ON THE STABILITY RADIUS OF SWITCHED POSITIVE LINEAR SYSTEMS

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Abstract: *This article investigates the stability radius based on exponentially stable of switched positive linear systems. A lower bound and upper bound for this radius with respect to structured affine positive perturbations of the system's parameters are established under the assumption that all its positive subsystems have a common linear copositive Lyapunov functional. An example is provided for illustrating the result.*

Keywords: Switched linear systems; Switched positive linear systems; Robust stability.

1. INTRODUCTION

A *switched system* is described by a family of subsystems and a rule that controls the switching between them. In the mathematical setting, such a system with time continuous can be described by a time-varying differential equation of the form:

$$\dot{x}(t) = f_{\sigma(t)}(x(t)),\ t \geq 0,\ \sigma \in \Sigma, \tag{1}$$

where $F := \{f_k(x) : k \in \underline{N}\},\ \underline{N} := \{1, 2, ..., \mathrm{N}\},$ is a given finite family of Lipschitz continuous vector fields and Σ is a set of switching signals $\sigma: [0, +\infty) \to \underline{N}$, which are piece-wise constant right-side continuous functions with points of discontinuity $t_i, i = 1, 2,...$ (known as the *switching instances*) satisfying $\tau(\sigma) := \inf_{i\in\mathbb{N}}(t_{i+1} - t_i) > 0$. It implies that $t_i \to +\infty$ as $i \to +\infty$ and t_i has no accumulated point in $\mathbb{R}$. Switched systems have numerous applications in control of mechanical systems, the automotive industry, air traffic and aircraft control, switching power converters, and in many other fields. The books [8] and [15] contain reports on various theoretical developments for switched systems as well as their applications in some of these areas.

Perhaps the most important of these developments related to the stability/ stabilizability of switched systems in which several significant results have been obtained over the last several decades (see e.g. the survey papers [9, 11] and the references therein). One of the basic problems of stability analysis of switched systems is to find conditions for stability/stabilizability under arbitrary switching. There are several methods to investigate this problem such as the Lyapunov function method, differential inclusions, linear matrix inequalities (LMIs) and Lie algebra. In the case of switched linear systems in K^n, which are of the form:

$$\dot{x}(t) = A_{\sigma(t)}(x(t)),\quad t \geq 0,\quad \sigma \in \Sigma,$$

$$A_{\sigma(t)} \in \mathrm{A} := \{A_k \in K^{n\times n}, k \in \underline{N}\},\ t \geq 0, \tag{2}$$

where, A is a given family of $n \times n$ matrices with elements in K, $K = \mathbb{C}$ or $\mathbb{R}$. It is well established (see [8]) that the zero solution of the system (2) is exponentially stable for any switching signal if all subsystems:

$$\dot{x}(t) = A_k x(t),\ t \geq 0,\ k \in \underline{N} \tag{3}$$

have a common quadratic Lyapunov function (or QLF, for short) of the form $V(x) = x^T Px$ or (see [2]) a common co-positive Lyapunov linear function of the form $V(x) = v^T x$.

In papers of Son-Ngoc (see [13]) and Thuan-Ngoc (see [14]), a stability radius to estimate bound radii for switched linear systems and periodically switched linear systems was introduced. The stability radii of the positive linear system by Son-Hinrichsen (see [12]) has read radius, which is equal to the complex radius. While the positive switched linear system, that is interested by many authors and given conditions stable (see [2], [3], [4], [10], [16]) have not yet studied robust stability.

Alongside the mentioned above studies, the aspect of developing *robustness measures of stability* for linear uncertain systems with state space description has received as well significant attention in system and control theory during the last decades.

The layout of the paper is as follows. In section 2, we introduce the notation and review some preliminaries that are useful for the development of the result of this paper. In Section 3, the notion of stability radius for the switched positive linear system is introduced and some formula for estimating this radius are derived, by using the solutions comparison principle. Finally, the conclusion is given in Section 4.

