Synthesis of control command for single-channel flying equipment by pulse width - amplitude modulation method

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ABSTRACT

The article examines the method of synthesizing control commands of single-channel flying equipment (TBBMK) with infrared homing heads currently in use; These methods require the rudder angle to be able to change from the maximum negative value to the maximum positive value continuously or in steps. Through analyzing the mathematical model describing the flight dynamics of TBBMK based on the assumption of actual flight conditions, the authors propose to control TBBMK according to the pulse width - amplitude modulation (PWAM) method.

Keywords: Single-channel flying equipment; Command control; Infrared seeker; Modulation method.

1. INTRODUCTION

In the current air defense system, TBBMK infrared seeker plays a critical role in air defense operations,... Many countries in the world have been promoting investment, research and development. To meet the requirements of destroying targets at long range, actively attacking the enemy from all directions, with high precision, researching control systems to create flexible, resilient, high-quality TBB generations is an extremely urgent issue for us. Studies [1, 2] have presented a comprehensive overview of the features, navigation-control algorithms of the TBBMK and technological solutions, requiring optimal design to achieve efficiency. The work [3, 4] presented has proven the law of steering control following the sign function that Russian and American scientists have applied to design the TBBMK control system that controls according to the error combined with the principle of fast action always keeping the steering vane at either the maximum or minimum position. The above works have mentioned and solved the problems in the old generation single-channel flying equipment (Strela-2, Igla-1, Igla) with relay-type steering vane with a control command structure based on the principle of limited pulse width modulation, regardless of the magnitude of the control command provided, the steering vane always deviates to the maximum angle leading to energy loss the TBB is subjected to a series of vibrations with maximum amplitude during the flight. Works [5-8] indicate a novel control strategy combining both pulse width and amplitude modulation (PWAM) strategies has been proposed for a novel selective harmonic rejection power converter (SHEPWAM) for drive applications and to minimize the power dissipation of the power converter.

The flight control system guides the TBBMK to the target using a proportional approach. In the construction of the equipment on board, the TBBMK control principle is used when the TBB rotates around the longitudinal axis and there is an actuator (steering machine) working in relay mode (PM) and in linear mode (continuously moving steering vane) allowing the rotation of the TBB to create a force to control the TBB in all directions of space. To improve the quality of actuator control, modern control command synthesis methods are applied. In this paper, the authors will focus on the application of pulse width amplitude modulation (PWAM) method in synthesizing TBBMK control commands.

2. RESEARCH MODELS AND METHODS

2.1. Single-channel flying equipment model

A single-channel flying equipment is a missile that has only one control mechanism that produces the angle of attack or glide angle. In this type of missile, the aerodynamic steering vane mechanism rotates through an angle in the OXYZ coordinate system is show in figure 1. For missiles using aerodynamic effects, the rotation of the steering mechanism creates aerodynamic force, the aerodynamic force creates moment of power that rotates the missiles. When the missile rotates, the angle of attack changes, causing a normal force to appear, changing the flight direction.

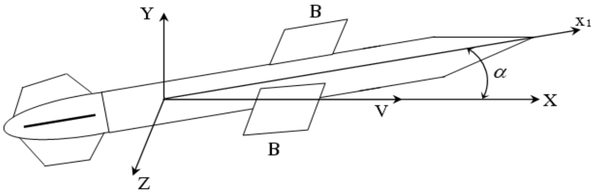

Figure 1. *Single channel control flying equipment type.*

According to documents [3], the general system of equations describing the motion of the center of mass of the TBBMK is as follows:

$$\begin{cases} mV\dfrac{d\theta}{dt} = P(\sin\alpha\,.\cos\gamma_c + \cos\alpha\,.\sin\beta\,.\sin\gamma_c) + Y\cos\gamma_c - Z\sin\gamma_c - G\cos\theta \\ -mV\cdot\cos\theta\cdot\dfrac{d\Psi}{dt} = P(\sin\alpha\,.\sin\gamma_c - \cos\alpha\,.\sin\beta\,.\cos\gamma_c) + Y\sin\gamma_c + Z\cos\gamma_c \\ \dfrac{dx}{dt} = V.\cos\theta\,.\cos\Psi \\ \dfrac{dh}{dt} = V\sin\theta \\ \dfrac{dz}{dt} = -V\cos\theta\,.\sin\Psi \end{cases}$$

Transform the above system of equations to get:

$$\begin{cases} mV\frac{d\theta}{dt} = \left(P\cdot\frac{C_y^\delta}{C_y^\alpha} + K_\alpha\right).\delta_B.\cos\omega_x t - G\cos\theta = K_\theta.\delta_B.\cos\omega_x t - G\cos\theta \\ -mV\cdot\cos\theta\cdot\frac{d\Psi}{dt} = (P\cdot\frac{C_y^\delta}{C_y^\alpha} + K_\alpha)\cdot\delta_B\cdot\sin\omega_x t = K_\Psi\cdot\sin\omega_x t\cdot\delta_B \\ \frac{dx}{dt} = V.\cos\theta\,.\cos\Psi \\ \frac{dh}{dt} = V\sin\theta \\ \frac{dz}{dt} = -V\cos\theta\,.\sin\Psi \end{cases} \quad \text{(I)}$$

