# Calculation and simulation of low Earth satellite orbit using least-squares method and on-board GPS data: case study of VNREDSat-1 satellite

Ngo Duy Tan, Bui Quang Huy*, Nguyen Duc Hieu, Truong Tuan Anh, Tran Kim Chi, Phung Duc Thuong, Pham Anh Tuan

Space Technology Institute, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam;

*Corresponding author: bqhuy@sti.vast.vn

*Received 07 Oct. 2024; Revised 20 Nov. 2024; Accepted 02 Dec. 2024; Published 25 Dec. 2024.*

**DOI:** https://doi.org/10.54939/1859-1043.j.mst.100.2024.54-61

## ABSTRACT

*The article focuses on describing the results of precise calculation and simulation of low Earth observation satellite orbits using Least-Squares Method and GPS data collected on-board satellite. The calculation and simulation tool is implemented on Matlab and tested for real data of the VNREDSat-1 satellite. The purpose of this tool is to improve the accuracy of calculating low Earth observation satellite orbits compared to the method of using Two-line Element Set (TLE) files according to the standards of NORAD (North American Aerospace Defense Command). The results achieved will contribute significantly to the process of mastering the technology of ground control stations for Earth observation satellites, especially the Flight Dynamics Terminal which is responsible for precise orbit determination and maneuver and supporting mission planning.*

**Keywords:** Least squares; Low earth orbit satellite; Ground control station; VNREDSat-1.

## 1. INTRODUCTION

The Least Squares Method (LSM) has been widely applied in various fields such as economics, engineering, geography, and aerospace. In economic analysis, this method plays an important role in linear regression problems, helping to estimate the relationship between economic variables, predict market trends, and analyze financial data. Another specific application is in signal processing and solving inverse problems, where this method is used for data reconstruction, singularity analysis, as well as model building in linear or Hilbert spaces [1-3]. In addition, LSM is also applied in measurement and optimization, such as GPS position estimation and experimental data processing to minimize errors.

Currently, to calculate and simulate the orbits of satellites orbiting the Earth, scientists often use satellite orbit description files in the Two-Line Element Set (TLE) format according to the standard of the North American Aerospace Defense Command (NORAD). TLE is currently the only publicly available source of orbit information [4]. However, TLE satellite orbit information has some major drawbacks: low accuracy, mistagging, and undefined corrections [5]. Precise orbit calculation based on onboard GPS data using LSM has been applied to ground control station to support orbit maneuvers and mission planning [6, 7].

This paper describes the method and development of a Matlab based simulation software tool applying LSM and onboard GPS data for precise orbit determination of a low Earth orbit satellite; the tool is performed with actual VNREDSat-1 satellite data. The outcome of this paper are expected to contribute significantly to the development of Flight Dynamics software tools for the ground control station for Vietnam's low Earth observation satellites.

## 2. LOW EARTH ORBIT SATELLITE ORBIT CALCULATION USING LEAST SQUARES METHOD AND ONBOARD GPS DATA

### 2.1. Orbital dynamics model

The dynamic orbit determination problem is fundamentally nonlinear. Orbital motion is described in an inertial reference frame by a system of differential equations [8-10].

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} + \mathbf{N} \quad (1)$$

Where $\mathbf{r} = (Px, Py, Pz)$ is the position vector, $\mu$ is the gravity parameter, and **N** is modelled perturbations. Thus, the state vector to be estimated is defined by $\mathbf{x} \equiv [\mathbf{r} \;\; \mathbf{v} \;\; \mathbf{b}]^t$, where **v** is the velocity vector and **b** is the bias parameter. The state transition matrix linking the state at times $t_k$ and $t_{k+1}$ can be calculated by the equation:

$$\dot{\mathbf{\Phi}}(t, t_k) = \mathbf{F}(\mathbf{x}, t)\mathbf{\Phi}(t, t_k) \quad (2)$$

