Developing new model order reduction algorithms to enhance the efficiency of large-scale electrical and electronic circuit simulation: Mixed Balanced Truncation and Riccati–Lyapunov Mixed Balanced Truncation

Dao Huy Du[1], Nguyen Thanh Tung[2*], Vu Ngoc Kien[1],
Ho Mau Viet[2], Hoang Van Ta[3], Trieu Viet Phuong[4]

[1]Thai Nguyen University of Technology, 3-2 Street, Tich Luong Ward, Thai Nguyen City, Vietnam;
[2]University of Information and Communication Technology, Z115 Street, Quyet Thang Commune, Thai Nguyen City, Vietnam;
[3]College of Technology and Trade, Luong Son Ward, Thai Nguyen City, Vietnam;
[4]Vietnam National Standards and Quality Institute, 8 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam.
[*]Corresponding author: nttung@ictu.edu.vn

Received 26 Oct. 2024; Revised 05 Dec. 2024; Accepted 05 Feb. 2025; Published 25 Feb. 2025.
DOI: https://doi.org/10.54939/1859-1043.j.mst.101.2025.13-22

ABSTRACT

This paper investigates model order reduction (MOR) techniques for simulating large-scale electrical and electronic systems, aiming to reduce computational cost and optimize performance while preserving essential physical properties. In particular, two novel reduction algorithms—Mixed Balanced Truncation (MBT) and Riccati–Lyapunov Mixed Balanced Truncation (MRLBT)—are developed to improve efficiency compared to standard Balanced Truncation (BT) and Positive Real Balanced Truncation (PRBT) methods. Both the MBT and MRLBT algorithms maintain the stability and passivity of the original system. The paper details the implementation steps of the algorithms and compares their performance on RLC circuits through simulations that include error analysis as well as time- and frequency-domain response evaluations. The results indicate that MBT strikes a balance between accuracy and computational cost, with reduction errors positioned between those of BT and PRBT, while MRLBT achieves the best overall performance and meets the model reduction requirements most effectively among the four methods considered.

Keywords: Model Order Reduction; Balanced Truncation; Positive Real Balanced Truncation; Mixed Balanced Truncation; Riccati–Lyapunov Mixed Balanced Truncation; Large-Scale Circuits.

1. INTRODUCTION

The advancement of microelectronics and high-speed integrated circuits has created a demand for precise electromagnetic modeling. However, these models can involve numerous variables and elements, making simulation on tools such as SPICE, Modelsim, or Matlab increasingly challenging as the system scales up. Moreover, with the growing complexity of electrical networks, employing detailed mathematical models for system analysis becomes impractical. High-order systems increase computational complexity, requiring large amounts of memory and slow processing times. To address this issue, several solutions-such as using more powerful microprocessors, parallel simulation, or distributed computing-have been implemented. Nevertheless, Model Order Reduction (MOR) [1] has attracted significant interest, as it enables the creation of low-order systems that mimic the original system's response while demanding lower computational and storage resources, making them suitable for real-time applications. Additionally, MOR facilitates dynamic system analysis, thereby reducing costs and enhancing simulation efficiency.

In surveying the model reduction methods for electrical and electronic circuits, the author observed that each technique possesses its own strengths and weaknesses, and no single method meets all the requirements for every reduction target. The current techniques still have many limitations. Specifically, methods based on Krylov subspaces [2-5] and those based on Proper

Orthogonal Decomposition (POD) [6, 7] boast low computational costs, with the reduced model's response closely matching that of the original system and yielding small reduction errors; however, the resulting low-order models do not necessarily preserve stability and passivity. Modal Truncation (MT) methods [8-10] have the advantage of retaining key eigenvalues, poles, or zeros to maintain the original system's stability, but they do not guarantee the preservation of passivity. Among the Gramian-based balanced truncation methods, the Balanced Truncation (BT) algorithm [11-18] and the Positive-Real Balanced Truncation (PRBT) method [19-24] have garnered considerable attention and are generally regarded as superior to the aforementioned techniques, since BT preserves stability, while PRBT preserves both stability and passivity. However, PRBT incurs a higher computational cost and produces larger reduction errors (in the H∞ norm) than BT.