Where:

$$K_\theta = (P\frac{C_y^\delta}{C_y^\alpha} + K_\alpha) = K_{p\alpha} \quad (1)$$

$$K_\Psi = -(P\frac{C_y^\delta}{C_y^\alpha} + K_\alpha) = -K_{p\alpha} \quad (2)$$

The essence of the control process is the process of intentionally changing the center of mass of the missile. Changing the position of the control mechanism causes the missile to rotate. Missile rotation causes the thrust vector to change direction and change the angle of attack, leading to a change in lift. This change is the fundamental factor to create a change in the total force acting on the center of mass of the missile. The synthesis of control commands to change the center of mass of a single-channel missile according to the altitude channel and the direction channel is presented according to the methods below.

2.2. Method of synthesizing control commands for single-channel flying equipment

Depending on the specified guidance method, the generation of commands for controlling the center of mass via the altitude channel and the direction channel is determined by the following functions:

$$u_1 = f_h(t) \quad (3)$$

$$u_2 = f_z(t) \quad (4)$$

The functions $f_h(t)$ and $f_z(t)$ are formed on the basis of information about the target and information about the missile itself. If we assume that $f_h(t)$ and $f_z(t)$ change slowly, then the cutoff frequency of their characteristic spectrum is much greater than the frequency of rotation around the longitudinal axis of the missile ω_x. Then, for one period of rotation, $f_h(t)$ and $f_z(t)$ are considered constant. we will consider the problem of controlling a missile flying in a small cone. Then the model describing the process of changing the center of mass of the missile is represented by a system of equations (I). It is seen that to control the altitude h it is necessary to change the orbital inclination angle θ, and to change Z it is necessary to change the orbital direction angle Ψ. There is relative independence here, a change in θ does not affect Ψ and vice versa. However, the missile has only one control channel (the mechanism δ_B), so changing δ_B will change both the orbital inclination angle θ and the orbital direction angle Ψ.

** Controlling TBBMK by pulse amplitude variation method:*

If we assume that the signals $f_h(t)$, $f_z(t)$ change to the limit, then the control structure changes continuously in the range from the negative value $-\delta^*$ to the positive value $+\delta^*$. It is assumed that the control process δ_B is such that the change in θ is proportional to $f_h(t)$=a, the change in Ψ is proportional to $f_z(t)$=b, which can be done using the following solution:

- When $2l\pi - \tau \le \omega_x t \le 2l\pi + \tau$ then $\delta_B(t) = k.u_1$;
- When $(2l+1)\pi - \tau \le \omega_x t \le (2l+1)\pi + \tau$ then $\delta_B(t) = -k.u_1$;
- When $\frac{\pi}{2} + 2l\pi - \tau \le \omega_x t \le \frac{\pi}{2} + 2l\pi + \tau$ then $\delta_B(t) = k.u_2$;
- When $-\frac{\pi}{2} + 2l\pi - \tau \le \omega_x t \le -\frac{\pi}{2} + 2l\pi + \tau$ then $\delta_B(t) = -k.u_2$; With l=0, 1, 2,...
- When $\omega_x t$ different from the above expressions, then $\delta_B(t) = 0$;

The average change of θ during one missile rotation around the longitudinal axis is proportional to $f_h(t)$ and the change of Ψ is proportional to $f_z(t)$. The change diagram of δ_B corresponding to $u_1 \ge 0, u_2 \ge 0$ is shown in figure 2.

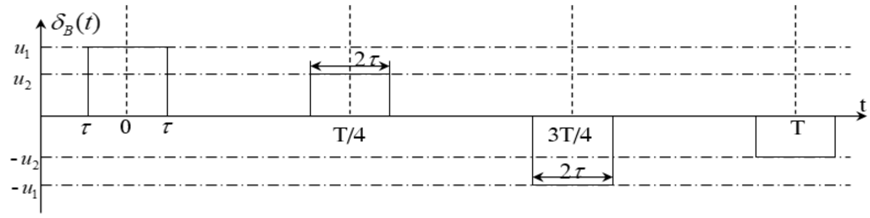

***Figure 2.** The rule for changing the rudder angle when a>0, b>0.*

From equation 1 of system (I) we have:

$$\frac{d\theta}{dt} = \frac{K_\theta}{mV} cos\omega_x\, t\,.\,\delta_B(t) - \frac{g}{V} \cos\theta$$

