Quadrotor control applying backstepping algorithm

Le Hoang Anh, Le Huu Toan*, Tran Duc Thuan

Vinh Long University of Technology Education, 73 Nguyen Hue, Vinh Long City, Vinh Long.
*Corresponding author: toanlh@vlute.edu.vn

Received: 06 Sep. 2024; Revised 27 Oct. 2024; Accepted 12 Nov. 2024; Published 25 Nov. 2024.
DOI: https://doi.org/10.54939/1859-1043.j.mst.99.2024.99-108

ABSTRACT

The paper presents the application of a backstepping controller for quadrotor tracking. The backstepping controller is specially designed to accommodate the nonlinearity of the system. The results show that the controller has achieved the ability to guide the quadrotor to the target position with small error and maintain stability when tracking the predetermined trajectory. Moreover, the controller also shows good performance in tracking the trajectory under the influence of disturbances.

Keywords: Rolling control; Flight trajectory; UAV; Quadrotor.

1. INTRODUCTION

In recent years, there have been various studies on Quadrotor flight trajectory control and various methods have been proposed. In the simulation studies conducted, the performance of Quadrotors following various trajectories has been studied [1]. Muhammad Maaruf et al. discussed the PID, sliding mode, backstepping, MPC and fuzzy control techniques for Quadrotor systems [2]. The study of Quadrotor trajectory tracking includes system stability analysis, evaluation of the performance of controllers and algorithms, and their anti-interference ability [3]. Rafael Guardeño et al. applied PID linear and quadratic control methods to keep Quadrotor stable in noisy environments [4]. Tianpeng Huang et al. designed an adaptive sliding mode controller to achieve attitude stability and path tracking capability for UAV [5-7] analyzed Quadrotor in detail in a simulation environment and used control methods such as backstepping control, linear quadratic controller and type 2 fuzzy logic. In this study, the authors applied backstepping control theory to design motion control commands for quadrotor UAV.

2. CONTENT TO BE RESOLVED

2.1. System modeling

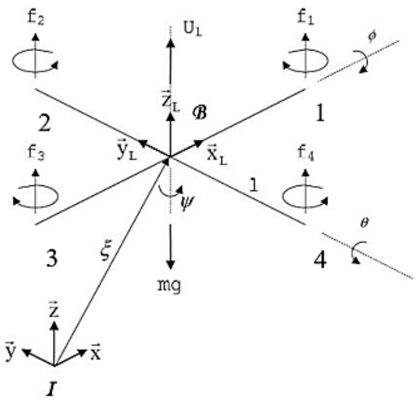

Figure 1. *Quadrotor position and orientation [8].*

In which, the moving force of the rotors creates the quadrotor to move around the roll Φ, pitch θ, yaw angles Ψ. Fixed coordinate system ***I*** and center of mass coordinate system ***B***.

The model used in the paper is taken from [8].

$$\begin{cases} \ddot{x} = \dfrac{u_1}{m}\left(cos\phi\, sin\,\theta cos\psi + sin\,\phi\, sin\,\psi\right) & \ddot{\phi} = \dfrac{I_{yyy} - I_{zzz}}{I_{xxx}}\dot{\phi}\dot{\psi} - \dfrac{j_{tp}}{I_{xxx}}\dot{\theta}\Omega + \dfrac{lu_2}{I_{yyy}} \\ \ddot{y} = \dfrac{u_1}{m}\left(cos\phi\, sin\,\theta cos\psi - sin\,\phi cos\psi\right) & \ddot{\theta} = \dfrac{I_{zzz} - I_{xxx}}{I_{yyy}}\dot{\phi}\dot{\psi} - \dfrac{j_{tp}}{I_{yyy}}\dot{\phi}\Omega + \dfrac{lu_3}{I_{yyy}} \\ \ddot{z} = \dfrac{u_1}{m}(\,cos\phi cos\theta\,) - g & \ddot{\Psi} = \dfrac{u_4}{I_{zzz}} + \dot{\phi}\dot{\theta}\dfrac{I_{xxx} - I_{yyy}}{I_{zzz}} \end{cases} \tag{1}$$

In which u_1, u_2, u_3, u_4 are the altitude control signals respectively. Ω are functions of the angular velocities of the rotors. $\ddot{x},\ddot{y},\ddot{z}$ are the accelerations in the direction x,y,z. $\ddot{\Phi},\ddot{\theta},\ddot{\Psi}$ are the original accelerations of the Quadrotor.