2. PRELIMINARIES

We begin by recalling some notations and concepts which will be frequently used later. $\mathbb{R}^{n\times m}$, $\mathbb{R}_+^{n\times m}$ denote the set of all n ×m matrices with elements in $\mathbb{R}$, $\mathbb{R}_+$, respectively. A matrix (in particular, a vector) A with entries in is *nonnegative*, and if so we adopt the notation $A \geq 0$. If, in addition, it has at least one positive entry, A is *positive* ($A > 0$), while if all its entries are positive it is *strictly positive* ($A \gg 0$). Given two matrices A and B, of the same size $A \geq B$, $A > B$ and $A \gg B$ are synonymous of $A - B \geq 0$,

$A - B > 0$ and $A - B \gg 0$, respectively. For any matrix $A \in \mathbb{R}^{n\times n}$, the spectral radius and the spectral abscissa of A is denoted by $\rho(A) = \max\{|\lambda| : \lambda \in \sigma(A)\}$, and $\mu(A) = \max\{\mathrm{Re}\lambda :\ \lambda \in \sigma(A)\}$, respectively, where $\sigma(A) := \{z \in \mathbb{C}:\ \det(zI_n - A) = 0\}$ is the set of all eigenvalues of A. A matrix $A \in \mathbb{R}^{n\times n}$ is called Hurwitz (Schur) if and only if μ(A) < 0 (ρ(A) < 1), respectively. A matrix $A \in \mathbb{R}^{n\times n}$ is called a Metzler matrix if all off-diagonal elements of A are nonnegative.

Let us recall some basic results from the robustness analysis for linear systems to be used in the next sections. Consider a linear continuous-time system:

$$\dot{x}(t) = Ax(t);\ x \in \mathrm{K}^n;\ t \geq 0. \tag{4}$$

Assume that the system (4) is Hurwitz stable that is $\mu(A) < 0$ and is subjected to structured affine perturbations of the form:

$$\dot{x}(t) = (A + D\Delta E)x(t);\ x \in \mathrm{K}^n;\ t \geq 0, \tag{5}$$

where $\Delta \in \mathrm{K}^{l\times q}$ is unknown disturbance matrix, $D \in K^{n\times l}$, $E \in \mathrm{K}^{q\times n}$ are given matrices defining the structure of perturbations. We have the well-known notion of structured stability radius [5] which is defined, for $K = \mathbb{R}, \mathbb{C}$, as:

$$r_K(A; D; E) := \inf\{\|\Delta\| : \Delta \in K^{l\times q},\ A + D\Delta E \text{ not Hurwitz stable}\}.$$

In particular, as has been shown in [12], if $A \in \mathbb{R}^{n\times n}$ is a Hurwitz stable Metzler matrix and $D \in \mathbb{R}_+^{n\times l}; E \in \mathbb{R}_+^{q\times n}$, then the real and the complex structured stability radii of the linear system (4) (which is then a positive system) coincide and can be calculated as:

$$r_{\mathbb{C}}(A, D, E) = r_{\mathbb{R}}(A, D, E) = \frac{1}{\|EA^{-1}D\|}. \tag{6}$$

The following two theorems summarize some spectral properties of Metzler matrices which will be used in that section (see [12]).

Theorem 2.1 (see [1]): Suppose that $A \in \mathbb{R}^{n\times n}$ is a Metzler matrix. Then

(i) (Perron-Frobenius) $\mu(A)$ is an eigenvalue of A and there exists a non-negative eigenvector $x \neq 0$ such that $Ax = \mu(A)x$.

(ii) Given $\alpha \in \mathbb{R}$, there exists a nonzero vector $x \geq 0$ s.t $Ax \geq \alpha x$ if and only if $\mu(A) \geq \alpha$.

(iii) $(tI_n - A)^{-1}$ exists and is nonnegative if and only if $t > \mu(A)$.

The following result is immediated from Theorem 2.1.

Theorem 2.2: Let $A \in \mathbb{R}^{n\times n}$ be a Metzler matrix. Then the following statements are equivalent:

(i) $\mu(A) < 0$;

(ii) $Ap \ll 0$ for some $p \in \mathbb{R}_+^n$, $p \gg 0$;

(iii) A is invertible and $A^{-1} \leq 0$.

3. LOWER BOUND OF STABILITY RADIUS OF SWITCHED POSITIVE LINEAR SYSTEMS

Now, consider a continuous-time switched positive linear system in $\mathbb{R}^{n\times n}$ described by (2). This ensures that if $x(0) = x_0$ belongs to the positive orthant $\mathbb{R}_+^n$, then any switching signal $\sigma \in \Sigma$ the system (2) admits a unique solution $x(t, x_0, \sigma)$, $t \geq 0$ (see [2]).

Definition 3.1 (see [4]): The switched linear system (2) is said to be positive if $x_0 \geq 0$ implies that $x(t, x_0, \sigma) \geq 0$ for all $t \geq 0$.