The solution of the above differential equation is the sum of two solutions of the following differential equations:

$$\frac{d\theta}{dt} = \frac{K_\theta}{mV} cos\omega_x\, t\,.\,\delta_B(t) \quad (5)$$

$$\frac{d\theta}{dt} = -\frac{g}{V}\cos\theta \tag{6}$$

The right side of (5) is an AC signal due to the cos(.) function. According to the theory of differential equation the solution (5) will be:

$$\theta = \frac{K_\theta}{mV}\int_0^t cos\omega_x\,\tau_1 \cdot \delta_B(\tau_1)\cdot d\tau_1 \tag{7}$$

Expression (7) is the integration term, has harmonic filtering properties and reacts strongly to slowly varying signals. So the right side (5) can be replaced by the average quantity. That is, (5) will be replaced by the following equivalent equation:

$$\frac{d\theta}{dt} = \frac{K_\theta}{mV}.\delta_{BTB}(t) - \frac{g}{V}\cos\theta \tag{8}$$

In which $\delta_{BTB}(t)$ is calculated as follows:

$$\delta_{BTB}(t) = \frac{1}{T}\int_{(i-1)T}^{iT} cos\omega_x\,\tau_1\cdot\delta_B(\tau_1)\cdot d\tau_1,\ \text{if } iT \le t \le (i+1)T$$

With the change rule of $\delta_B(t)$ as above (figure 2), $\delta_{BTB}(t)$ is calculated as follows:

$$\delta_{BTB}(t) = \frac{1}{T}[u_1\int_{(i-1)T}^{(i-1)T+\tau} cos\omega_x\,\tau_1\cdot d\tau_1 + u_2\int_{(i-1)T+\frac{T}{4}-\tau}^{(i-1)T+\frac{T}{4}+\tau} cos\omega_x\,\tau_1\cdot d\tau_1$$
$$-u_1\int_{(i-1)T+\frac{T}{2}-\tau}^{(i-1)T+\frac{T}{2}+\tau} cos\omega_x\,\tau_1\cdot d\tau_1 - u_2\int_{(i-1)T+\frac{3T}{4}-\tau}^{(i-1)T+\frac{3T}{4}+\tau} cos\omega_x\,\tau_1\cdot d\tau_1$$
$$+u_1\int_{(i-1)T+T-\tau}^{(i-1)T+T} cos\omega_x\,\tau_1\cdot d\tau_1] = \frac{4\cdot u_1}{T.\omega_x}\cdot sin\omega_x\,\tau = \frac{2.u_1}{\pi}sin\omega_x\,\tau \tag{9}$$

Carrying out a similar transformation, the right side of the second equation of system (I) can be replaced by the average quantity, that is:

$$\frac{d\Psi}{dt} = \frac{K_\Psi}{mV}\delta_{HTB} \tag{10}$$

In there: $\delta_{HTB}(t) = \frac{1}{T}\int_{(i-1)T}^{iT} sin\omega_x\,\tau_1\cdot\delta_B(\tau_1)d\tau_1$, if $iT \le t \le (i+1)T$

Similar transformation: $\delta_{HTB} = \frac{2.u_2}{\pi}cos\left(\frac{\pi}{2} - \omega_x\tau\right)$ (11)

In the case of fixed τ, from expressions (9) and (11), it can be seen that the change in the control command of the tilt angle and the trajectory angle in a missile rotation cycle is proportional to the input control values u_1 and u_2. Since there is one channel, the largest possible value of τ is T/8, that is: $\tau \le \frac{T}{8}$, So:

$$sin\omega_x\,\tau \le sin\,\pi/4 = \sqrt{2}/2,\ cos\left(\frac{\pi}{2} - \omega_x\tau\right) \le cos\,\pi/4 = \sqrt{2}/2 \tag{12}$$

Thus, we can consider the two equations 1 and 2 of system (I) equivalent to two independent stages with input signals u_1 and u_2:

$$mV\cdot\frac{d\theta}{dt} = K_\theta\cdot\frac{2\,sin\omega_x\,\tau}{\pi}\cdot u_1 - G\cdot\cos\theta \tag{13}$$

$$mV\cdot\frac{d\Psi}{dt} = K_\Psi\cdot\frac{2\,cos\left(\frac{\pi}{2}-\omega_x\tau\right)}{\pi}\cdot u_2 \tag{14}$$

In the above pulse encoding command structure, the width is kept the same, the pulse amplitude changes according to the altitude command and the direction command. The above model can be linearized by the averaging method as shown in figure 3.