In this paper, the backstepping control method is applied to realize the adjustment of the attitude, heading, and altitude of the quadrotor. This method enables the quadrotor to operate stably in diverse environments and complex situations. The backstepping algorithm uses a recursive control technique, whose working principle is to construct intermediate control laws for the state variables of the system. These state variables are clearly defined and play an important role in ensuring that the quadrotor can track the desired trajectory smoothly and without interruption. These control laws are designed to optimize the control performance, and are detailed as follows: $x_1 = \phi$, $x_2 = \dot{x}_1$, $x_3 = \theta$, $x_4 = \dot{x}_2$, $x_5 = \psi$ $x_6 = \dot{x}_5$, $x_7 = z$, $x_8 = \dot{z}$, $x_9 = x$, $x_{10} = \dot{x}_9$, $x_{11} = y$, $x_{12} = \dot{x}_{11}$, $\lambda_1 = \dfrac{I_{yyy} - I_{xxx}}{I_{xxx}}$, $\lambda_2 = \dfrac{I_{zzz} - I_{xxx}}{I_{yyy}}$, $\lambda_3 = \dfrac{I_{xxx} - I_{yyy}}{I_{zzz}}$ $\beta_1 = \dfrac{1}{I_{xxx}}$, $\beta_2 = \dfrac{1}{I_{yyy}}$, $\beta_3 = \dfrac{1}{I_{zzz}}$.

Applying the state variables and kinematic model parameters to equation (1) is rewritten:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} x_2 \\ \lambda_1 x_4 x_6 + \beta_1 u_2 \\ x_4 \\ \lambda_2 x_2 x_6 + \beta_2 u_3 \\ x_6 \\ \lambda_3 x_2 x_4 + \beta_3 u_4 \\ x_8 \\ -g + \left(c\,x_1\, c\,x_3\right)u_1 / m \\ x_{10} \\ \left(s\,x_1\, s\,x_5 + c\,x_1\, c\,x_5\, s\,x_3\right)u_1 / m \\ x_{12} \\ \left(-c\,x_5\, s\,x_1 + c\,x_1\, c\,x_3\, s\,x_5\right)u_1 / m \end{bmatrix} \tag{2}$$

2.2. Controller design

The model developed according to equation (2) is used to design a backstepping controller to stabilize the angles. The stabilization of roll, pitch, and yaw angles is related to the control signals u_2, u_3, and u_4.

Roll angle regulation controller

The state variables x_1 and x_2 represent the roll angle and the roll rate, respectively:

The Lyapunov function is chosen:

$$T_1 = 0.5z_1^2 \tag{3}$$

z_1 is the error between the desired roll angle and the actual roll angle:

$$z_1 = \left(\dot{x}_{1c} - \dot{x}_1\right)$$

The derivative (3) is:

$$\dot{T}_1 = z_1\dot{z}_1 = z_1\left(\dot{x}_{1c} - \dot{x}_1\right) = z_1\left(\dot{x}_{1c} - x_2\right) \tag{4}$$

The positive definite limit function is chosen to be T_1 given in the equation:

$$\dot{T}_1 = z_1\left(\dot{x}_{1c} - x_2\right) \le -c_1z_1^2 \tag{5}$$

Let c_1 be a positive constant that satisfies inequality (5) we have the control input as:

$$\left(x_2\right)_d = \dot{x}_{1c} + c_1z_1 \tag{6}$$

The error variable z_2 is defined as the difference between x_2 and its desired value:

$$z_2 = x_2 - \dot{x}_{1c} - c_1z_1 \tag{7}$$

Rewrite equation (4):

$$\dot{T}_1 = z_1\dot{z}_1 = z_1\left(\dot{x}_{1c} - x_2\right) = z_1\left(\dot{x}_{1c} - \left(z_2 + \dot{x}_{1c} + c_1z_1\right)\right) \Rightarrow \dot{T}_1 = -z_1z_2 - c_1z_1^2 \tag{8}$$

Enhance the Lyapunov function T_1 by incorporating a quadratic term associated with the variable z_2 to obtain T_2:

$$T_2 = T_1 + 0.5z_2^2 \Rightarrow \dot{T}_2 = -z_1z_2 - c_1z_1^2 + z_2\left(\dot{x}_2 - \ddot{x}_{1c} - c_1\dot{z}_1\right) \tag{9}$$

Select a limit function with a positive value and substitute it into $\dot{x}_2$, yielding:

$$-z_1z_2 - c_1z_1^2 + z_2\left(\lambda_1x_4x_6 + \beta_1u_2 - \ddot{x}_{1c} - c_1\dot{z}_1\right) \le -c_1z_1^2 - c_2z_2^2 \tag{10}$$

Using the case of equality equation (10) we have the system will be stable according to Lyapunov:

$$u_2 = \frac{1}{\beta_1}\left(\ddot{x}_{1d} + c_1\dot{z}_1 - \lambda_1x_4x_6 + z_1 - c_2z_2\right) \tag{11}$$

Angle controller pitch

The state variables x_3 and x_4 correspond to the pitch angle and the pitch rate, respectively:

The Lyapunov function is chosen as follows:

$$T_3 = 0.5z_3^2 \tag{12}$$

The error variable z_3 represents the difference between the desired pitch angle and the actual pitch angle: $z_3 = \left(\dot{x}_{3c} - \dot{x}_3\right)$

The derivative of equation (12) is:

$$\dot{T}_3 = z_3\dot{z}_3 = z_3\left(\dot{x}_{3c} - \dot{x}_3\right) = z_3\left(\dot{x}_{3c} - x_4\right) \tag{13}$$

The positive definite limit function is selected $\dot{T}_3$ given in the equation:

$$\dot{T}_3 = z_3\left(\dot{x}_{3c} - x_4\right) \le -c_3 z_3^2 \tag{14}$$

Here c_3 is a positive constant satisfying inequality (14) the control input is:

$$x_{4c} = \dot{x}_{3c} + c_3 z_3 \tag{15}$$

The error variable z_4 is defined as the difference between x_4 and its desired value:

$$z_4 = x_4 - \dot{x}_{3c} - c_3 z_3 \tag{16}$$

Rewrite equation (13):

$$\dot{T}_3 = z_3\dot{z}_3 = z_3\left(\dot{x}_{3c} - x_3\right) = z_3\left(\dot{x}_{3c} - \left(z_4 + \dot{x}_{3c} + c_3 z_3\right)\right) \Rightarrow \dot{T}_3 = -z_3 z_4 - c_3 z_3^2 \tag{17}$$

Enhance the Lyapunov function T_3 by incorporating a quadratic term associated with the variable z_4 to obtain T_4:

$$T_4 = T_3 + 0.5 z_4^2 \Rightarrow \dot{T}_4 = -z_3 z_4 - c_3 z_3^2 + z_4\left(\dot{x}_4 - \ddot{x}_{3c} - c_3\dot{z}_3\right) \tag{18}$$

Select a positive definite candidate Lyapunov function and substitute $\dot{x}_4$ the resulting expression:

$$-z_3 z_4 - c_3 z_3^2 + z_4\left(a_2 x_2 x_6 + \beta_2 u_3 - \ddot{x}_{3c} - c_3\dot{z}_3\right) \le -c_3 z_3^2 - c_4 z_4^2 \tag{19}$$

Using the equality case of equation (19) we have that the system will be stable according to Lyapunov:

$$u_3 = \frac{1}{\beta_2}\left(\ddot{x}_{3c} + c_3\dot{z}_3 - a_2 x_2 x_6 + z_3 - c_4 z_4\right) \tag{20}$$

Yaw angle controller

The state variables x_5 and x_6 correspond to the yaw angle and the yaw rate, respectively:

The Lyapunov function is chosen:

$$T_5 = 0.5 z_5^2 \tag{21}$$

The error variable z_5 represents the difference between the desired yaw angle and the actual yaw angle: $z_5 = x_{5c} - x_5$

Derivative of equation (21):

$$\dot{T}_5 = z_5\dot{z}_5 = z_5\left(\dot{x}_{5c} - \dot{x}_5\right) = z_5\left(\dot{x}_{5c} - x_6\right) \tag{22}$$

Positive definite limit function $\dot{T}_5$ given in the equation:

$$\dot{T}_5 = z_5\left(\dot{x}_{5c} - x_6\right) \le -c_5 z_5^2 \tag{23}$$

Here c_5 is a positive constant satisfying inequality (23) and the control input is:

$$x_{6c} = \dot{x}_{5c} + c_5 z_5 \tag{24}$$

The error variable z_6 is defined as the difference between x_6 and its desired value:

$$z_6 = x_6 - \dot{x}_{5c} - c_5 z_5 \tag{25}$$

Rewriting equation (22) we have:

$$\dot{T}_5 = z_5\dot{z}_5 = z_5\left(\dot{x}_{5c} - x_5\right) = z_5\left(\dot{x}_{5c} - \left(z_6 + \dot{x}_{5c} + c_5 z_5\right)\right) \Rightarrow \dot{T}_5 = -z_5 z_6 - c_5 z_5^2 \tag{26}$$

Enhance the Lyapunov function T_5 by incorporating a quadratic term in the variable z_6 to obtain T_6:

$$T_6 = T_5 + 0.5 z_6^2 \Rightarrow \dot{T}_6 = -z_5 z_6 - c_5 z_5^2 + z_6\left(\dot{x}_6 - \ddot{x}_{5c} - c_5\dot{z}_5\right) \tag{27}$$

Choosing a positive definite limit function and substituting $\dot{x}_6$ leads to:

$$-z_5 z_6 - c_5 z_5^2 + z_6\left(\lambda_3 x_2 x_4 + \beta_3 u_4 - \ddot{x}_{5c} - c_5\dot{z}_5\right) \le -c_5 z_5^2 - c_6 z_6^2 \tag{28}$$

