

## Method for improving accuracy of trajectory correction using lateral pulse jets

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### ABSTRACT

*The paper presents methods for trajectory correction using lateral pulse jets on flying devices using global navigation systems (GNSS). In the control algorithm, the activation time between two consecutive pulses, instead of being a given constant value, is determined according to the characteristics of the flying device and trajectory parameters. Results of simulation and comparison between cases show that the proposed method of trajectory correction is better than the remaining methods because it requires the smallest number of pulses within the threshold of impact point deviation.*

**Keywords:** Lateral pulse jets; Reference trajectory tracking; Impact point prediction.

### 1. INTRODUCTION

One of the most current tendencies in the field of precision-guided artillery research is to extend the range and use low-cost control mechanisms and guidance systems to reduce the number of projectiles consumed and minimize damage to surrounding civilian infrastructures. An appropriate solution of control for correcting the trajectory of projectiles is using one-time lateral pulse jets.

The GNSS system, with its specialized structure, is used to determine position, velocity, and rotational motion around the symmetry axis (roll angle, roll rate) through a single-patch antenna structure [1, 2]. Studies and experiments have shown that the error in measuring rotational angles around the vertical axis is less than  $5^{\circ}$ . However, the drawback of the GNSS system is the large startup time (up to several tens of seconds), so due to the influence of dynamic noise, at the start time of control significant deviations of rocket trajectory compared to the reference trajectory may occur. This feature of the GPS navigation system has not been taken into account in published studies.

Another difficulty is that the pulse jets control mechanism has low control capability due to the limitation of the number of pulse jets. To overcome this difficulty, many studies have focused on building control methods divided into two main directions: reference trajectory tracking and impact point prediction.

The reference trajectory tracking method [3, 4] proposes an algorithm to activate the appropriate lateral pulse jet when the flying device's position deviation from the reference trajectory exceeds the required threshold. The disadvantage of this method is that it requires the use of a large number of pulse jets for the flying device to follow the reference trajectory throughout the flight, especially when large deviations appear in the initial correction stage (from the time the GNSS system has been operating stably).

To reduce the number of required pulses, Pavkovic, Pavic and Cuk [5, 6] added an active damping control mechanism to calculate the appropriate time between two pulses, however, this study has not overcome the disadvantage of the GNSS system in the initial correction stage. Nguyen Van Khoi [7] proposed an improved reference trajectory tracking method, which added

a pulse jet activation condition. In this case, the control signal will be activated if two conditions are simultaneously satisfied: The deviation exceeds the threshold and this deviation lies outside the the line connecting point  $i$  with the target MT (figure 3). However, this study only examined cases where the time between two pulses is a constant value.

The impact point prediction method uses ballistic linear theory to determine and correct the predicted impact point error relative to the target location [8-10]. However, the disadvantages of this method are:

- High demands on the on-board control computer to calculate the predicted impact point;
- More information from the navigation system is needed to solve the problem of determining the predicted impact point;
- The accuracy of the predicted impact point depends not only on the linear ballistic model but also on the accuracy of the GNSS system (the accuracy of the initial conditions for solving the linear ballistic model after each trajectory correction).

From the above analysis, it can be seen that published articles haven't considered both factors in the study: Calculating the reasonable time interval between two pulses and the characteristics of the GNSS system in building the trajectory correction algorithm using the lateral pulse jets. Therefore, in this paper, the authors conduct a comparative simulation of the published methods, thereby proposing a method to improve the trajectory correction accuracy by combining the advantages of the proposed methods.

## 2. PROBLEM

### 2.1. Model of lateral pulse jets controller

Due to the one-time use characteristic of the pulse engine, the control system needs to activate the corresponding pulse engine when satisfying the following conditions [7]:

1) Only generates activation signals for previously unused pulse jets.

Suppose,  $S$  is a pulse jet matrix with  $N_J$  rows and  $M_J$  columns where  $N_J$  – Total number of pulses in a face and  $M_J$  – Total number of pulse generating faces.