Some studies have attempted to combine BT and PRBT methods to overcome their respective limitations, as discussed in [22, 25-33]. However, these combined approaches are mainly applicable to standard linear time-invariant systems and cannot be directly applied to Linear Time-Invariant Continuous-Time Descriptor (LTI CTD) systems (1) commonly encountered in electrical and electronic circuit modeling. Moreover, the scientific articles [26, 31-33] do not provide an upper bound for the global reduction error, and the studies [25, 29, 30] involve high computational costs when solving the two Riccati equations.

Based on this analysis, the author focuses on developing two new model reduction algorithms to overcome the drawbacks and leverage the advantages of BT and PRBT. These algorithms are applied to the LTI CTD system (1) with the following corresponding properties:

$$\mathbf{G}(s):\begin{cases}\mathbf{E}\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t)=\mathbf{C}\mathbf{x}(t)+\mathbf{D}\mathbf{u}(t)\end{cases}\Leftrightarrow \mathbf{G}(s)=\mathbf{C}(s\mathbf{E}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D} \tag{1}$$

Where $\mathbf{E}\in\mathbb{R}^{nxn},\mathbf{A}\in\mathbb{R}^{nxn},\mathbf{B}\in\mathbb{R}^{nxp},\mathbf{C}\in\mathbb{R}^{pxn},\mathbf{D}\in\mathbb{R}^{pxp}$, respectively, are the descriptor matrix, the state-space matrix, the input matrix, the output matrix, and the direct transmission matrix; $\mathbf{x}(t)\in\mathbb{R}^{n},\mathbf{u}(t)\in\mathbb{R}^{p},\mathbf{y}(t)\in\mathbb{R}^{p}$ are, respectively, the state variable vector, the control input vector, and the output vector; and *n* and p denote, respectively, the system order (complexity) and the number of inputs (which equals the number of outputs). In large-scale electrical and electronic circuits, *n* is very large, while *p* is relatively small ($n \gg p$). The system (1) is characterized as minimal, stable, and passive, with the initial condition defined by zero state, input, and output vectors. Both matrices **E** and **A** are non-singular, full-rank, and positive definite, $\mathbf{B} = \mathbf{C}^T$, and ($\mathbf{D} + \mathbf{D}^T$) is guaranteed to be non-negative.

The newly developed algorithm must preserve the intrinsic physical characteristics of electrical and electronic circuits-namely stability and passivity-while ensuring low reduction error and computational cost, and it provides an expression that determines an upper bound for the global reduction error.

2. THE MIXED BALANCED TRUNCATION (MBT) ALGORITHM

The MBT algorithm is also based on Gramian balancing, similar to BT and PRBT; however, it differs in that the controllability and observability Gramians in MBT are not obtained from two Lyapunov equations or two positive-real Riccati equations, but rather are solved from one Lyapunov equation and one Riccati equation. This approach reduces computational cost compared to PRBT while achieving a reduction error that is intermediate between BT and PRBT. A notable advantage of MBT is its ability to guarantee passivity in the reduced system-a feature that BT does not ensure-while preserving the original system's characteristics and maintaining the inherent stability and passivity of electrical circuits. Furthermore, this method enables the determination of an upper bound on the reduction error based on the Hankel singular values of the mixed balanced system. The MBT algorithm is implemented through the following steps:

Inputs: Electrical circuits are characterized and modeled with an input-output relationship represented as an LTI CTD as shown in (1). Enter the order of the system to be reduced to: *r*.

- Step 1. Determine the pair of controllability and observability Gramians ($\mathbf{P}$, $\mathbf{R}_o$) from (2) and (3) or the Gramian pair ($\mathbf{R}_c$, $\mathbf{Q}$) from (4) and (5).

$$\mathbf{APE}^T + \mathbf{EPA}^T + \mathbf{BB}^T = \mathbf{0} \tag{2}$$

$$\mathbf{A}^T\mathbf{R}_o\mathbf{E} + \mathbf{E}^T\mathbf{R}_o\mathbf{A} + (\mathbf{E}^T\mathbf{R}_o\mathbf{B} - \mathbf{C}^T)(\mathbf{D} + \mathbf{D}^T)^{-1}(\mathbf{E}^T\mathbf{R}_o\mathbf{B} - \mathbf{C}^T)^T = \mathbf{0} \tag{3}$$

$$\mathbf{AR}_c\mathbf{E}^T + \mathbf{ER}_c\mathbf{A}^T + (\mathbf{ER}_c\mathbf{C}^T - \mathbf{B})(\mathbf{D} + \mathbf{D}^T)^{-1}(\mathbf{ER}_c\mathbf{C}^T - \mathbf{B})^T = \mathbf{0} \tag{4}$$

$$\mathbf{A}^T\mathbf{QE} + \mathbf{E}^T\mathbf{QA} + \mathbf{C}^T\mathbf{C} = \mathbf{0} \tag{5}$$