Using the case of equality equation (28) we have the system will be stable according to Lyapunov:

$$u_4 = \frac{1}{\beta_3}\left(\ddot{x}_{5c} + c_5\dot{z}_5 - \lambda_3 x_2 x_4 + z_5 - c_6 z_6\right) \tag{29}$$

Position controller

In this section, the control signal u_1 is generated by the four blades. It is directly related to the motion in the x, y, and z directions. In addition, the motions in the x and y directions are affected by u_x and u_y because they depend on the roll angle and pitch angle. To design a controller by correcting the error in the z direction in the system, we set

$$z_7 = x_{7c} - x_7 \tag{30}$$

The Lyapunov function is chosen:

$$T\left(z_7\right) = 0.5\left(z_7\right)^2 \tag{31}$$

Take the derivative (31) to get the equation:

$$\dot{T}\left(z_7\right) = z_7\dot{z}_7 \tag{32}$$

Equation (33) is obtained after substituting formula (30) into (32):

$$\dot{T}\left(z_7\right) = z_7\left(\dot{x}_{7c} - \dot{x}_7\right) = z_7\left(\dot{x}_{7c} - x_8\right) \tag{33}$$

If the expression in Formula (33) decreases with time, the error will also gradually converge to 0.

$$x_8 = \dot{x}_{8c} + c_7 z_7 ; c_7 > 0 \tag{34}$$

Thus z_7 will converge to 0.

$$z_8 = x_{8c} - x_8 = \dot{x}_{7c} + c_7 z_7 - x_8 \tag{35}$$

The Lyapunov function is chosen:

$$T\left(z_7, z_8\right) = 0.5\left(z_7\right)^2 + 0.5\left(z_8\right)^2 \tag{36}$$

If the chosen Lyapunov function is negative and decreasing as the derivative is taken with respect to time, then z_7 and z_8 will also decrease with respect to time:

$$\dot{T}\left(z_7, z_8\right) = z_7\dot{z}_7 + z_8\dot{z}_8 \tag{37}$$

The resulting expression:

$$\dot{z}_7 = z_8 - c_7 z_7 \tag{38}$$

$$\dot{z}_8 = \ddot{x}_{7c} + c_7 z_7 - \dot{x}_8 \tag{39}$$

If we substitute Formulas (38) and (39) into (37), we get:

$$\dot{T}\left(z_7, z_8\right) = z_7\left(z_8 - c_7 z_7\right) + z_8\left(\ddot{x}_{7c} + c_1 z_7 - \dot{x}_8\right) \tag{40}$$

Substitute $\dot{x}_8$ into formula (40) we get:

$$\dot{T}\left(z_7, z_8\right) = z_7 z_8 - c_7 z_7^2 + z_8 \ddot{x}_{7c} + c_7 z_8^2 - c_7^2 z_7 z_8 + g z_8 - z_8\left(cos\left(x_1\right)cos\left(x_3\right)\right)\frac{u_1}{m} \tag{41}$$

Choosing u_1 appropriately will lead to $\dot{T}\left(z_7, x_8\right) < 0$, with ($c_7$, $c_8 > 0$):

$$u_1 = \left(m / \left(cos\left(x_1\right)cos\left(x_3\right)\right)\right)\left(z_7 + \ddot{x}_{7c} + c_7 z_8 - c_7^2 z_7 + g + c_8 z_8\right) \tag{42}$$

$$\dot{T}\left(z_7, x_8\right) = -c_7^2 z_7 - c_8^2 z_8 \tag{43}$$

In the equation, z_7, $z_8 \neq 0$. According to Lyapunov theory, the errors (z_7, z_8) converge to 0 and the system is stable. By using the same method, the controller is synthesized from the x and y directions, giving the following results:

$$u_x = \left(\frac{m}{u_1}\right)\left(z_9 + \ddot{x}_{9c} + c_9 z_{10} - c_9^2 z_9 + c_{10} z_{10}\right) \tag{44}$$

$$u_y = \left(\frac{m}{u_1}\right)\left(z_{11} + \ddot{x}_{11c} + c_{11} z_{12} - c_{11}^2 z_{11} + c_{12} z_{12}\right) \tag{45}$$