In this case, the  $i^{\text{th}}$  pulse of the  $j^{\text{th}}$  face satisfies the following condition:

- +)  $S_{ij} = 0$  - The pulse is not activated;
- +)  $S_{ij} = 1$  - The pulse is activated.

2) The time for the next activation of the pulse jet, measured from the time of the nearest previous activation, must be greater than a specified threshold, that is:

$$t - t^* > \Delta t_{Thres} \quad (1)$$

where,  $t^*$  the time of the nearest activation of the pulse jet;  
 $\Delta t_{Thres}$  - The threshold for the time between two pulse firings.

3) To ensure accuracy in the phase of the jet pulse, there must be a relationship between the selected pulse jet for activation and the phase error that satisfies the following condition (figure 1):

$$|\phi_{Ji} + \varphi_r + \varphi_d - \pi - \gamma| \leq \delta_{Thres} \quad (2)$$

where,  $\delta_{Thres}$  - The threshold for the phase error of the selected pulse jet.

The phase  $\gamma$  of the selected pulse is determined in the body

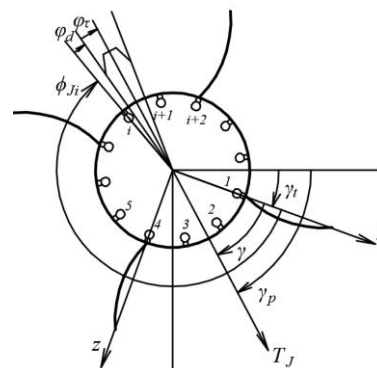


Figure 1. Illustration selected pulse jet for activation.

frame according to the following expression:

$$\gamma = \text{mod}\left((\gamma_p - \gamma_t), 2\pi\right); \quad \gamma_t = \text{mod}(\phi, 2\pi) \quad (3)$$

where,  $\phi$  – Roll angle.

From figure 1, the phase of the selected pulse jet is determined by the following expression:

$$\phi_{ji} = \frac{2\pi(i-1)}{N_j} \quad (4)$$

where  $N_j$  is the total number of pulses in one face.

The angle  $\varphi_\tau$  is determined by half of the rotation angle during the pulse duration of a pulse jet. The angle  $\varphi_d$  is determined by the activation delay of the pulse jet. Therefore,

$$\varphi_\tau + \varphi_d = \frac{\omega_x(\tau + \tau_d)}{2} \quad (5)$$

where,  $\tau$  - The pulse duration of a pulse jet,;  $\tau_d$  - The activation delay of the pulse jet.

Expanding condition (2) results in the expression for determining the position of the pulse jet to be activated:

$$i_1 \leq i \leq i_2 \quad (6)$$

where,

$$i_1 = \begin{cases} 1 + \frac{N_j}{2\pi} \left( \pi + \gamma - \omega_x \frac{\tau_j}{2} \right) - \frac{N_j}{2\pi} \delta_{Thres} & \text{if } \gamma \leq \pi \\ 1 + \frac{N_j}{2\pi} \left( -\pi + \gamma - \omega_x \frac{\tau_j}{2} \right) - \frac{N_j}{2\pi} \delta_{Thres} & \text{if } \gamma > \pi \end{cases};$$

$$i_2 = \begin{cases} 1 + \frac{N_j}{2\pi} \left( \pi + \gamma - \omega_x \frac{\tau_j}{2} \right) + \frac{N_j}{2\pi} \delta_{Thres} & \text{if } \gamma \leq \pi \\ 1 + \frac{N_j}{2\pi} \left( -\pi + \gamma - \omega_x \frac{\tau_j}{2} \right) + \frac{N_j}{2\pi} \delta_{Thres} & \text{if } \gamma > \pi \end{cases}$$

Based on the trajectory correction method, the pulse jet controller selects and activates the appropriate pulse jet to reduce the guidance error.