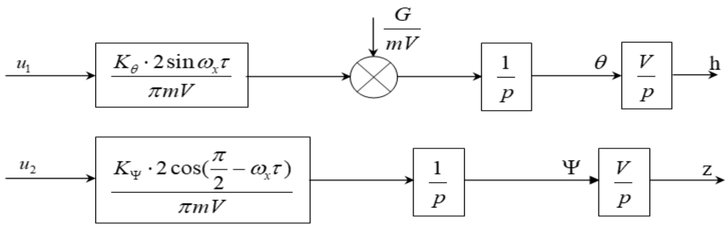

Figure 3. *Model of the altitude and direction control process of TBBMK.*

** Controlling TBBMK by changing the amplitude and phase of the aerodynamic steering vane angle:*

TBBMK control method based on the idea: When TBB rotates the plane containing the steering vane coincides with the horizontal direction, the steering vane rotation angle is proportional to the altitude command. When the steering vane plane coincides with the vertical direction, the steering vane rotation angle is proportional to the lateral drift command. The signs of those angles depend on whether the command is up or down or left or right. According to document [3] it is stated:

Lemma 1: *If in a rotation cycle, the steering vane maintains a constant value $\delta_B = \delta^* = const$, then the average value of the control command for the yaw angle and the trajectory angle is 0, TBB oscillates around the ballistic trajectory.*

The ballistic trajectory is the trajectory determined by the forces of thrust, gravity and aerodynamic drag. That is, the angles of attack α and the glide angles β are always 0 ($\alpha \equiv \beta \equiv 0$). In there, the normal force due to the buoyancy effect and the aerodynamic effect is absent, the aerodynamic force is purely a drag force.

Consider two equations (8) and (10). Indeed, for all $i = 0,1,2,\dots n,$ the average orbital tilt angle control command generated by the control effect in one rotation cycle is:

$$\delta_{BTB}(t) = \int_{(i-1)T}^{iT} cos\omega_x \tau_1 \cdot \delta_B(\tau_1)d\tau_1 = \delta^* \int_{(i-1)T}^{iT} cos\omega_x \tau_1 . d\tau_1 = \delta^* \int_0^T cos\omega_x \tau_1 . d\tau_1 = 0$$

Similarly, the average orbital angle control command over one rotation period is:

$$\delta_{HTB}(t) = \int_{(i-1)T}^{iT} sin\omega_x \tau_1 . \delta_B(\tau_1). d\tau_1 = \int_{(i-1)T}^{iT} \delta^* sin\omega_x \tau_1 . d\tau_1 = \delta^* \int_0^T sin\omega_x \tau_1 . d\tau_1 = 0$$

Thus the trajectory of TBB oscillates around the ballistic trajectory:

$$mV \frac{d\theta}{dt} = K_\theta \, cos\omega_x \, t \, . \delta_B(t) - G \, cos \, \theta = K_\theta \delta_{BTB} - G \, cos \, \theta = -G \, cos \, \theta \quad (15)$$

$$mV. cos \, \theta \cdot \frac{d\Psi}{dt} = K_\Psi \, sin\omega_x \, t \, . \delta_B(t) = K_\Psi \delta_{HTB} = 0 \quad (16)$$

Lemma 2*: If in a rotation cycle, the steering vane angle changes sign once, the time interval at positive value is equal to the time interval at negative value and equal to 1/2 of the cycle, then the average normal force amplitude depends on the value δ^*, the ratio of lift force Y and yaw force Z depends on the phase of the steering vane rotation command.*

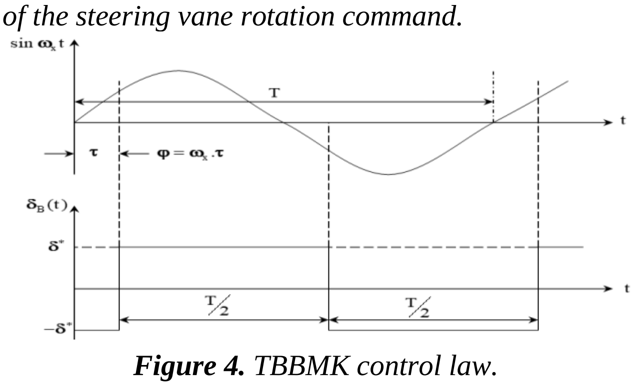

Figure 4. *TBBMK control law.*

The time point is calculated when the symmetry plane of the TBB coincides with the vertical plane, then the control law according to the rule has the form as shown in figure 4.

The average elevation force (controlling the orbital tilt angle) during one TBB rotation cycle is:

$$F_{hTB} = mV\frac{d\theta}{dt}\Big|_{TB} = K_\theta co\omega_x t.\delta_b(t)|_{TB} = K_{p\alpha}.\delta_{BTB} = K_{p\alpha}.\frac{1}{T}\int_{(i-1)T}^{iT} cos\omega_x\,\tau_1\,.\,\delta_B(\tau_1).d\tau_1$$

$$= -\int_{(i-1)T}^{(i-1)T+\tau} K_{p\alpha}.\delta^* cos\omega_x\,\tau_1\,.\,d\tau_1 + \int_{(i-1)T+\tau}^{(i-1)T+\tau+\frac{T}{2}} K_{p\alpha}.\delta^* cos\omega_x\,\tau_1\,.\,d\tau_1 -$$
$$\int_{(i-1)T+\tau+\frac{T}{2}}^{(i-1)T+T} K_{p\alpha}.\delta^* cos\omega_x\,\tau_1\,.\,d\tau_1 = -4K_{p\alpha}.\delta^* sin\omega_x\,\tau \quad (17)$$