With: (c_9, c_{10}, c_{11}, $c_{12} > 0$) and

$$u_x = \left(sin\, x_1\, sin\, x_5 + cosx_1 cosx_5\, sin\, x_3\right) \tag{46}$$

$$u_y = \left(-cosx_5\, sin\, x_1 + cosx_1 cosx_3\, sin\, x_5\right) \tag{47}$$

Besides:

$$z_9 = x_{9c} - x_9 \tag{48}$$

$$z_{10} = x_{10c} - x_{10} \tag{49}$$

$$z_{11} = x_{11c} - x_{11} \tag{50}$$

$$z_{12} = x_{12c} - x_{12} \tag{51}$$

The reference parameters x_{9c}, x_{11c} for the x and y position controllers are determined based on Equations (44) and (45). These reference parameters are calculated according to Equations (49) and Equations (51) as follows:

$$x_{10c} = x_{9c} - c_9 z_9 \tag{52}$$

$$x_{12c} = x_{11c} - c_{11} z_{11} \tag{53}$$

3. RESULTS AND DISCUSSION

The quadrotor trajectory control test was performed for a circular trajectory. The path started at a height of 5 meters above the ground, oscillated from 0 meters to 10 meters along the x-axis, 0 meters to 10 meters along the y-axis, and oscillated from 0 meters to 5 meters along the z-axis. This trajectory was created to provide simultaneous control of the Quadrotor along the x, y, and z axes.

The goal of the control in this study is to track a circular trajectory accurately and stably. To do this, figure 2 shows the SIMULINK model of the quadrotor, including the position and attitude control blocks integrated into the control loop. This model allows to simulate and analyze the control performance of the quadrotor in real-world situations. The system was tested in two conditions: one in a disturbance-free environment and the other in a disturbance-containing environment. The disturbance was simulated from the effect of wind, with a speed of 1 m/s, to evaluate the resilience of the control system in real-world operating conditions.

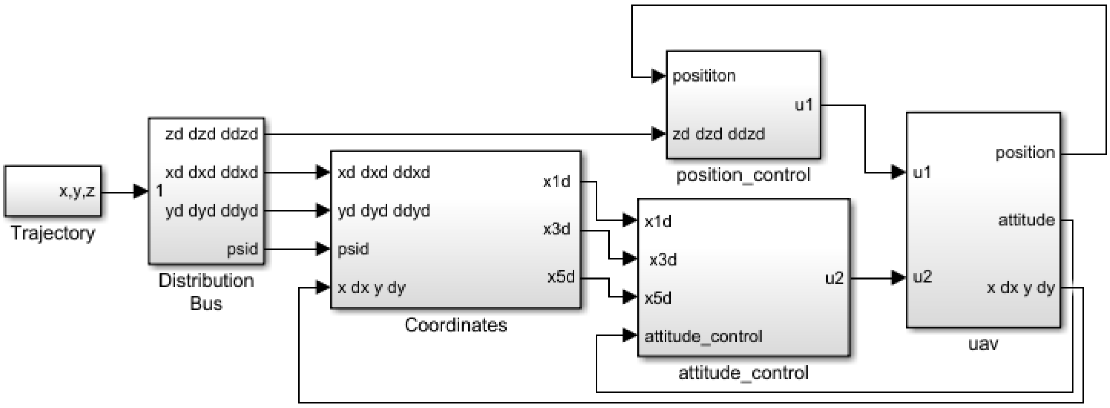

Figure 2. *Simulink model used in simulation.*

3.1. System simulation without noise

Figures 3, 4, 5 show the 2D, 3D trajectories, position, direction, control input and trajectory error without interference. This shows that the system tracks the set trajectory very well and is stable without interference. The trajectory tracking error is very small and the system operates stably.