## 2.2. Method for improving accuracy of trajectory correction

### 2.2.1. Reference trajectory tracking method (TT)

According to [3], the positional and phase deviations of the flying device from the reference trajectory are determined by the following expressions:

$$G = \sqrt{\Delta y_n^2 + \Delta z_n^2}; \quad \gamma_p = \text{mod}\left(\arctan 2\left(\frac{\Delta z_n}{\Delta y_n}\right), 2\pi\right) \quad (7)$$

The controller activates the appropriate pulse jet when the deviation magnitude exceeds the threshold, i.e.:

$$G > G_{Thres} \quad (8)$$

where,  $G_{thres}$  – Threshold of pulse activation.

### 2.2.2. Impact point prediction method (IPP)

According to [8], model of impact point prediction has the following form:

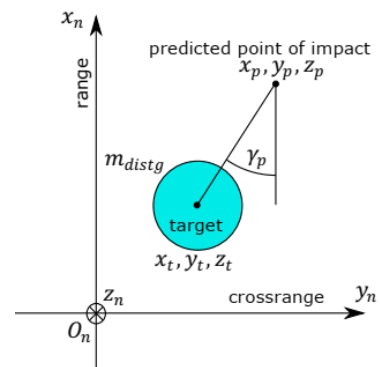


Figure 2. Error of predicted impact point by method IPP.

$$\ddot{x}_n = -\frac{\rho S C_{x0} V_0}{2m} \dot{x}_n, \quad \ddot{y}_n = -\frac{\rho S C_{x0} V_0}{2m} \dot{y}_n, \quad \ddot{z}_n = -\frac{\rho S C_{x0} V_0}{2m} \dot{z}_n + g. \quad (9)$$

The positional and phase deviations of the predicted impact point from the target position are determined by the following expression (figure 2):

$$G = \sqrt{\Delta x_n^2 + \Delta y_n^2}; \quad \gamma_p = \text{mod} \left( -\arctan 2 \frac{\Delta y_n}{\Delta x_n}, 2\pi \right) \quad (10)$$

### 2.2.3. Modified reference trajectory tracking method (MTT)

According to the TT method, the controller activates the lateral pulse jets when the deviation magnitude begins to exceed the threshold. This is only true when the trajectory deviation from the reference trajectory is small. In practice, due to the characteristics of the GNSS system, the lateral pulse jets only activates after a period of time (up to several tens of seconds) after launch, that is, the deviation magnitude from the reference trajectory is much larger than the threshold. To reduce the number of pulse jets at this stage, the following additional condition is proposed.

The controller activates the appropriate pulse jet when the deviation magnitude exceeds the threshold and the flying device tends to move away from the reference trajectory (In figure 3, at the calculation time  $i+1$ , the flying device's position deviation is outside the line connecting point  $i$  with the target MT), that is:

$$G_{i+1} > G_{Thres} \quad \text{v\grave{a}} \quad G_{i+1} > G_i \frac{x_{mt} - x_{n,i+1}}{x_{mt} - x_{n,i}} \quad (11)$$

where,  $G_{thres}$  – Threshold of pulse activation;  $i$  – The number of sampling time  $t_i$ ;  $x_{n,i}$ ,  $x_{mt}$  – Flying device and target positions.

### 2.2.4. Proposed method for improving accuracy of trajectory correction (IMTT)

From the published results, it can be seen that the MTT method has taken into account the characteristics of the GNSS system. However, due to the limited number of pulses, to ensure the number of pulses and the accuracy of trajectory correction, the time between two pulses needs to be calculated reasonably. In the initial and terminal stages, the number of pulses used needs to be larger because it is necessary to correct the trajectory to ensure the accuracy of the flying device, in the intermediate stage, the number of pulses used is smaller to maintain the trajectory error within the allowable threshold.