- Step 2. Compute the Cholesky decompositions of $\mathbf{P}$ and $\mathbf{R}_o$ as in (6) and (7), where $\mathbf{J}$ and $\mathbf{K}$ are lower triangular and invertible matrices.

$$\mathbf{P} = \mathbf{JJ}^T \tag{6}$$

$$\mathbf{R}_o = \mathbf{KK}^T \tag{7}$$

- Step 3. Perform the singular value decomposition (SVD) of $\mathbf{K}^T\mathbf{J}$ as in (8).

$$\mathbf{K}^T\mathbf{J} = \mathbf{UM}_B\mathbf{V}^T = \begin{bmatrix}\mathbf{U}_r & \mathbf{U}_{n-r}\end{bmatrix}\begin{bmatrix}\mathbf{M}_{B_r} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{B_{n-r}}\end{bmatrix}\begin{bmatrix}\mathbf{V}_r \\ \mathbf{V}_{n-r}\end{bmatrix} \tag{8}$$

- Step 4. Compute the equivalent balanced transformation matrices $\mathbf{T}$ and $\mathbf{T}^{-1}$ according to (9) and (10).

$$\mathbf{T} = \mathbf{JVM}_B^{-1/2} \tag{9}$$

$$\mathbf{T}^{-1} = \mathbf{M}_B^{-1/2}\mathbf{U}^T\mathbf{K}^T \tag{10}$$

- Step 5. Determine the matrices of the equivalent mixed balanced system as in (11).

$$\mathbf{E}_{MB} = \mathbf{T}^{-1}\mathbf{ET}; \mathbf{A}_{MB} = \mathbf{T}^{-1}\mathbf{AT}; \mathbf{B}_{MB} = \mathbf{T}^{-1}\mathbf{B}; \mathbf{C}_{MB} = \mathbf{CT}; \mathbf{D}_{MB} = \mathbf{D} \tag{11}$$

- Step 6. Truncation of the last *n* – *r* rows and *n* – *r* columns from the matrices $(\mathbf{E}_{MB}, \mathbf{A}_{MB}, \mathbf{B}_{MB}, \mathbf{C}_{MB}, \mathbf{D}_{MB})$

Output: A reduced-order system of order *r* that preserves stability and passivity, with $\mathbf{E}_r \in \mathbb{R}^{rxr}, \mathbf{A}_r \in \mathbb{R}^{rxr}, \mathbf{B}_r \in \mathbb{R}^{rxp}, \mathbf{C}_r \in \mathbb{R}^{pxr}, \mathbf{D}_r \in \mathbb{R}^{pxp}$.

The reduced-order system obtained using the MBT algorithm has an H∞ norm error bound with respect to the original system, as specified in (12).

$$\left\|\mathbf{G}(s) - \mathbf{G}_r(s)\right\|_{H_{inf}} \le 2\sum_{r+1}^{n} m_i \tag{12}$$

where $m_1 > m_2 > ... > m_n > 0$ denotes the Hankel singular values of the mixed balanced system, determined as in (13).

$$\mathbf{T}^{-1}\mathbf{PT}^{-T} = \mathbf{T}^T\mathbf{R}_o\mathbf{T} = \mathbf{T}^{-1}\mathbf{R}_c\mathbf{T}^{-T} = \mathbf{T}^T\mathbf{QT} = diag\left(m_1, m_2, ..., m_n\right) \tag{13}$$