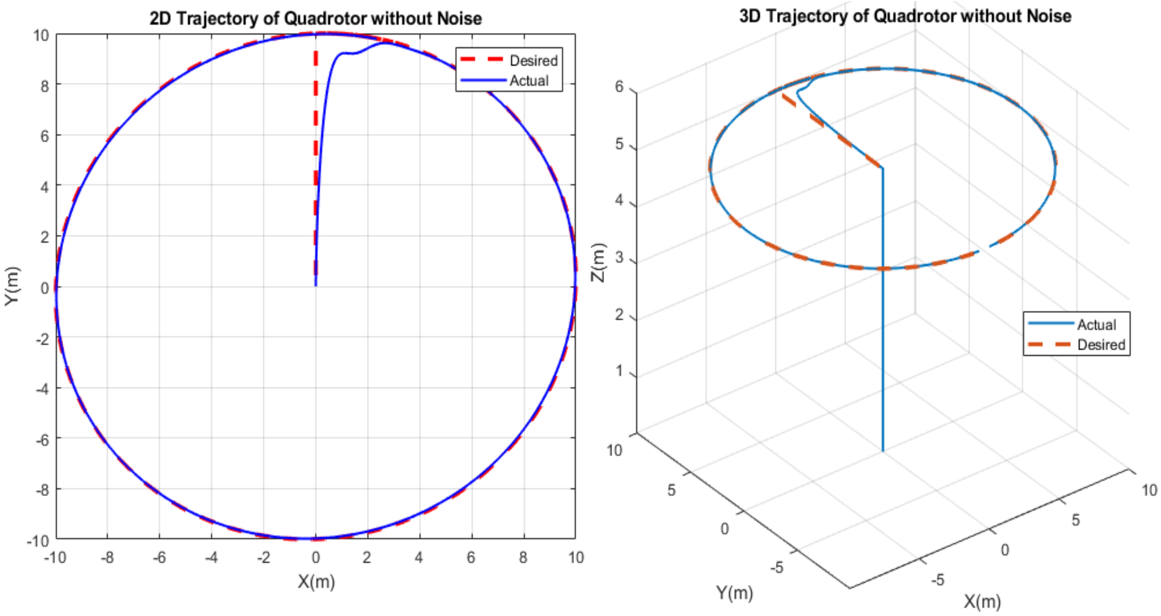

Figure 3. *2D and 3D circular orbits without interference.*

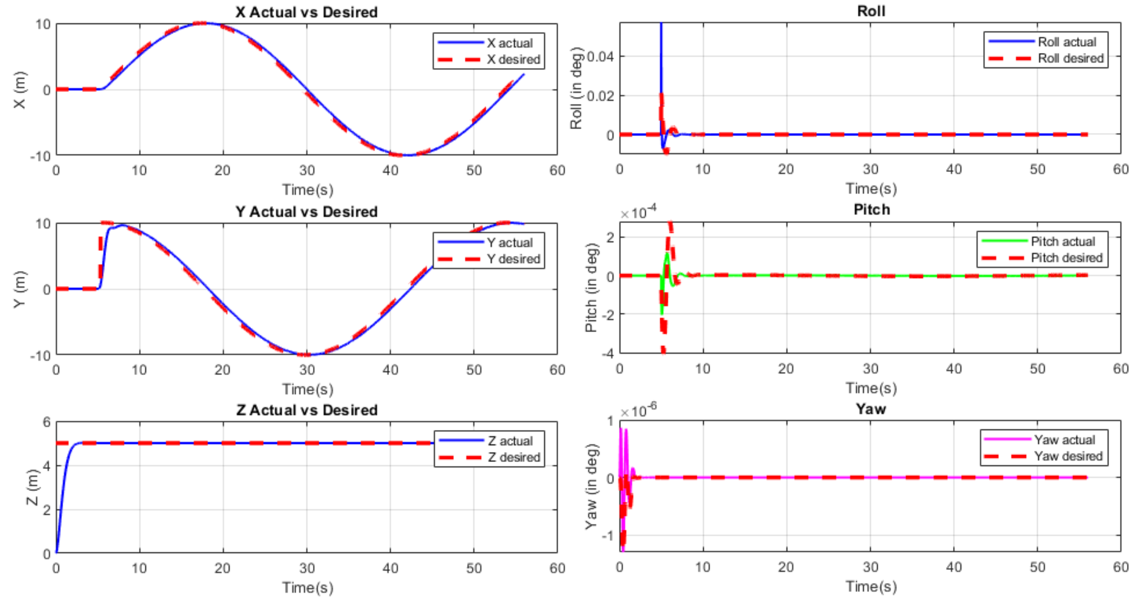

Figure 4. *Position and direction.*

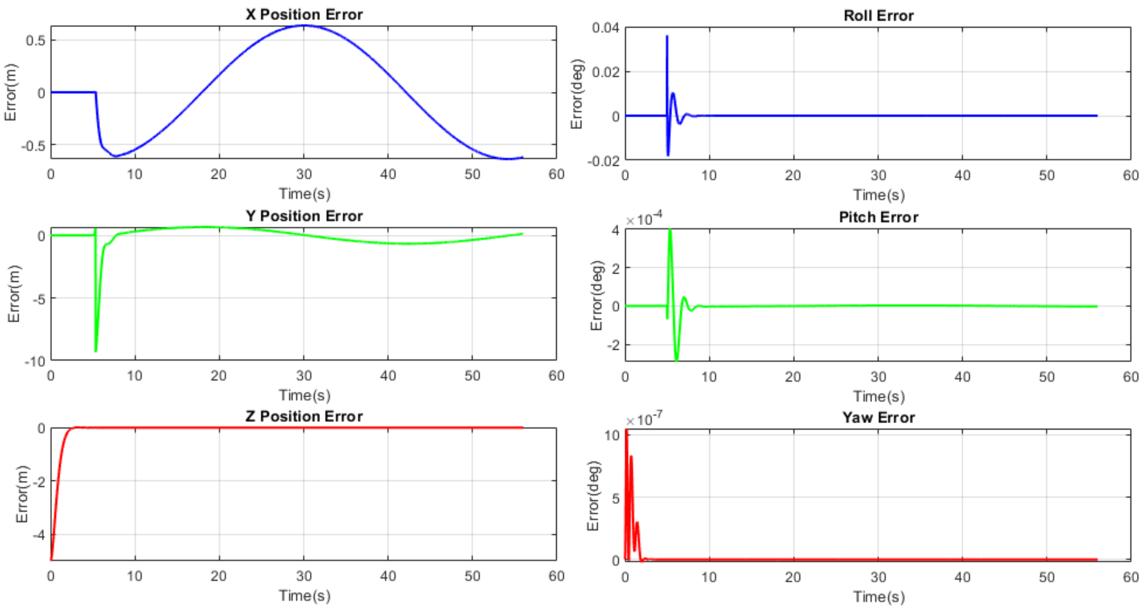

Figure 5. *Errors x, y, z and the angles Roll, Pitch, Yaw.*

3.2. System simulation with noise

The perturbation used is wind with a velocity of 1 m/s in the x, y directions at time t = 5 s. The 2D and 3D plots in Figures 6 and 7, 8 illustrate the trajectories, positions, directions, control inputs and tracking errors when the perturbation is applied. From the plot data, it can be seen that the controller is operating with a significant error. The results show that the error rate decreases as the values vary linearly along the xy axis. The controller generates rapidly changing reference angle signals to prevent vibration and oscillation, depending on the specific environmental conditions in which the controller is operating.