According to [5], in the lateral pulse jet model, the activation time threshold, instead of being a constant value ( $\Delta t_{Thres} = const$ ), is now a function dependent on the cutoff frequency  $\omega_c$ , given by the expression:

$$\Delta t_{Thres} = \pi / \omega_c \quad (11)$$

The cutoff frequency  $\omega_c$  depends on the flying device's characteristics (such as reference area  $S$ , reference length  $D$ , moment of inertia  $I_z$ , derivative of the moment coefficient of attack angle, velocity  $V$  and air density  $\rho$ ) as expressed in the following equation [11].

$$\omega_c = V \sqrt{\frac{-m_z^\alpha \rho S D}{2I_z}} \quad (12)$$

Thus, the proposed IMTT method in the paper is a method that combines MTT method with expression (11), calculating the reasonable time interval between two pulses in depending on cutoff frequency of the flying device.

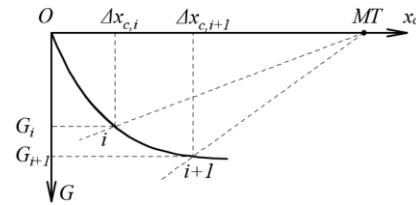


Figure 3. Illustration of error in the method MTT.

### 3. SIMULATION AND DISCUSSION

#### 3.1. Input data

The 6DOF flight dynamics model is determined according to [7]. The simulation program is built using Matlab/Simulink 2022b software (figure 4). The parameters for the simulation include:

1) Basic characteristics of the flying device: Length 3996 mm, diameter 160 mm, mass 135 kg, center of gravity at 2.017 m, inertia  $I_x = 0.539 \text{ kg}\cdot\text{m}^2$ ,  $I_z = 159.35 \text{ kg}\cdot\text{m}^2$ , pulse engine 12000 N, burn time 1.9 s.

2) Control system parameters:  $G_{Thres} = 5 \text{ m}$ .

3) Pulse jet parameters:  $N_J = 40$ ;  $M_J = 2$ ;  $T_J = 100 \div 1000 \text{ N}$ ;  $\tau_d = 5 \text{ ms}$ ;  $\tau = 40 \text{ ms}$ .

4) Noise model of the navigation system GNSS: Information update frequency 10 Hz, position deviation  $3\sigma = 5 \text{ m}$ ; velocity deviation  $3\sigma = 0.1 \text{ m/s}$ ; angle deviation  $3\sigma = 5^\circ$ .

5) Noise model of launch angles:  $\delta\theta_0 = 0,1^\circ$ ;  $\delta\psi_0 = 0,1^\circ$ .

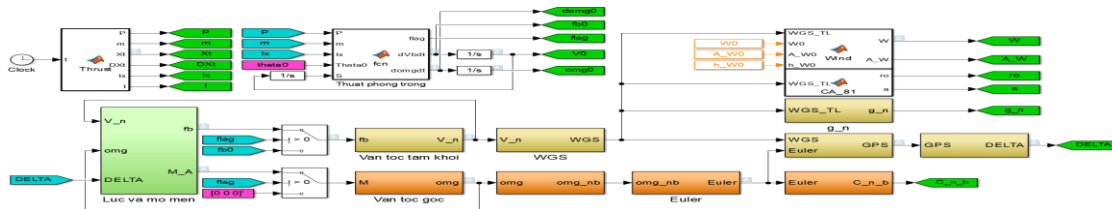


Figure 4. Interface of the trajectory correction simulation program.

#### 3.2. Simulation results and comments

Position: Flying device (N11057,209'; E109016,201'; h = 20 m); target (N11054,303; E109034,127'; h = 6 m). Based on the firing table built from the simulation program to determine the flying device's reference trajectory, the determined firing element is the flying device's launch angle: The launch angle is  $68.89^\circ$ , the azimuth angle is  $99.37^\circ$ .

The simulation determine the error value G and the number of pulses required to activate for the process of trajectory correction by TT, IPP and MTT methods. Simulations performed for both cases: when  $\Delta t_{Thres} = 1,0 \text{ s}$  and when  $\Delta t_{Thres}$  determined according to expression (11). Results of simulation are shown in the following figures.

