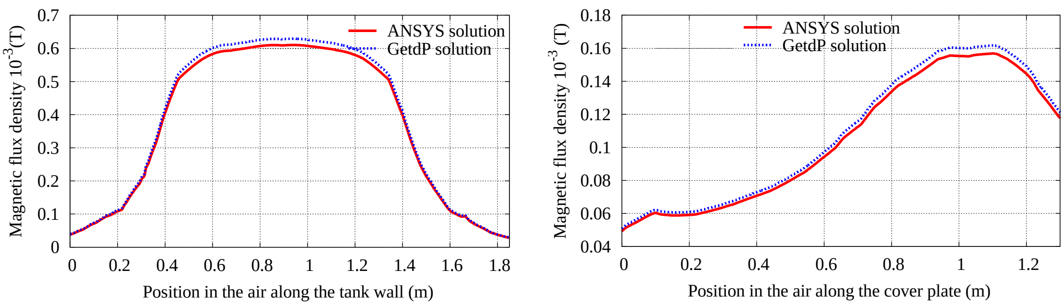

Figure 5. *Distribution of magnetic flux densities in the air along the tank wall (left) and along with the cover plate (right).*

It can be seen that the magnetic induction distribution in the tank wall mostly focuses on the region from 0.4 m to 1.4 m (Fig. 5, *left*) and from 0.8 m to 1.2 m along with the cover plate (Fig. 5, *right*). It should be noted that the obtained results are checked to be close to the results in the computation from the GetDP and GMSH softwares [10], [11]. These are open softwares that everyone can access to adapt/write source-codes for solving practical problems.

5. CONCLUSION

A numerical technique has been successfully proposed for modeling of the distribution transformers with the magnetic vector potential formulations to compute and simulate the local fields such as the magnetic flux density along the tank wall and along with the cover plate, as well as radius electromotive forces for the rated and short circuit modes. In particular, the developments of the method have been successfully applied to the practical problem. The results obtained from the commercial software (ANSYS) are compared to be similar to the solutions in the calculation from the Gedp [10], [11].

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TÓM TẮT

MÔ HÌNH HOÁ MÁY BIẾN ÁP PHÂN PHỐI BẰNG PHƯƠNG PHÁP SỐ

Mô hình các thông số điện từ (từ thông, từ thông rò/tản, tổn hao dòng điện xoáy và lực điện từ) trong các máy biến áp phân phối đóng vai trò cực kỳ quan trọng đối với các nhà nghiên cứu và các nhà thiết kế trong việc tính toán và chế tạo các thiết bị điện nói chung và máy biến áp nói riêng. Trong phạm vi của nghiên cứu này, sự phân bố của từ trường dọc theo vách thùng và mặt bích, từ thông tản/rò trong vùng không khí và lực điện từ cho cả hai trường hợp định mức và ngắn mạch trong máy biến áp sẽ được tính toán/mô phỏng bằng phương pháp số với mô hình 3-D. Phương pháp được phát triển với công thức véc tơ từ thế và được áp dụng thông qua bài toán thực tế với máy biến áp phân phối 630kVA-22/0,4kV.

Từ khoá: Từ thông; Từ trường; Từ thông tản và từ thông rò; Lực điện từ; Phương pháp số.

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