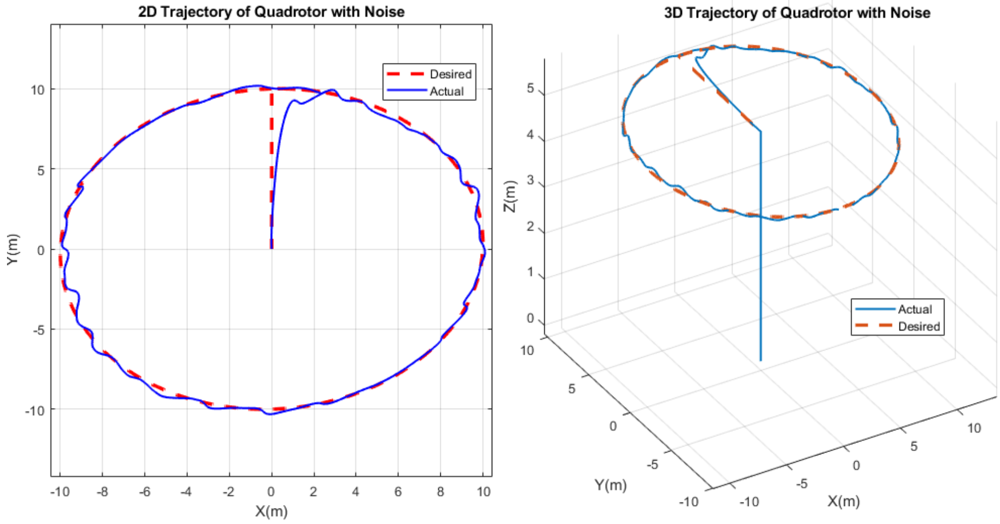

Figure 6. *2D and 3D circular trajectories with noise.*

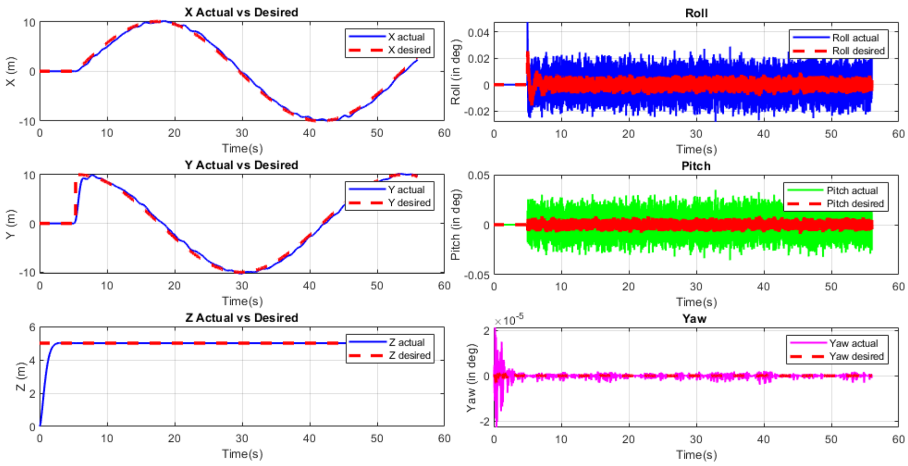

Figure 7. *Position and Direction vs. Time [Noise at 5th second].*

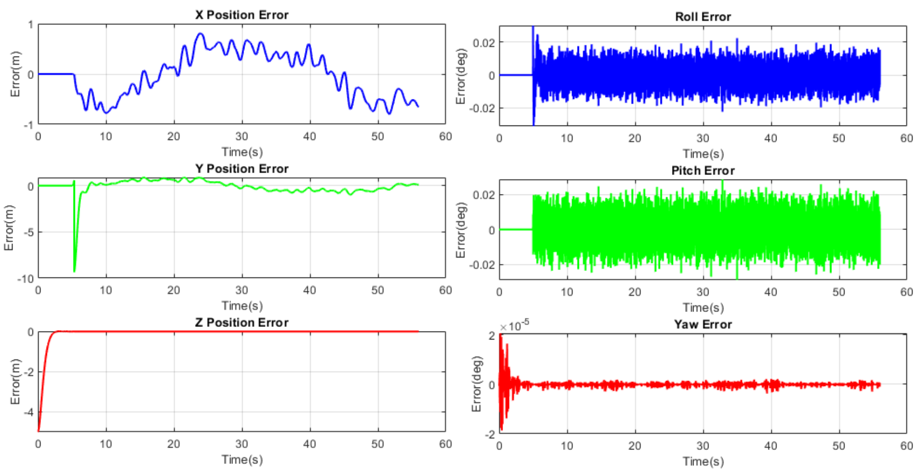

Figure 8. *Errors of x, y, z and angles of Roll, Pitch, Yaw [There is noise at 5th second].*

4. CONCLUSIONS

The Backstepping algorithm has determined the direction and attitude control laws of the proposed orbit tracking quadrotor to ensure Lyapunov stability. The control laws have been determined mathematically. The simulation results show that the proposed algorithm brings stable performance to the system. Although the system does not lose stability when there is a disturbance, there is a significant error. A possible solution is to use a sliding mode controller to eliminate the nonlinearity of the system.

REFERENCES

[1]. Imane Siti, Mostafa Mjahed, Hassan Ayad, and Abdeljalil El Kari, *"New Trajectory Tracking Approach for a Quadcopter Using Genetic Algorithm and Reference Model Methods,"* Applied Sciences, vol. 1780, no. 9, p. 21, (2019).

[2]. Muhammad Maaruf, Magdi Sadek Mahmoud, Alfian Ma'arif, *"A Survey of Control Methods for Quadrotor UAV,"* International Journal of Robotics and Control Systems, vol. 2, no. 4, pp. 652-665, (2022).

[3]. Marco Rinaldi, Stefano Primatesta, and Giorgio Guglieri, *"A Comparative Study for Control of Quadrotor UAVs,"* Applied Sciences, vol. 3464, p. 13, (2023).