##### 1) Trajectory correction by method TT:

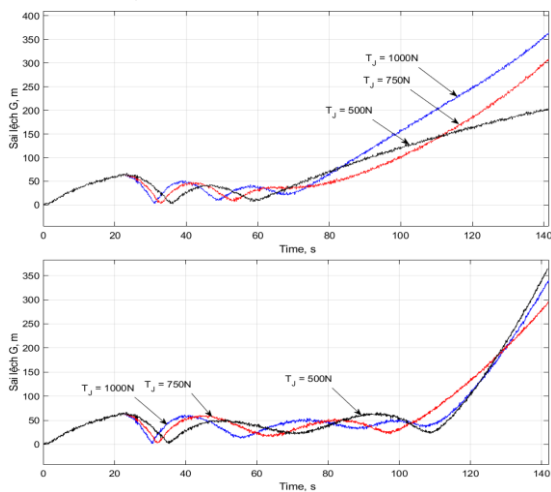


Figure 5. Deviations by TT method (top) and TT combined with (11) (bottom).

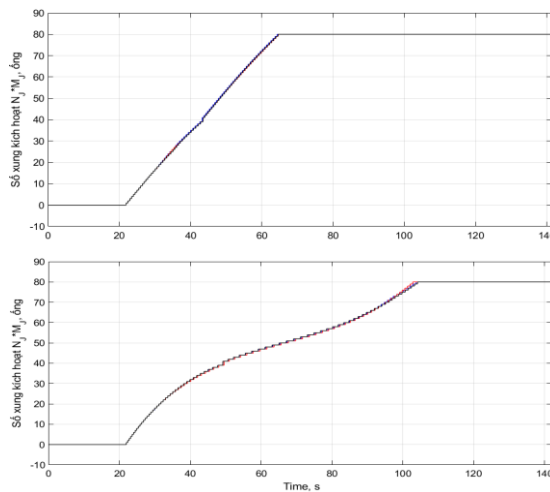


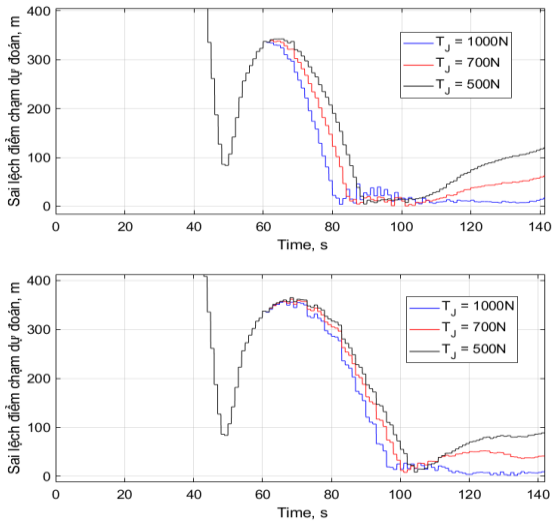
Figure 6. The number of used pulses by TT (top) and TT combined with (11) (bottom).

Discussion:

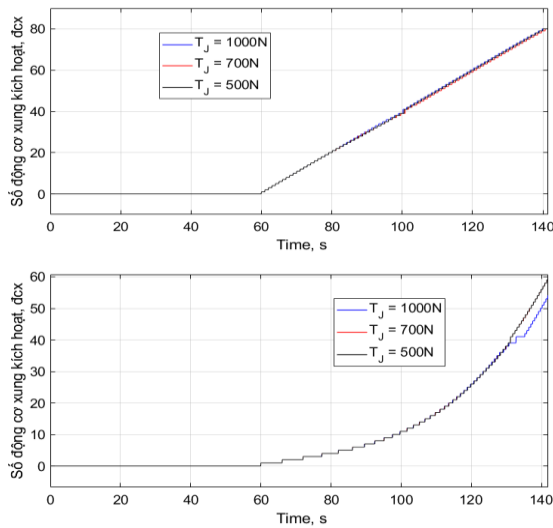
- At the beginning of the control (after 22 s), the deviation from the reference trajectory is large. The TT method quickly reduces the deviation to the threshold (10 m), leading to sliding and fluctuation around the reference trajectory. Therefore, the total number of pulses (80 pulse jets) for the entire tracking process is insufficient (figure 5).