The average drift force (force controlling the trajectory angle) during one TBB rotation cycle:

$$F_{ngTB} = mV\frac{dV}{dt}\Big|_{TB} = K_\Psi\, sin\omega_x\, t\,.\,\delta_B(t)|_{TB} = K_{p\alpha}\delta_{HTB}$$
$$= K_{p\alpha}\int_{(i-1)T}^{iT} sin\omega_x\,\tau_1\,.\,\delta_B(\tau_1).\,d\tau_1 = 4K_{p\alpha}.\delta^* cos\omega_x\,\tau \quad (18)$$

So the average normal force controlling the trajectory of the missile center of mass will be:

$$\vec{F}_{TB} = \overrightarrow{F_{hTB}} + \vec{F}_{ngTB} \quad (19)$$

So: $F_{TB} = \sqrt{{F_{hTB}}^2 + {F_{ngTB}}^2}$ then $F_{TB} = 4K_{p\alpha}.\delta^*$ (20)

The angle between vector F and the horizontal plane is the angle: $\Delta\varphi = \omega_x\tau$ (21)

From (20) and (21) it can be seen that the average change of the normal force vector is proportional to the control blade rotation angle, the phase of the average vector is the phase of the control command. Easy to see $\Delta\varphi = 0$ then TBB moves up, $\Delta\varphi = \pi$ then TBB moves down, When $\Delta\varphi = \frac{\pi}{2}$ or $\Delta\varphi = \frac{3\pi}{2}$ then TBB moves left or right. Thus, the TBBMK control command synthesis is performed by simultaneously changing the phase and amplitude of the steering vane pulse.

$$\left.\begin{array}{l} \delta^* = \sqrt{u_1^2 + u_2^2} \\ \Delta\varphi = arccos\frac{u}{\sqrt{u_1^2+u_2^2}} \end{array}\right\} \quad (22)$$

Based on the values of δ^* and $\Delta\varphi$, the control pulse for the steering vane blade are generated as shown in figure 4. The model diagram of the control process of the orbital tilt angle and the orbital direction angle is shown in figure 5.

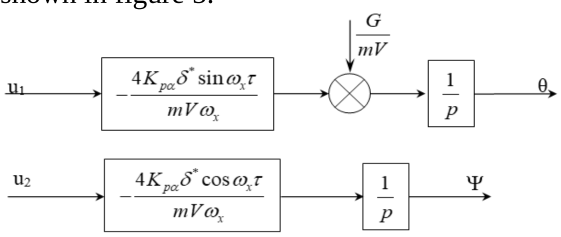

Figure 5. *Tilt and trajectory angle control model.*

** Controlling TBBMK by pulse width modulation method:*

According to [3, 4] the average deviation value F_{hTB}

$$F_{hTB} = mV\frac{d\theta}{dt}\Big|_{TB} = K_{p\alpha}co\omega_x t.\delta_B(t)\Big|_{TB} = K_{p\alpha}\delta_{BTB} = K_{p\alpha}\frac{1}{T}\int_{iT}^{iT+T} cos\omega_x\,\tau_1\,.\,\delta_B(\tau_1)d\tau_1$$

Let $x = \omega_x t$, with $dx = \omega_x dt$, let $\varphi = \omega_x\phi$, $\Delta\varphi = \omega_x\tau$ it is easy to see $(\omega_x T) = 2\pi$. With the above setting, the expression to calculate F_{hTB} will have the following form:

$$F_{hTB} = \frac{K_{p\alpha}\delta^*}{2\pi}\sin\varphi.\cos\Delta\varphi \quad (23)$$

The average horizontal control force has the following form:

$$F_{ngTB} = -\frac{K_{p\alpha}\delta^*}{2\pi} cos\,\varphi\, cos\,\Delta\,\varphi \quad (24)$$

The general force will be: $F = \sqrt{F_{hTB}^2 + F_{ngTB}^2} = \frac{K_{p\alpha}\delta^*}{2\pi} cos\,\Delta\varphi$ (25)

From expressions (23), (24) and (25) see that:

- The magnitude of the force acting on a flying object depends only on the value of $\Delta\varphi$, independent of angle φ. The direction of the force depends on the angle φ. So, changing φ and $\Delta\varphi$ simultaneously can create the desired flying control forces F_{hTB}, F_{ngTB}.