Where the state transition matrix $\mathbf{\Phi}(t,t_k)$ is used to link the state of the system at two time points $t_k$ and $t$, represented by:

$$x(t) = \Phi(t, t_k).x(t_k) \quad (3)$$

Where $x(t_k)$ is the state at the initial time $t_k$, $\mathbf{\Phi}(t,t_k)$ is the state transition matrix at the initial time (which is the identity matrix). With the initial condition:

$$\Phi(t_k, t_k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The Jacobian matrix $\mathbf{F}(\mathbf{x},t)$ can be written as:

$$\mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{pmatrix} \mathbf{0}_{3x3} & \mathbf{I}_{3x3} & \mathbf{0}_{3x3} \\ \mathbf{A}_{3x3} & \mathbf{0}_{3x3} & \mathbf{0}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} & \mathbf{0}_{3x3} \end{pmatrix} \quad (4)$$

Where **f** is the acceleration acting on the satellite, $\mathbf{0}_{3x3}$ is the 3×3 zero matrix, $\mathbf{I}_{3x3}$ is the 3×3 identity matrix, and $\mathbf{A}_{3x3}$ is the 3×3 gravity gradient matrix calculated as follows:

$$\mathbf{A}_{3x3} = \frac{\partial \mathbf{v}'}{\partial \mathbf{r}} \quad (5)$$

## 2.2. Orbit determination using the least-squares method and on-board GPS data

Assuming that:

- N is the number of GPS measurements.
- $\mathbf{y}_i = [P_{xi}, P_{yi}, P_{zi}]$ are the actual GPS coordinates of the satellite at time $i$ (GPS data).
- $f(\mathbf{x})$ is the desired orbit model to be estimated, where **x** is a vector containing the parameters to be optimized, including the initial position ($P_{x0}, P_{y0}, P_{z0}$), initial velocity ($V_{x0}, V_{y0}, V_{z0}$), and acceleration factors.

The objective of the least squares is to find the parameters in **x** such that the error between the GPS data and the estimated orbit is minimized [11-13]. That is, the following function must be minimized:

$$\mathbf{x}_{\mathbf{est}} = \min_{\mathbf{x}} \sum_{i=1}^{N} [\![ \mathbf{y}_i - f(\mathbf{x}, t_i) ]\!]^2 \quad (6)$$

Where $t_i$ is the time of the $i$-th measurement, $f(\mathbf{x},t_i)$ is the estimated position of the satellite at time $t_i$ based on the parameters in **x**, $[\![ \mathbf{y}_i - f(\mathbf{x}, t_i) ]\!]^2$ is the squared distance between the actual GPS position and the estimated position. To model the satellite's orbit, the fundamental equation of motion is used:

$$\mathbf{P}(t) = \mathbf{P_0} + \mathbf{V_0}.t + \frac{1}{2}\mathbf{a}.t^2 \quad (7)$$

Where $\mathbf{P}_t$ is the position of the satellite at time $t$, $\mathbf{P_0}$ and $\mathbf{V_0}$ are the initial position and velocity, and **a** is the acceleration (due to gravity and environmental factors acting on the satellite).

*a) Linear least squares solution method*

Assume the initial state vector is: $\mathbf{x} = \left[P_{x0}; P_{y0}; P_{z0}; V_{x0}; V_{y0}; V_{z0}\right]$, then the system of equations has the form:

$$\mathbf{Y} = \mathbf{H}.\mathbf{x} + \boldsymbol{\varepsilon} \quad (8)$$

Where **Y** is the vector containing the measured GPS positions at different times, **H** is the design matrix containing the time values (calculated from the $t$ and $t^2$ components), and $\boldsymbol{\epsilon}$ is the noise or measurement error vector.

To determine the optimal value for **x**, the least-squares method is employed.

$$\mathbf{x} = (\mathbf{H^T}.\mathbf{H})^{-1}.\mathbf{H^T}.\mathbf{Y} \quad (9)$$

Here, $\mathbf{H^T}.\mathbf{H}$ represents the square matrix derived from the design matrix, and $\mathbf{H^T}.\mathbf{Y}$ represents the product of the transposed design matrix and the vector of measured GPS positions.