- The TT method combined with (11) increases the tracking time along the reference trajectory (over 100 s). However, the total number of pulses required for the entire correction process (over 140 s) is insufficient (figure 6).

**2) Trajectory correction by method IPP:**



**Figure 7.** Deviations by IPP method (top) and IPP combined with (11) (bottom).



**Figure 8.** The number of used pulses by IPP (top) and IPP combined with (11) (bottom).

Discussion:

- The method IPP with constant value of the activation time threshold does not guarantee the accuracy of the impact point due to the insufficient number of pulse jets for the entire correction process. This is due to the errors in the GNSS system and the impact point prediction model. As a result, after each update, the predicted impact point position varies, requiring frequent correction by the pulse jet controller to correct this discrepancy (figure 7).

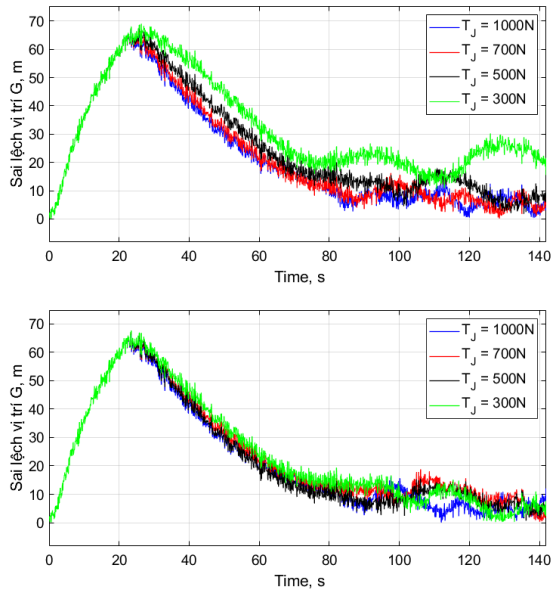
- The method IPP combined with (11) reduces the required number of pulse jets for the correction process (around 55 units). However, to ensure the accuracy of the impact point position, the activation pulse value needs to satisfy  $J_T \geq 1000 N$ . This value becomes more challenging for the pulse engines design problem as it increases (figure 8).

**3) Trajectory correction by method MTT:**

Discussion:

- The trajectory correction method MTT, in both cases of  $\Delta t_{Thres} = 1,0s$  and when combined with (11), produces deviations within the threshold (10 m) when the pulse value  $J_T \geq 500 N$ . The number of required pulse jets for the correction process is much smaller compared to the method TT và IPP (around 30 pulse engines).

- The number of required pulse jets for the trajectory correction process using the method IMTT is smaller compared to when the activation time threshold is fixed, especially at the pulse value  $J_T = 500 N$  (24 pulse engines compared to 32 pulse engines).



**Figure 9.** Deviations by MTT method (top) and IMTT method (bottom).

- At the pulse value  $J_T = 300 N$ , only the method IMTT keeps the deviation of the predicted collision point within the threshold. This can be explained as follows: according to figure 11, the value of activation time threshold is changed over time. In the middle phase of the trajectory, this value is larger so fewer pulses are used. In the terminal phase, we have  $\Delta t_{Thres} < 1,0 s$ , therefore, the pulse jet controller will activate more pulse jets to reduce the deviation of the impact point.