[4]. Rafael Guardeño, Manuel J. López and Víctor M. Sánchez, *"MIMO PID Controller Tuning Method for Quadrotor Based on LQR/LQG Theory,"* Robotics, vol. 36, p. 8, (2019).

[5]. Tianpeng Huang, Deqing Huang, Zhikai Wang, and Awais Shah, *"Robust Tracking Control of a Quadrotor UAV Based on Adaptive Sliding Mode Controller,"* Research Article | Open Access, p. 15, (2019).

[6]. H. Bouadi, F. Mora-Camino, *"Direct Adaptive Backstepping Flight Control for Quadcopter Trajectory Tracking,"* IEEE, vol. 2018 IEEE/AIAA 37th Digital Avionics Systems Conference (DASC), (2018).

[7]. Samir Zeghlache, Ali Djerioui, Loutfi Benyettou, Tarak Benslimane, Hemza Mekki, Abderrahmen Bouguerra, *"Fault tolerant control for modified quadrotor via adaptive type-2 fuzzy backstepping subject to actuator faults,"* ScienceDirect, vol. 95, pp. 330-345, (2019).

[8]. Ernesto Paiva, Juan Soto, Julio Salinas and William Ipanaqué, *"Modeling and PID cascade control of a Quadcopter for trajectory tracking,"* IEEE, vol. 22, p. 7, (2016).

TÓM TẮT

Điều khiển quadrotor ứng dụng giải thuật backstepping

Bài báo trình bày ứng dụng của bộ điều khiển backstepping để bám theo quỹ đạo của quadrotor. Bộ điều khiển backstepping được thiết kế để thích nghi phù hợp với tính phi tuyến của hệ thống. Kết quả cho thấy bộ điều khiển đã đạt được khả năng dẫn quadrotor đến vị trí mục tiêu với lỗi nhỏ và duy trì sự ổn định khi theo dõi quỹ đạo đã được xác định trước. Hơn nữa, bộ điều khiển cũng thể hiện hiệu suất tốt trong việc bám quỹ đạo khi có sự tác động của nhiễu.

Từ khóa: Điều khiển cuốn chiếu; Quỹ đạo bay; UAV; Quadrotor.