3. THE MIXED RICCATI–LYAPUNOV BALANCED TRUNCATION (MRLBT) ALGORITHM

The MBT reduction algorithm offers several advantages over BT and PRBT, such as reducing computational cost while preserving the system's stability and passivity. However, the H∞ norm error of MBT is typically intermediate between that of BT and PRBT, which raises the issue of needing improved accuracy in the reduction process. To minimize the error between the original and reduced systems relative to both BT and PRBT methods, the author has developed a new model reduction algorithm called Mixed Riccati–Lyapunov Balanced Truncation (MRLBT). Similar to MBT, MRLBT determines the controllability Gramian and the observability Gramian from one Lyapunov equation and one positive-real Riccati equation. However, the key difference in MRLBT is that the method is implemented on the matrices of a new system-referred to as the Riccati–Lyapunov mixed balanced system-which is derived from the original system according to the expressions in (17). Consequently, the equations (2) to (5) take on the corresponding forms in (18) to (21). The MRLBT algorithm is implemented through the following steps:

Input: Similar to the input of the MBT algorithm, an LTI CTD system (1) is given along with the desired reduced order *r*.

- Step 1. Introduce auxiliary variables and transform the system matrices: the state-space matrix $\mathbf{A}_n$, the input matrix $\mathbf{B}_n$, and the output matrix $\mathbf{C}_n$, as shown in (14).

$$\mathbf{F} = (\mathbf{D} + \mathbf{D}^T)^{-1}; \mathbf{A}_n = (\mathbf{A} - \mathbf{BFC}); \mathbf{B}_n = \mathbf{BF}^{1/2}; \mathbf{C}_n = \mathbf{F}^{1/2}\mathbf{C} \quad (14)$$

- Step 2. Determine the pair of controllability and observability Gramians ($\mathbf{L}_c$, $\mathbf{R}_o$) from (15) and (16) or the Gramian pair ($\mathbf{R}_c$, $\mathbf{L}_o$) from (17) and (18).

$$\mathbf{A}_n\mathbf{L}_c\mathbf{E}^T + \mathbf{E}\mathbf{L}_c\mathbf{A}_n^T + \mathbf{B}_n\mathbf{B}_n^T = \mathbf{0} \quad (15)$$

$$\mathbf{A}_n^T\mathbf{R}_o\mathbf{E} + \mathbf{E}^T\mathbf{R}_o\mathbf{A}_n + \mathbf{E}^T\mathbf{R}_o\mathbf{B}_n\mathbf{B}_n^T\mathbf{R}_o\mathbf{E} + \mathbf{C}_n^T\mathbf{C}_n = \mathbf{0} \quad (16)$$

$$\mathbf{A}_n\mathbf{R}_c\mathbf{E}^T + \mathbf{E}\mathbf{R}_c\mathbf{A}_n^T + \mathbf{E}\mathbf{R}_c\mathbf{C}_n^T\mathbf{C}_n\mathbf{Q}_c\mathbf{E}^T + \mathbf{B}_n\mathbf{B}_n^T = \mathbf{0} \quad (17)$$

$$\mathbf{A}_n^T\mathbf{L}_o\mathbf{E} + \mathbf{E}^T\mathbf{L}_o\mathbf{A}_n + \mathbf{C}_n^T\mathbf{C}_n = \mathbf{0} \quad (18)$$

- Step 3. Perform the Cholesky decompositions of $\mathbf{L}_c$ and $\mathbf{R}_o$ as in (19) and (20), where $\mathbf{J}_n$ and $\mathbf{K}$ are lower triangular and invertible matrices.

$$\mathbf{L}_c = \mathbf{J}_c\mathbf{J}_c^T \quad (19)$$

$$\mathbf{R}_o = \mathbf{K}\mathbf{K}^T \quad (20)$$

- Step 4. Compute the singular value decomposition (SVD) of $\mathbf{K}^T\mathbf{J}_c$ as in (21).

$$\mathbf{K}^T\mathbf{J}_c = \mathbf{XMZ}^T = \begin{bmatrix}\mathbf{X}_r & \mathbf{X}_{n-r}\end{bmatrix}\begin{bmatrix}\mathbf{M}_r & \mathbf{0}\\ \mathbf{0} & \mathbf{M}_{n-r}\end{bmatrix}\begin{bmatrix}\mathbf{Z}_r\\ \mathbf{Z}_{n-r}\end{bmatrix} \quad (21)$$

- Step 5. Calculate the balanced transformation matrices $\mathbf{T}_n$ and its inverse (or the equivalent transformation) according to (22) and (23).