3. SYNTHESIS OF CONTROL COMMAND BY PULSE WIDTH - AMPLITUDE MODULATION

Based on the study of methods for synthesizing TBBMK control commands, a mathematical description of TBBMK control according to the pulse width - amplitude modulation method is built. Specifically:

Suppose the control lever of single-channel flying equipment has a range channel proportional to some quantity Δh and a direction channel proportional to some quantity Δz. The values Δh, Δz can be false information provided by the control station or by the homing head. Those average forces acting on TBB are the normal forces that change the curvature of the orbit. In the altitude channel, there is also a gravitational force (mg) acting on TBB but only consider two components that make up the normal force. The basic problem is to perform the δ_B instruction synthesis so that:

$$F_{hTB} = k_1 \cdot u_1 \quad (26)$$

$$F_{ngTB} = k_2 \cdot u_2 \quad (27)$$

The total force acting on TBB is:

$$\vec{F} = \vec{F}_{hTB} + \vec{F}_{ngTB} \quad (28)$$

If the angle between the horizontal force component vector and the resultant vector is φ, then:

$$sin\,\varphi = \frac{F_{hTB}}{F} \quad (29)$$

Thus, it is necessary to synthesize the control command δ_B so that the average resultant force is equal to the value F, and the angle between it and the horizontal is φ.

Consider the control command $\delta_B(t)$ has the form as shown in figure 6. The mathematical description is as follows:

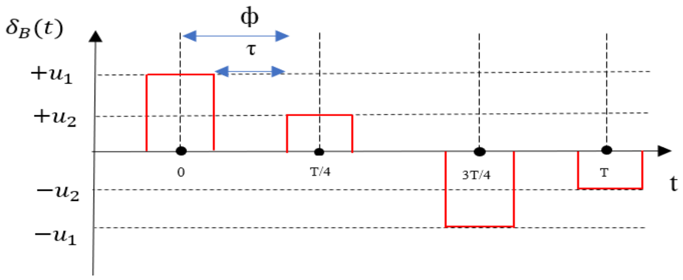

Figure 6. *TBBMK control command format.*

- When $iT - (\phi - \tau) \le t \le iT + (\phi - \tau)$ then $\delta_B(t) = k.u_1$;
- When $iT + \frac{T}{4} - (\phi - \tau) \le t \le iT + \frac{T}{4} + (\phi - \tau)$ then $\delta_B(t) = -k.u_1$;

- When $iT + \frac{T}{2} - (\phi - \tau) \le t \le iT + \frac{T}{2} + (\phi - \tau)$ then $\delta_B(t) = k.u_2$;
- When $iT + \frac{3T}{4} - (\phi - \tau) \le t \le iT + \frac{3T}{4} + (\phi - \tau)$ then $\delta_B(t) = -k.u_2$;
- When t is different from the above expressions then $\delta_B(t) = 0$;

From the system of equations (I) transformed similarly to expressions (5), (6), (7) and (8) received:

$$\delta_{BTB}(t) = \frac{1}{T}\int_{(i-1)T}^{iT} cos\omega_x \tau_1 \cdot \delta_B(\tau_1) \cdot d\tau_1, \text{ if } iT \le t \le (i+1)T \quad (30)$$

With the change rule of $\delta_B(t)$ as shown in Fig. 6, $\delta_{BTB}(t)$ is calculated as follows:

$$\delta_{BTB}(t) = \frac{1}{T}[u_1 \int_{(i-1)T}^{(i-1)T+(\phi-\tau)} cos\omega_x \tau_1 \cdot d\tau_1 + u_2 \int_{(i-1)T+\frac{T}{4}-(\phi-\tau)}^{(i-1)T+\frac{T}{4}+(\phi+\tau)} cos\omega_x \tau_1 \cdot d\tau_1$$
$$- u_1 \int_{(i-1)T+\frac{T}{2}-(\phi-\tau)}^{(i-1)T+\frac{T}{2}+(\phi-\tau)} cos\omega_x \tau_1 \cdot d\tau_1 - u_2 \int_{(i-1)T+\frac{3T}{4}-(\phi-\tau)}^{(i-1)T+\frac{3T}{4}+(\phi-\tau)} cos\omega_x \tau_1 \cdot d\tau_1$$
$$+ u_1 \int_{(i-1)T+T-(\phi-\tau)}^{(i-1)T+T} cos\omega_x \tau_1 \cdot d\tau_1]$$
$$= \frac{u_1}{T.\omega_x}(sin\omega_x(\phi - \tau) - sin\,0) + \frac{u_2}{T.\omega_x}\left[sin(\frac{\pi}{2} + \omega_x(\phi - \tau)) - sin(\frac{\pi}{2} - \omega_x(\phi - \tau))\right]$$
$$- \frac{u_1}{T.\omega_x}[sin(\pi + \omega_x(\phi - \tau)) - sin(\pi - \omega_x(\phi - \tau))]$$
$$- \frac{u_2}{T.\omega_x}\left[sin(\frac{3\pi}{2} + \omega_x(\phi - \tau)) - sin(\frac{3\pi}{2} - \omega_x(\phi - \tau))\right]$$
$$+ \frac{u_1}{T.\omega_x}[sin\,2\pi - sin(2\pi - \omega_x(\phi - \tau))]$$

So: $sin(\frac{\pi}{2} + \omega_x(\phi - \tau)) = sin(\frac{\pi}{2} - \omega_x(\phi - \tau))$ and $sin(\frac{3\pi}{2} + \omega_x(\phi - \tau)) = sin(\frac{3\pi}{2} - \omega_x(\phi - \tau))$

So the 2nd and 4th terms of the above expression are 0.