After calculation, **x** will contain the estimated values of the initial position and velocity. Based on these values, the satellite's trajectory at other times can be predicted [14].

*b) Nonlinear Least-Squares Method*

Nonlinear LSM utilizes an iterative optimization procedure (such as Gauss-Newton or Levenberg-Marquardt) to adjust **x** through iterations, minimizing the error function. This process comprises the following steps:

- Initialize an initial value $\mathbf{x_0}$ for the parameters **x** (for example, from the initial position and velocity values based on GPS data).
- Repeat the following steps until convergence is achieved:

\- Calculate the error between the actual GPS position and the predicted position from the orbital model.

$$\mathbf{residual}_i = \mathbf{y}_i - \mathrm{f}(\mathbf{x}, t_i) \quad (10)$$

\- Construct the Jacobian matrix **J**, where each element $\mathbf{J}_{ij}$ is the partial derivative of the error with respect to each parameter $\mathbf{x}_j$.

$$\mathbf{J}_{ij} = \frac{\partial(\mathbf{residual}_i)}{\partial \mathbf{x}_j} \quad (11)$$

Update **x** using an optimization method like Gauss-Newton or Levenberg-Marquardt:

$$\mathbf{x} \leftarrow \mathbf{x} + \Delta\mathbf{x} \text{ with } \Delta\mathbf{x} = -(\mathbf{J^T}.\mathbf{J})^{-1}.\mathbf{J^T}.\mathbf{residuals} \quad (12)$$

Where $\mathbf{J^T}.\mathbf{J}$ is the coefficient matrix, and $\mathbf{J^T}.\mathbf{residuals}$ is the error vector.

- Stop when the error is sufficiently small or when the change in **x** between iterations is negligible.
- Finally, the formula for nonlinear optimization of the satellite orbit with GPS data can be expressed as follows:

$$\mathbf{x} = \arg\min_{\mathbf{x}} \sum_{i=1}^{N} [\![ \mathbf{y}_i - \mathrm{f}(\mathbf{x}, t_i) ]\!]^2 \quad (13)$$

The **x** update process is repeated until convergence. Both linear and nonlinear least-squares methods facilitate finding the optimal parameters for the satellite's position and velocity, thereby accurately modeling the satellite's orbit based on measured GPS data [15].

### 2.3. Kalman Filter

The Kalman Filter is employed to estimate the position and velocity of an object based on GPS data. The Kalman Filter, an optimal filtering method for state estimation of a dynamic system in a noisy environment, consists of two primary steps:

- **Prediction:** Predicting the next state and the error covariance.

$$\mathbf{x_{pred}} = \mathbf{A}.\mathbf{x} \tag{14}$$

Where $\mathbf{x_{pred}}$ is the predicted state, **x** is the current state, and **A** is the state transition matrix.

$$\mathbf{P_{pred}} = \mathbf{A}.\mathbf{P}.\mathbf{A}^{\mathbf{T}} + \mathbf{Q} \tag{15}$$

Where $\mathbf{P_{pred}}$ is the predicted error covariance, **P** is the current estimated error covariance, and **Q** is the system noise.

- **Update:** Updating the state with measurements. Based on the new measurement, calculate the measurement error, the Kalman Gain matrix, and update the estimated state as well as the error covariance.

$$\mathbf{y} = \mathbf{z} - \mathbf{H}.\mathbf{x} \tag{16}$$

Where y is the measurement error, **z** is the actual measured value, **H** is the measurement matrix, mapping the system state (including position and velocity) to the measurement space (including only position), and **H.x** is the predicted measurement value.

$$\mathbf{S} = \mathbf{H}.\mathbf{P}.\mathbf{H}^{\mathbf{T}} + \mathbf{R} \tag{17}$$

Where **S** is the measurement error covariance matrix, and **R** is the measurement noise matrix [16].