$$\mathbf{T}_n = \mathbf{J}_c\mathbf{Z}\mathbf{M}^{-1/2} \quad (22)$$

$$\mathbf{T}_n^{-1} = \mathbf{M}^{-1/2}\mathbf{X}^T\mathbf{K}^T \quad (23)$$

- Step 6. Determine the matrices $(\mathbf{E}_M, \mathbf{A}_M, \mathbf{B}_M, \mathbf{C}_M, \mathbf{D}_M)$ of the equivalent mixed balanced system corresponding to MRLBT as in (24).

$$\mathbf{E}_M = \mathbf{T}^{-1}\mathbf{E}\mathbf{T}; \mathbf{A}_M = \mathbf{T}^{-1}\mathbf{A}_n\mathbf{T}; \mathbf{B}_T = \mathbf{T}^{-1}\mathbf{B}_n; \mathbf{C}_M = \mathbf{C}_n\mathbf{T}; \mathbf{D}_M = \mathbf{D} \quad (24)$$

- Step 7. Truncation of the last $n-r$ rows and $n-r$ columns from the matrices $(\mathbf{E}_M, \mathbf{A}_M, \mathbf{B}_M, \mathbf{C}_M, \mathbf{D}_M)$

Output: A reduced-order system of order r that preserves stability and passivity $\mathbf{G}_{n_r}(s):\left(\mathbf{E}_{n_r}, \mathbf{A}_{n_r}, \mathbf{B}_{n_r}, \mathbf{C}_{n_r}, \mathbf{D}_{n_r}\right)$ with $\mathbf{E}_{n_r} \in \mathbb{R}^{rxr}, \mathbf{A}_{n_r} \in \mathbb{R}^{rxr}, \mathbf{B}_{n_r} \in \mathbb{R}^{rxp}, \mathbf{C}_{n_r} \in \mathbb{R}^{pxr}, \mathbf{D}_{n_r} \in \mathbb{R}^{pxp}$.

The reduced-order system obtained using the MRLBT algorithm has an H_∞ norm error bound given by expression (25):

$$\left\|\mathbf{G}(s) - \mathbf{G}_r(s)\right\|_{H_{\inf}} \le 2\sum_{r+1}^{n} m_i \quad (25)$$

with, $\mu_1 > \mu_2 > ... > \mu_n > 0$ are the Hankel singular values of the Riccati–Lyapunov mixed balanced system, determined according to (26).

$$\mathbf{T}^{-1}\mathbf{L}_c\mathbf{T}^{-T} = \mathbf{T}^{T}\mathbf{R}_o\mathbf{T} = \mathbf{T}^{-1}\mathbf{R}_c\mathbf{T}^{-T} = \mathbf{T}^{T}\mathbf{L}_o\mathbf{T} = diag\left(\mu_1, \mu_2, ..., \mu_n\right) \quad (26)$$

4. SIMULATION, COMPUTATION, AND DISCUSSION

Consider the RLC electrical network shown in figure 1 [34, 35]. This single-input single-output (SISO) system has 51 nodes (*k = 51*) and is of order *n = 101*. The resistances are given as *RS = 0.2, RE = 0.5, R = 1*, while all inductors and capacitors share the same value $L_j = C_i$. The voltage *u* is the input, and the current *y* is the output. The state variables are defined such that x_{2i-1} represents the voltage across the i-th capacitor, and x_{2j} represents the current through the j-th inductor $(i = \overline{1:k};\ j = \overline{1:(k-1)})$.

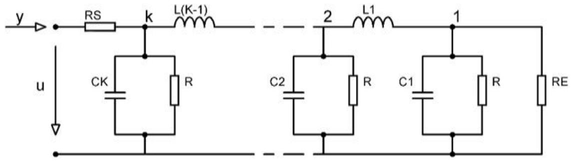

Figure 1. *Stable and passive RLC network.*

From the error plots in figure 2, the following observations can be made:

- The reduction error increases as the reduced order r decreases for all four algorithms, which is consistent with both theory and practice-since lowering the order naturally diminishes accuracy.

- There are minor fluctuations in the error values for all methods at each order, attributable to the inherent complexity of the system and the characteristic approximations of each model reduction technique.

- Over the entire range of reduced orders, the maximum H∞ norm error is consistently observed for PRBT (blue curve), while the smallest error is achieved by MRLBT (black curve). The error from BT (red curve) is the second smallest, and the error of MBT (green curve) falls between BT and PRBT.