So: $sin(\pi + \omega_x(\phi - \tau)) = -sin\omega_x(\phi - \tau)$

$$sin(\pi - \omega_x(\phi - \tau)) = sin\omega_x(\phi - \tau)$$
$$sin(2\pi - \omega_x(\phi - \tau)) = -sin\omega_x(\phi - \tau)$$

Then: $\delta_{BTB} = \frac{u_1}{T.\omega_x}.[sin\omega_x(\phi - \tau) + 0 + sin\omega_x(\phi - \tau) + sin\omega_x(\phi - \tau) + 0 + sin\omega_x(\phi - \tau)]$

$$= \frac{4 \cdot u_1}{T.\omega_x} \cdot sin\omega_x(\phi - \tau) = \frac{4 \cdot u_1}{T.2\pi.f} \cdot sin\omega_x(\phi - \tau) = \frac{2.u_1}{\pi} sin\omega_x(\phi - \tau) = \frac{2.u_1}{\pi} sin(\varphi - \Delta\varphi) \quad (31)$$

Carrying out the same transformation as above, the right side of the second equation of system (I) can be replaced by the average quantity, that is: $\frac{d\Psi}{dt} = \frac{K_\Psi}{mV}\delta_{HTB}$ (32)

Where: $\delta_{HTB}(t) = \frac{1}{T}\int_{(i-1)T}^{iT} sin\omega_x \tau_1 \cdot \delta_B(\tau_1) d\tau_1,$ nếu $iT \le t \le (i+1)T$

$$= \frac{1}{T}[u_1 \int_{(i-1)T}^{(i-1)T+(\phi-\tau)} sin\omega_x \tau_1\, d\tau_1 + u_2 \int_{(i-1)T+\frac{T}{4}-(\phi-\tau)}^{(i-1)T+\frac{T}{4}+(\phi-\tau)} sin\omega_x \tau_1 \cdot d\tau_1$$
$$- u_1 \int_{(i-1)T+\frac{T}{2}-(\phi-\tau)}^{(i-1)T+\frac{T}{2}+(\phi-\tau)} sin\omega_x \tau_1 \cdot d\tau_1 - u_2 \int_{(i-1)T+\frac{3T}{4}-(\phi-\tau)}^{(i-1)T+\frac{3T}{4}+(\phi-\tau)} sin\omega_x \tau_1 \cdot d\tau_1$$
$$+ u_1 \int_{(i-1)T+T-(\phi-\tau)}^{(i-1)T+T} sin\omega_x \tau_1 \cdot d\tau_1]$$

$$= \frac{1}{T.\omega_x}\left\{u_1(-cos\omega_x(\phi-\tau)+1)+u_2\left[-cos\left(\frac{\pi}{2}+\omega_x(\phi-\tau)\right)+cos\left(\frac{\pi}{2}-\omega_x(\phi-\tau)\right)\right]\right.$$
$$-u_1[-cos(\pi+\omega_x(\phi-\tau))+cos(\pi-\omega_x(\phi-\tau))]$$
$$-u_2\left[-cos\left(\frac{3\pi}{2}+\omega_x(\phi-\tau)\right)+cos\left(\frac{3\pi}{2}-\omega_x(\phi-\tau)\right)\right]$$
$$\left.+u_1[-cos\,2\,\pi+cos(\,2\pi-\omega_x(\phi-\tau))]\right\}$$

So: $cos\,\pi = 1, cos(\,2\pi-\omega_x(\phi-\tau)) = cos\omega_x(\phi-\tau)\,, cos(\pi+\omega_x(\phi-\tau)) = cos(\pi-\omega_x(\phi-\tau))$

$$cos\left(\frac{\pi}{2}-\omega_x(\phi-\tau)\right) = -\,cos\left(\frac{\pi}{2}+\omega_x(\phi-\tau)\right) = cos\left(\frac{3\pi}{2}+\omega_x(\phi-\tau)\right) = -\,cos\left(\frac{3\pi}{2}-\omega_x(\phi-\tau)\right)$$