## 3. DEVELOPMENT OF CALCULATION TOOL AND SIMULATION RESULTS OF VNREDSat-1 SATELLITE ORBIT

### 3.1. Input data

The input data is stored in a file in.LMF (Lunar Mission Manager File), containing information about the orbit of the VNREDSat-1 satellite. This data includes measured values of position coordinates ($P_x$, $P_y$, $P_z$) and velocity ($V_x$, $V_y$, $V_z$) of the VNREDSat-1 satellite. When the LMF formatted file is input into the calculation program, the time information (yyyy/mm/dd hh:mm:sss), data columns $P_x$, $P_y$, $P_z$, $V_x$, $V_y$, $V_z$, and their corresponding values are formatted as presented in table 1.

***Table 1.*** *Example of information data contained in LMF file format.*

| Time | $P_x$ | $P_y$ | $P_z$ | $V_x$ | $V_y$ | $V_z$ |
|---|---|---|---|---|---|---|
| 2019/08/03 15:25:26.00 | -2019.30062 | 3738.2887 | -5651.1760 | -0.9325 | 6.1156 | 4.3808 |
| 2019/08/03 15:25:26.00 | -2156.6600 | 5359.4490 | -4074.53425 | 0.01371 | 4.5935 | 6.0416 |
| 2019/08/03 15:25:26.00 | -1645.4051 | 6870.3625 | 111.37307 | 1.5691 | 0.2459 | 7.4350 |

The information within the file is utilized for analysis and determination of the actual orbit of the VNREDSat-1 satellite.

### 3.2. Calculation and simulation tool

The tool constructed in this paper is based on MATLAB, combined with LSM and GPS data of the VNREDSat-1 satellite for testing. Figure 1 illustrates the flowchart of the orbit calculation tool. In this flowchart, module G is responsible for calculating the satellite orbit using LSM.