The BT, PRBT, MBT, and MRLBT algorithms were implemented in Matlab to reduce the order of this system from *n* to a desired order *r*. The absolute error curves between the original system and the reduced systems (of order *r*), measured in the H_∞ norm, are depicted in figure 2.

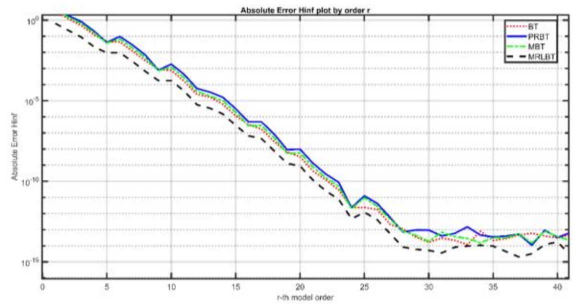

Figure 2. *H_∞ norm error plot for BT, PRBT, MBT, and MRLBT.*

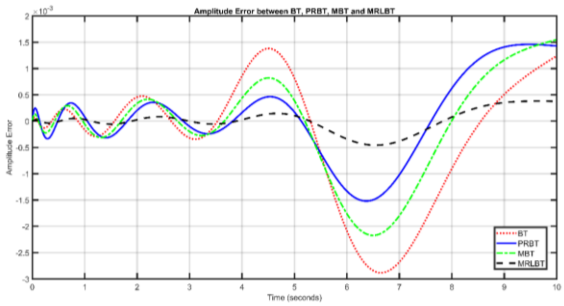

Figure 3. *Amplitude error between the original system and the order-7 reduced system using BT, PRBT, MBT, and MRLBT.*

Based on these plots, it is observed that when *r = 7* the reduced systems obtained by all four algorithms exhibit relatively small errors; therefore, the author selects an order of 7 for reducing the original system. Time-domain and frequency-domain simulations were then carried out, resulting in the error plots shown in figures 3, 4, and 5.

From the amplitude error plot in figure 3, it can be seen that:

- The amplitude error in the time domain increases for all four algorithms (BT, PRBT, MBT, and MRLBT) when the system is reduced to order 7.

- Over the time interval considered, MRLBT consistently exhibits the smallest error, followed by PRBT, while BT shows the largest error; MBT's error lies between those of BT and PRBT.

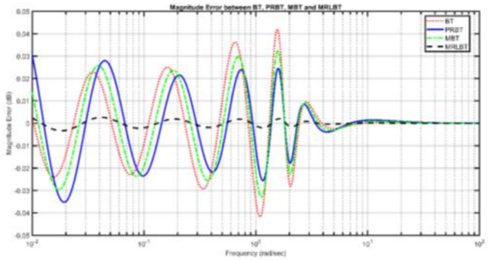

Figure 4. *Magnitude error between the original system and the order-7 reduced system using BT, PRBT, MBT, and MRLBT.*

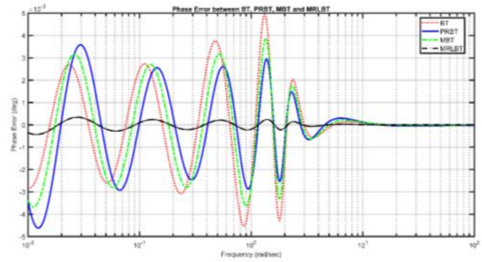

Figure 5. *Phase error between the original system and the order-7 reduced system using BT, PRBT, MBT, and MRLBT.*

From the magnitude and phase error plots in figures 4 and 5, we observe that:

- At frequencies below 10^{-1} rad/s, the order-7 reduced system using BT exhibits a smaller error than that using PRBT; in the frequency range from 10^{-1} rad/s rad/s to 10^{1} rad/s, the error for PRBT becomes smaller than that for BT.

- At frequencies above 10^{1} rad/s, all four algorithms (BT, PRBT, MBT, and MRLBT) yield errors that are approximately zero.

- Across the entire frequency range, the error for the order-7 reduced system is always smallest for MRLBT, with MBT's performance falling between BT and PRBT.

From the magnitude error chart, the error of phase by frequency as shown in figure 4, figure 5, we can see:

- In the frequency range less than 10^{-1} rad/s, the error of BT is smaller than PRBT, in the frequency range from 10^{-1} rad/s to 10^{1} rad/s, the error of PRBT is smaller than BT.