Definition 3.2 (see [2]): The switched positive linear system (2) is said to be exponentially stable if there exist real constants $C > 0$ and $\beta > 0$ such that all the solutions of (2) satisfy

$$\|x(t, x_0, \sigma)\| \leq Ce^{-\beta t} \|x_0\|, \tag{7}$$

for every $x_0 \in \mathbb{R}_+^n$, $t \geq 0$ an for all switching signal $\sigma \in \Sigma$.

Lemma 3.1 (see [3]): The switched positive linear system (2) is positive if and only if A_k, $k \in \underline{N}$ is a Metzler matrix.

Lemma 3.2 (see [2]): Consider a switched linear positive systems described by (2). If there exists $v \in \mathbb{R}_+^n, v \gg 0$ such that:

$$v^T A_k \ll 0, \quad \forall k \in \underline{N}, \tag{8}$$

then the positive switched linear system (2) is exponentially stable.

According to Lemma 2 in [13], we immediately have a lemma:

Lemma 3.3 (see [13]): Consider a switched positive linear systems described by (2). If there exists $p \in \mathbb{R}_+^n$, $p \gg 0$ such that:

$$A_k p \ll 0, \ \forall k \in \underline{N}, \tag{9}$$

then the switched positive linear system (2) is exponentially stable. Given Theorem 2.2 (ii), the preceding result immediately implies.

Corollary 3.1 (see [13]): If there exists a Hurwitz stable Metzler matrix A_0 such that:

$$A_k \leq A_0, \forall k \in \underline{N}, \tag{10}$$

then the switched positive linear system (2) is exponentially stable for any $\sigma \in \Sigma$.

We assume that the switched positive linear system (2) is exponentially stable and the matrices A_k, $k \in \underline{N}$ of positive subsystems (3) are subjected to affine positive perturbations of the form:

$$A_k \to \hat{A}_k := A_k + D_k \Delta_k E_k, \ k \in \underline{N}, \tag{11}$$

where $D := \{D_k, \ D_k \in \mathbb{R}_+^{n \times l_k}\}$, $\varepsilon := \{E_k, \ E_k \in \mathbb{R}_+^{q_k \times n}\}$, $k \in \underline{N}$ are given matrices defining the structure of the perturbations and $\Delta_k \in \mathbb{R}_+^{l_k \times q_k}$, $k \in \underline{N}$ are unknown disturbances. Then the perturbed system is described by

$$\dot{x}(t) = \hat{A}_{\sigma(t)} x(t), \ \ t \geq 0, \ \sigma \in \Sigma,$$

$$\hat{A}_{\sigma(t)} \in \hat{A} := \left\{A_k + D_k \Delta_k E_k, \Delta_k \in \mathbb{R}_+^{l_k \times q_k}, k \in \underline{N}\right\}, \ t \geq 0. \tag{12}$$

An important question that arises in the robustness analysis of stability for the nominal system (2) subjected to parameter perturbations is how large perturbations $\Delta_k, k \in \underline{N}$ are allowable in the perturbed model (12), without destroying the exponential stability of the system (w.r.t. arbitrary switching signals). To deal with such a question, let us measure the size of perturbations $\Delta := (\Delta_1, \Delta_2, ..., \Delta_N) \in \mathbb{R}_+^{l_1 \times q_1} \times \mathbb{R}_+^{l_2 \times q_2} \times \ldots \mathbb{R}_+^{l_N \times q_N}$ by

$$\|\Delta\|_{\max} := \max_{k \in \underline{N}} \|\Delta_k\|. \tag{13}$$

Definition 3.3: If the switched positive linear system (2) is exponentially stable and is subjected to affine perturbations of the form (11). Then its structured stability radius is defined as:

$$\begin{aligned} r_{\mathbb{R}_+}(\mathrm{A}, D, \varepsilon) := \inf\{\|\Delta\|_{\max} : \Delta = (\Delta_1, \Delta_2, ..., \Delta_N), \Delta_k \in \mathbb{R}_+^{l_k \times q_k}, k \in \underline{N}, \\ \exists \sigma \in \Sigma \textit{ such that } (12) \textit{ is not } \exp\textit{onentially stable}\} \end{aligned} \tag{14}$$

where the norm $\|.\|_{\max}$ of perturbations Δ is defined by (13). If $D_k = E_k = I,\ k \in \underline{N}$ (the case of unstructured perturbations) then we put $r_{\mathbb{R}_+}(\mathrm{A}) = r_{\mathbb{R}_+}(\mathrm{A}, I, I)$.

It is well-known that if $A_0 \in \mathbb{R}^{n \times n}$ is a Hurwitz stable Metzler matrix then its complex and real structured stability radii are equal and can be calculated by a simple formula. Below we will use the Corollary 3.1 to get a more explicit formula for computing a lower bound of the real stability radius $r_{\mathbb{R}_+}(\mathrm{A}, D, \varepsilon)$ of the switched positive linear systems (2).