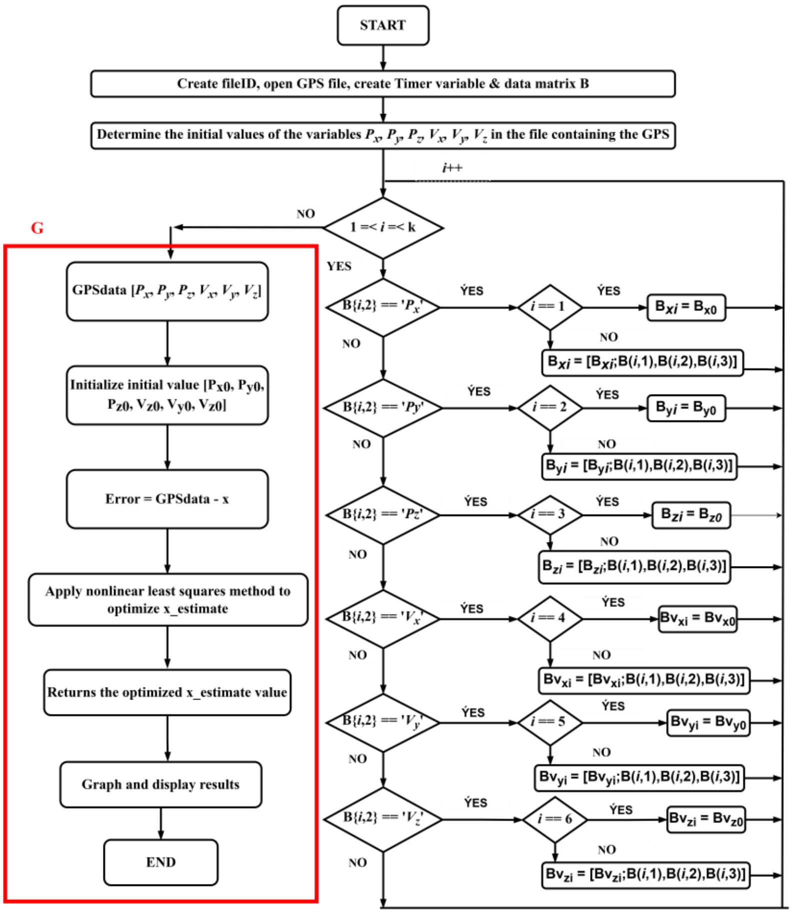


***Figure 1.*** *Flowchart for orbit calculation of low earth satellite by LSM.*

The program's first step involves reading, processing, and storing data from the LMF file and reorganizing the satellite's time, position, and velocity information (*Time*, $P_x$, $P_y$, $P_z$, $V_x$, $V_y$, $V_z$) into a data matrix named **GPSdata**. After the values are combined into a matrix B with information such as Time, $P_x$, $P_y$, $P_z$ (position), and $V_x$, $V_y$, $V_z$ (velocity), the next step is classifying and organizing the data within matrix **B** into different variables, based on the second column value of each row. Once the **GPSdata** matrix is complete, LSM is employed to estimate position and velocity. This process comprises the following steps:

- Initializing initial values for position and velocity: $P_{x0}$, $P_{y0}$, $P_{z0}$, $V_{x0}$, $V_{y0}$, $V_{z0}$;
- Utilizing LSM to estimate position and velocity.

After the **x_estimate** vector is determined, the Kalman filter is employed to estimate the position and velocity of the moving object, based on GPS data. This procedure includes the following steps:

- Initializing the Kalman matrix with crucial components such as the state transition matrix **A**, measurement matrix **H**, process noise matrix **Q**, measurement noise matrix **R**, and covariance matrix **P**;
- The program initializes variables to store the estimated position and velocity, as well as a variable *x* containing the initial estimated value;
- Starting from the estimated initial position **x_estimate**, the program utilizes a loop to update the estimated state for each GPS data point within the dataset;
- Within each state update, the following steps are executed:
  - o Predicting the next state based on the state transition matrix **A**;

- o Predicting the next covariance matrix **P** based on the state transition matrix **A**, the current covariance matrix, and the process noise matrix **Q**;
- o Retrieving data from GPS and updating the estimated state based on the measured data, measurement matrix **H**, measurement noise matrix **R**, covariance matrix, and Kalman gain matrix **K**;
- o Updating the covariance matrix according to the Kalman filter formula;
- o The estimated state (position and velocity) after each loop is stored in the **EstimatedState** matrix to track the changes in the state estimation process over time.
- o Vislalization of the results on a graph.