**Overall comments:**

- Comparing the simulation results of the TT, IPP and MTT trajectory correction methods, it is found that these methods use controller with activation time threshold according to formula (11) (instead of constant value) increases the trajectory correction accuracy (MTT method) and reduce the number of required pulses (all three methods).

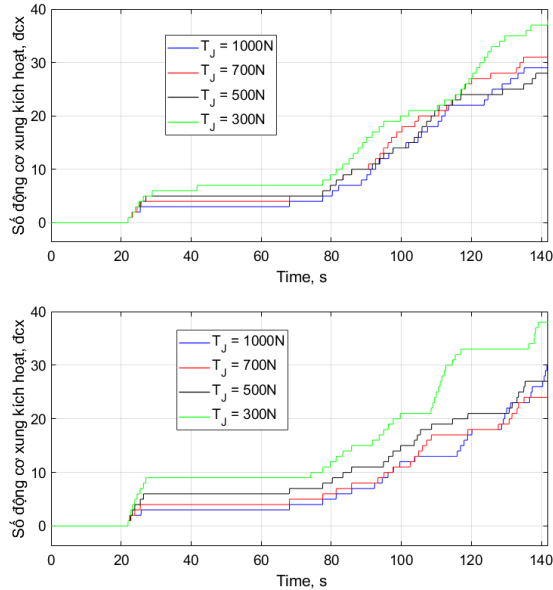
- The IMTT method proposed in the paper (MTT method combined with (11)) has advantages over the remaining methods, including accuracy under conditions of small pulse value ( $T_J = 300 N$ ) and the smallest number of pulses required for the correction process ( $N_j$ ).

**4. CONCLUSIONS**

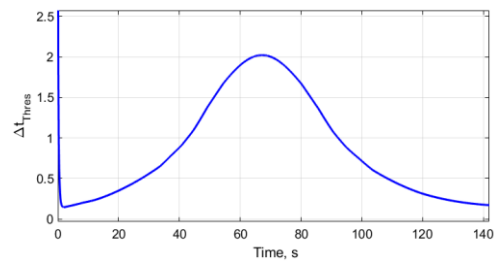
Based on the constructed mathematical model, the paper presented trajectory correction methods for flying devices using the Global Navigation Satellite System (GNSS) and the algorithm of the lateral pulse jet controller. From there, the entire controlled flight process of the flying device in 6DOF was simulated.

By comparing the simulation results, it was observed that the proposed method IMTT ensured that the deviation of the flying device's impact point from the target was less than 10 m, while requiring fewer pulse jets during the control process compared to the methods TT, IPP and MTT.

The research results provide a scientific basis for optimizing the pulse jet controller, as



**Figure 10.** The number of used pulses by MTT (top) and IMTT (bottom).



**Hình 11.** Activation time threshold according to (11).

well as designing control system for flying devices using the GNSS and a one-time lateral pulse jet controller.

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### ABSTRACT

#### **Phương pháp nâng cao độ chính xác hiệu chỉnh quỹ đạo bằng ma trận động cơ xung bên**

*Bài báo trình bày các phương pháp hiệu chỉnh quỹ đạo bằng ma trận động cơ xung trên thiết bị bay sử dụng hệ thống dẫn đường định vị toàn cầu (GNSS). Trong thuật toán điều khiển ma trận động cơ xung, thời gian kích hoạt giữa hai lần xung liên tiếp không được cố định ở một giá trị cho trước mà được xác định theo đặc trưng của thiết bị bay và tham số quỹ đạo. Kết quả mô phỏng và so sánh giữa các trường hợp cho thấy, phương pháp hiệu chỉnh quỹ đạo đề xuất trong bài báo có ưu điểm hơn các phương pháp còn lại khi yêu cầu số lượng xung và giá trị mỗi xung là nhỏ nhất để đảm bảo sai lệch điểm va chạm nằm trong ngưỡng cho trước.*

**Từ khóa:** Ma trận động cơ xung; Bám sát quỹ đạo tham chiếu; Dự đoán điểm va chạm.