Theorem 3.1: If the switched positive linear system (2) is exponentially stable and is subjected to affine perturbations of the form (11). Assume, moreover, that there exist a Hurwitz stable Metzler matrix $A_0 \in \mathbb{R}^{n \times n}$ such that:

$$A_k \le A_0,\ \forall k \in \underline{N}\ , \tag{15}$$

then the real stability radius of the switched positive linear system (2) satisfies the following estimation:

$$\frac{1}{\max\limits_{i,j \in \underline{N}} \|E_i A_0^{-1} D_j\|} \le r_{\mathbb{R}_+}(\mathrm{A}, D, \varepsilon) \le \min_{k \in \underline{N}} \left\{ r_{\mathbb{R}}\left(A_k, D_k, E_k\right) \right\}. \tag{16}$$

where, for each $k \in \underline{N}$, $r_{\mathbb{R}}\left(A_k, D_k, E_k\right) = \dfrac{1}{\|E_k A_k^{-1} D_k\|}$ is the structured stability radius of the subsystem (3). To prove the lower bound, or equivalently if

$$\|\Delta\|_{\max} < \frac{1}{\max\limits_{i,j \in \underline{N}} \|E_i A_0^{-1} D_j\|} \tag{17}$$

then the perturbed system (12) is still exponentially stable with any $\sigma \in \Sigma$.

We claim that $\mu(A_0 + D_i\Delta_i E_i) < 0, i \in \underline{N}.$

Since A_0 is a Metzler matrix and D_i, Δ_i, E_i are nonnegative for any $i \in \underline{N}$, $A_0 + D_i\Delta_i E_i$ is a Metzler matrix. We will show that $\mu_0 = \mu(A_0 + D_i\Delta_i E_i) < 0, \mathrm{i} \in \underline{N}$. Assume on the contrary that $\mu_0 \geq 0$. By using the Perron Frobenius Theorem 2.1 (i), there exists $x_0 \in \mathbb{R}_+^n, x_0 \neq 0$ such that:

$$(A_0 + D_i\Delta_i E_i)x_0 = \mu_0 x_0,\ i \in \underline{N}.$$

By assumption, $\mu(A_0) < 0.$ Thus $(\mu_0 I_n - A_0)$ is invertible and this implies:

$$(\mu_0 I_n - A_0)^{-1} D_i\Delta_i E_i x_0 = x_0,\ \ i \in \underline{N}. \tag{18}$$

Let i_0 be an index such that $\left\|E_{i_0}x_0\right\| = \max_{i\in\underline{N}}\left\|E_i x_0\right\|$. It follows from (18) that $\left\|E_{i_0}x_0\right\| > 0$. Multiply both sides of (18) from the left by E_{i_0} to get:

$$E_{i_0}(\mu_0 I_n - A_0)^{-1} D_i\Delta_i E_i x_0 = E_{i_0}x_0,\ i \in \underline{N}.$$

It follows that:

Since $\left\|\Delta\right\|_{\max} \geq \left\|\Delta_i\right\|$, for all $i \in \underline{N}$, we have:

$$\left\|E_{i_0}(\mu_0 I_n - A_0)^{-1} D_i\right\| \left\|\Delta\right\|_{\max} \left\|E_i x_0\right\| \geq \left\|E_i x_0\right\|,\ \ i \in \underline{N}.$$

Thus,

$$\max_{i,j\in\underline{N}}\left\|E_i(\mu_0 I_n - A_0)^{-1} D_j\right\| \left\|\Delta\right\|_{\max} \left\|E_{i_0}x_0\right\| \geq \left\|E_{i_0}x_0\right\|.$$

Equivalently,

$$\max_{i,j\in\underline{N}}\left\|E_i(\mu_0 I_n - A_0)^{-1} D_j\right\| \left\|\Delta\right\|_{\max} \geq 1, i \in \underline{N}. \tag{19}$$

On the other hand, the resolvent identity gives:

$$(-A_0)^{-1} - (\mu_0 I_n - A_0)^{-1} = \mu_0(-A_0)^{-1}(\mu_0 I_n - A_0)^{-1}$$

Since, $\mu_0 \geq 0, (-A_0)^{-1} \geq 0$ and $(\mu_0 I_n - A_0)^{-1} \geq 0$, we have:

$$E_i(-A_0)^{-1}D_j \geq E_i(\mu_0 I_n - A_0)^{-1}D_j \geq 0 \text{, for any } i, j \in \underline{N}\text{ .}$$