### 3.3. Simulation results

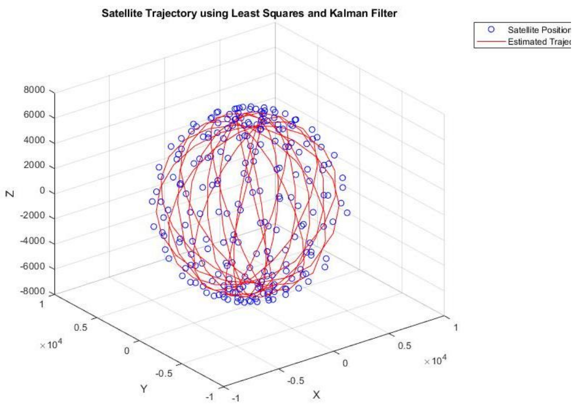


***Figure 2.*** *Simulated orbit using linear least squares.*

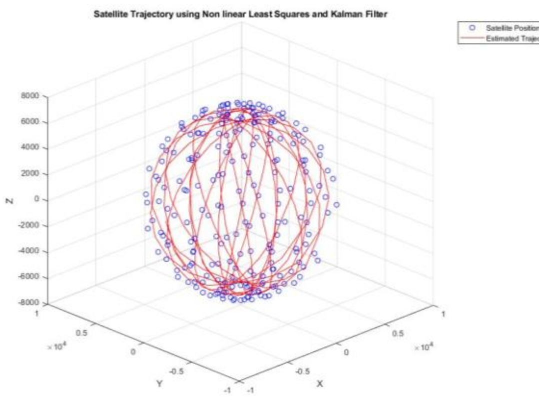


***Figure 3.*** *Simulated orbit using non-linear least squares.*

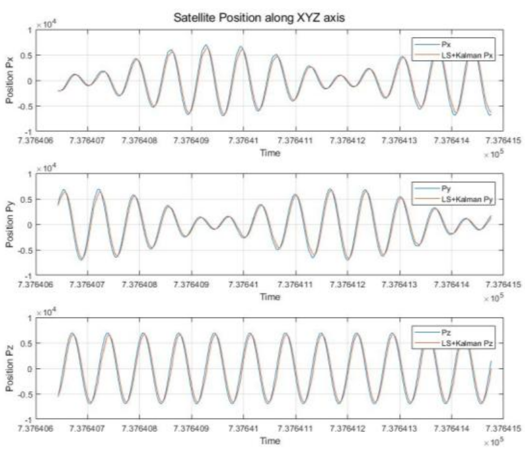


***Figure 4.*** *Coordinates along each axis between GPS measured values and estimated values using linear least squares combined with Kalman filter.*

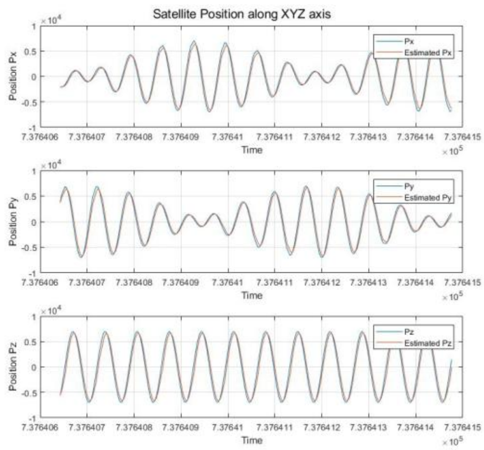


***Figure 5.*** *Coordinates along each axis between GPS measured values and estimated values using non-linear least squares combined with Kalman filter.*

Figures 2, 4, 6 illustrate the results obtained with the linear LSM and figures 3, 5, 7 illustrate the results obtained with the non-linear method. In general, the orbits illustrated by both LSMs are similar and represent the shape of the satellite's orbit around the Earth. This is clearly shown in figures 4 and 5 where both methods give small deviations in coordinates along the axes between the actual values (measured by GPS) and the estimated values (calculated by the model).

The discrepancies between the two computational models are illustrated in figures 6 and 7. In figure 6, the deviation between the actual value and the estimated value using the LSM combined with a Kalman filter is smaller (Root Mean Square (RMS) < 250 m) and unstable (increasing over time). The results calculated based on the non-linear LSM combined with a Kalman filter exhibit greater stability along each axis, but a larger deviation amplitude compared to the linear LSM (RMS < 1400 m) (see figure 7). The RMS errors of the two methods are listed in table 2.

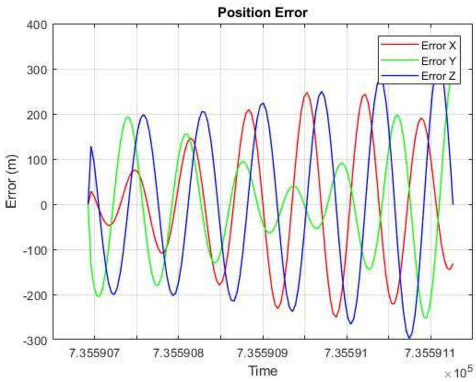


***Figure 6.*** *Positional deviation between actual values and estimated values using linear LSM combined with Kalman filter.*