Then: $\delta_{HTB} = \frac{2.u_2}{\pi}cos\left(\frac{\pi}{2}-\omega_x(\phi-\tau)\right) = \frac{2.u_2}{\pi}cos\left(\frac{\pi}{2}-(\varphi-\Delta\varphi)\right)$ (33)

$$F_{hTB} = mV\frac{d\theta}{dt}\Big|_{TB} = K_\theta co\omega_x t.\,\delta_B(t)|_{TB} = K_{p\alpha}\delta_{BTB} = K_{p\alpha}\frac{2.u_1}{\pi}sin(\varphi-\Delta\varphi) \quad (34)$$

$$F_{ngTB} = -mV\frac{d\Psi}{dt}\Big|_{TB} = -K_\Psi co\omega_x t.\,\delta_B(t)|_{TB} = K_{p\alpha}\frac{2.u_2}{\pi}cos\left(\frac{\pi}{2}-(\varphi-\Delta\varphi)\right) \quad (35)$$

The total force acting on the aircraft is:

$$F_{TB} = \sqrt{F_{hTB}^2+F_{ngTB}^2} = \frac{2K_{p\alpha}.\sqrt{u_1^2+u_2^2}}{\pi}.\,sin(\varphi-\Delta\varphi) = \frac{2.K_{p\alpha}\delta^*}{\pi}sin(\varphi-\Delta\varphi) \quad (36)$$

Comment: Expressions (31) and (33) command the TBBMK control according to the pulse width - amplitude modulation method according to the altitude channel and according to the direction channel.

In the case of fixed τ, ϕ from the two expressions (31) and (33), it can be seen that the change in the control command of the tilt angle and the equivalent trajectory direction angle in one rotation cycle of the missile is proportional to the input control values u_1 and u_2. Thus, the control commands δ_{BTB}, δ_{HTB} depend on the input signal amplitude, the missile rotation frequency and the deflection angle φ.

The magnitude of the force acting on the TBBMK depends on the value of $\sqrt{u_1^2+u_1^2}$, $\Delta\varphi$ and φ. Thus, simultaneously changing $\sqrt{u_1^2+u_1^2}$, $\Delta\varphi$ and φ can create the desired flying force F_{hTB}, F_{ngTB}.

When synthesizing TBBMK control commands using the pulse amplitude method, then ϕ=0, $\varphi = \omega_x.$ ϕ=0; then expression (31) becomes expression (9); expression (33) becomes expression (11).

When synthesizing the TBBMK control command by the pulse width method, then u_1=u_2=δ^* =const. The problem returns to the form of TBBMK control by the pulse width modulation method.

In the above TBBMK control methods: The TBBMK control method changes the amplitude and phase of the aerodynamic steering vane angle, and the pulse width - amplitude modulation modulation (PWAM) method has a total force F_{TB} acting on the TBB that is larger than the pulse amplitude change method and the pulse width modulation method. Therefore, the mobility of the TBBMK is better.

In practice, the determination of control commands for the TBBMK steering vane is based on the principle of adhesion. The basis for designing this adhesion system comes from the above explanations.

The calculation results are the basis for the task of analyzing and separating the design of new generation of military equipment, moving towards mastering and improving the technical and combat characteristics of weapons and equipment.

4. CONCLUSIONS

The above presents the method of synthesizing TBBMK control commands using the pulse width - amplitude modulation (PWAM) method. This method ensures error-sensitive control and the generated control command is capable of continuously varying from maximum negative value to maximum positive value, improving control quality and selective harmonic rejection for drive applications and minimizing power dissipation of the power converter.

On that basis, we analyzed and surveyed the methods of synthesizing control commands for TBBMK infrared homing head and used mathematical tools, basic transformations and arguments to synthesize control commands for infrared homing radars according to the new method of pulse width - amplitude modulation (PWAM). This is the result of expanding domestic academic research, because infrared self-guided TBBMK are being put into service and new generation missile systems are being prepared for purchase. The explanations of the article are the basis for the study of the synthesis of the tracking system and the automatic flight control system of the TBBMK.

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TÓM TẮT

Tổng hợp lệnh điều khiển cho thiết bị bay một kênh theo phương pháp biên độ - độ rộng xung

Bài báo khảo sát phương pháp tổng hợp lệnh điều khiển của thiết bị bay một kênh (TBBMK) đầu tự dẫn hồng ngoại đang sử dụng; Các phương pháp này đòi hỏi góc cánh lái phải có khả năng thay đổi từ giá trị âm tối đa đến giá trị dương tối đa một cách liên tục hoặc nhảy bậc. Thông qua việc phân tích mô hình toán mô tả động học bay của TBBMK trên cơ sở giả thiết điều kiện bay thực tế, nhóm tác giả đề xuất điều khiển TBBMK theo phương pháp điều chế biên độ - độ rộng xung (PWAM).

Từ khóa: Thiết bị bay một kênh; Lệnh điều khiển; Đầu tự dẫn hồng ngoại; Phương pháp điều chế.