By monotonicity of an operator norm associated with a given pair of monotonic vector norms, we have:

$$\left\|E_i A_0^{-1} D_j\right\| \geq \left\|E_i(\mu_0 I_n - A_0)^{-1} D_j\right\|, i, j \in \underline{N}. \tag{20}$$

From (19) and (20), we get:

$$\max_{i,j\in \underline{N}} \left\| E_i A_0^{-1} D_j \right\| \left\| \Delta \right\|_{\max} \geq 1.$$

It implies that: $\left\| \Delta \right\|_{\max} \geq \dfrac{1}{\max_{i,j\in \underline{N}} \left\| E_i A_0^{-1} D_j \right\|}$.

However, this conflicts with (17), which completes the proof.

Example 3.1: Consider the switched positive linear system (2) with $\underline{N} = \{1, 2\}$, subjected to affine perturbations of the form (11), where

$$A_1 = \begin{pmatrix} -1.03 & 0 & 0 \\ 0.12 & -1.04 & 0 \\ 0 & 2 & -2 \end{pmatrix}, D_1 = \begin{pmatrix} 1.01 \\ 0 \\ 1.02 \end{pmatrix}, E_1 = \begin{pmatrix} 1.01 & 0.01 & 0.33 \end{pmatrix};$$

$$A_2 = \begin{pmatrix} -2.01 & 0.1 & 0 \\ 0.02 & -1.04 & 0 \\ 0 & 1.02 & -2 \end{pmatrix}, D_2 = \begin{pmatrix} 1.03 \\ 0.01 \\ 0 \end{pmatrix}, E_2 = \begin{pmatrix} 0.12 & 0.21 & 1.03 \end{pmatrix}.$$

It is easy to verify that the Metzler matrices $A_k, k = 1, 2$ are Hurwitz stable and (15) holds for

$$A_0 = \begin{pmatrix} -1.03 & 0.1 & 0 \\ 0.12 & -1.04 & 0 \\ 0 & 2 & -2 \end{pmatrix}.$$

From Theorem 3.1, we obtain the following bound for the system's structured stability radius $0.8272 \leq r_{\mathbb{R}_+}(\mathrm{A}, D, \varepsilon) \leq 0.8353$.

By Theorem 3.1, the preceding result immediately implies.

Corollary 3.2: Assume that matrices $A_k \in \mathbb{R}^{n\times n}, k \in \underline{N}$ are all Hurwitz stable and subjected to unstructured perturbations:

$$A_k \to \hat{A}_k := A_k + \Delta_k, k \in \underline{N}. \tag{21}$$

If there exists a Hurwitz stable Metzler matrix A_0 such that $A_k \leq A_0, k \in \underline{N}$ and $\Delta_k \in \mathbb{R}_+^{nxn}$, then the stability radius of the switched positive linear system (2) with unstructured perturbations (21) is $r_{\mathbb{R}_+}(A) \geq r_{\mathbb{R}}(A_0)$.

Proof. We have $r_{\mathbb{R}_+}(A) = r_{\mathbb{R}_+}(A, I, I) \geq \dfrac{1}{\max_{i,j\in \underline{N}} \left\| E_i A_0^{-1} D_j \right\|} = \dfrac{1}{\left\| A_0^{-1} \right\|} = r_{\mathbb{R}}(A_0)$.

This completes the proof.

4. CONCLUSION

We have presented a new approach to study the robustness of exponential stability switched positive linear systems, based on the concept of stability radius with respect to structured affine of the subsystem's matrices. As the main contributions, we obtain formulas to estimate this radius for different approaches from the paper [13]. In the case of unstructured, the stability radius of positive switched linear system A_0 is just a lower bound.

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TÓM TẮT

BÁN KÍNH ỔN ĐỊNH CỦA HỆ CHUYỂN MẠCH TUYẾN TÍNH DƯƠNG

Bài báo nghiên cứu bán kính ổn định dựa trên ổn định mũ của hệ chuyển mạch tuyến tính dương. Kết quả chính đưa ra là chặn dưới và chặn trên của bán kính ổn định nhiễu dương có cấu trúc affin với giả thiết tất cả các hệ con dương có hàm Lyapunov đồng dương tuyến tính chung. Ví dụ đưa ra để minh họa cho kết quả đạt được.

Từ khóa: Hệ chuyển mạch tuyến tính; Hệ chuyển mạch tuyến tính dương; Ổn định vững.

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