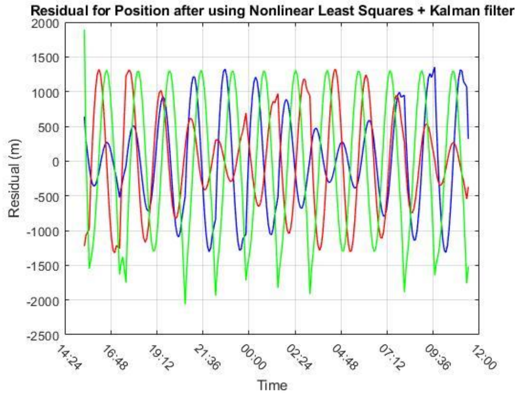


***Figure 7.*** *Positional deviation between actual values and estimated values using non-linear LSM combined with Kalman filter.*

***Table 2.*** *RMS Error Evaluation between the two models.*

| RMS Error Evaluation | Linear Least Squares (unit: m) | Non-linear Least Squares (unit: m) |
|---|---|---|
| RMS for x-axis | 134.0926 | 572.6617 |
| RMS for y-axis | 116.9565 | 759.9481 |
| RMS for z-axis | 169.9569 | 959.3636 |
| Average RMS | 246.0589 | 1351.2369 |

The error values in table 2 are calculated without considering perturbation factors such as air resistance, solar radiation pressure, gravitational forces, and non-spherical Earth effects. The obtained results demonstrate higher accuracy when using TLE data for low Earth orbit satellite calculations (see table 3).

***Table 3.*** *Comparison between TLE and least-squares1.*

| **Criteria** | **TLE** | **Least-squares method** |
|---|---|---|
| **Error** | Several kilometers to tens of kilometers | Below 0.25 km (linear LSM) and below 1.4 km (non-linear LSM) |
| **Stability** | Decreases over time | More stable due to GPS updates |
| **Calculation speed** | Fast, simple calculation | Slower, requires complex optimization |

## 4. CONCLUSIONS

The MATLAB-based low Earth orbit satellite calculation and simulation tool offers high flexibility for adjusting model parameters, enabling the study of various scenarios with factors influencing the orbit. Applying linear and non-linear LSM to VNREDSat-1's GPS data yields relatively similar results. This outcome indicates the tool's accurate predictive capability for monitoring the position and velocity of low Earth orbit satellites. It also demonstrates the potential applicability of this software tool in precise orbit calculation and prediction programs in the Flight Dynamics Terminal of satellite ground stations in Vietnam. This tool will continue to be tested with other GPS data from VNREDSat-1 satellite or other satellites managed and operated by Vietnam.

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## TÓM TẮT

### Tính toán và mô phỏng quỹ đạo vệ tinh tầm thấp sử dụng phương pháp bình phương tối thiểu và dữ liệu GPS: Ứng dụng cho vệ tinh VNREDSat-1


*Bài báo tập trung mô tả kết quả tính toán chính xác và mô phỏng quỹ đạo vệ tinh quan sát Trái Đất tầm thấp sử dụng phương pháp bình phương tối thiểu (least-squares) và dữ liệu GPS thu được trên vệ tinh. Phần mềm công cụ tính toán và mô phỏng được thực hiện trên công vụ Matlab và thử nghiệm cho dữ liệu thực tế của vệ tinh VNREDSat-1. Mục đích của của công cụ này nhằm nâng cao độ chính xác trong việc tính toán quỹ đạo vệ tinh tầm thấp so với phương thức sử dụng tập tin Two-line Element Set (TLE) theo tiêu chuẩn của NORAD (North American Aerospace Defense Command). Kết quả đạt được sẽ đóng góp quan trọng vào quá trình làm chủ công nghệ trạm điều khiển vệ tinh quan sát Trái Đất, cụ thể là phân hệ động lực học bay (Flight dynamics terminal) thực hiện nhiệm vụ xác định chính xác quỹ đạo và điều khiển quỹ đạo vệ tinh, hỗ trợ lập lịch chụp ảnh cho vệ tinh.*


**Từ khoá:** Bình phương tối thiểu; Quỹ đạo vệ tinh tầm thấp; Trạm điều khiển; VNREDSat-1.