- In the frequency range greater than 10^{1} rad/s, the 7th order reduction systems all give approximately 0 error.

- In the whole frequency, the MRLBT always has the smallest value and the MBT is always between BT and PRBT.

Table 1. *Comparison of absolute errors among the reduction methods.*

Algorithm	**H_∞ Error**	**H_2 error**
BT	0.011909372094279	0.005164057028704
PRBT	0.028399505655068	0.004450889932818
MBT	0.018438067397248	0.004584196872338
MRLBT	0.002711299200206	0.000956587026549

Table 1 presents the absolute error values for the BT, PRBT, MBT, and MRLBT reduction methods relative to the original system, based on the H∞ norm and H2 norm. From these values, we observe that:

- H_∞ norm error (measuring the maximum error over the frequency domain): MRLBT produces the smallest error (0.0027), indicating the best preservation of the original system's quality. BT has an error of 0.0119, which is lower than that of MBT (0.0184) but still significantly higher than

MRLBT. PRBT yields the largest error (0.0284).

- H_2 norm error (measuring the average error between the responses of the original and reduced systems based on input–output energy): MRLBT again demonstrates superiority with the smallest error (0.00096), markedly outperforming the other methods in minimizing energy-domain error. PRBT follows with an error of 0.00445, MBT is similar to PRBT with an error of 0.00458, and BT shows the highest error (0.00516).

Analyzing the original system G(s) and the order-7 reduced systems obtained using BT, PRBT, MBT, and MRLBT-denoted as G_BT(s), G_PRBT(s), G_MBT(s), G_MRLBT(s) respectively-we observe the following regarding poles, damping ratios, oscillation frequencies, and time constants:

- Poles, Damping Ratios, Oscillation Frequencies, and Time Constants:

+ G(s): The poles of G(s) indicate high stability, with most exhibiting a damping ratio of 1 and no overshoot. The complex poles with a real part of –5.09 and oscillatory imaginary parts suggest that the system has a lightly oscillatory response but stabilizes quickly with a time constant of approximately 2 seconds. At higher frequencies, the system may oscillate at frequencies above 1.9 rad/s, yet it maintains stability as the damping ratio decreases from 1 to 0.25.

+ G_BT(s): This system exhibits a very short time constant (as low as approximately 0.17 seconds), which allows for rapid stabilization at some high frequencies. However, with complex poles having a real part of –0.977 and an imaginary part of 1.38i, G_BT(s) tends to oscillate more strongly than G(s), with a damping ratio of only about 0.577.

+ G_PRBT(s): The poles of G_PRBT(s) are similar to those of G(s) but with a faster time constant-approximately 0.1 seconds at high frequencies.

+ G_MT(s): This system has poles at frequencies exceeding 5 rad/s, indicating a greater tendency for oscillation, and consequently requires a lower damping ratio to mitigate such oscillations.

+ G_MRLBT(s) exhibits multiple poles with a very short time constant (below 0.2 seconds) and at higher frequencies compared to the other systems, indicating extremely rapid stabilization.

The bandwidth of G(s) is 0.01852. G_BT(S) is closest to the original system, with a bandwidth of 0.01898. G_MT(S) and G_PRBT(s) have bandwidths of 0.01924 and 0.01956, respectively, which are approximately equal to that of the original system-demonstrating that G_BT(s), G_MT(s) and G_PRBT(s) are effective in preserving the dynamic characteristics. In contrast, G_MRLBT(s) has an infinite (Inf) bandwidth. This suggests that in terms of response speed to changes in the input signal-that is, the ability to quickly process variations and adjust promptly-the systems rank in the following order: G(s), G_BT(s), G_PRBT(s), G_MBT(s), G_MRLBT(s).

Table 2. *Time-domain response information for G(s), G_BT(s), G_PRBT(s), G_MBT(s) and G_MRLBT(s).*

Parameter	G(s)	G_BT(s)	G_PRBT(s)	G_MBT(s)	G_MRLBT(s)
TransientTime	831.639	762.186	697.785	734.209	551.537
SettlingTime	734.794	678.932	623.765	655.133	83.764
SettlingMin	1.1398	1.1394	1.1393	1.1393	4.3882
Overshoot (%)	9.434	9.720	10.118	9.877	0.964

- The time-domain response parameters are detailed in table 2, from which the following analysis can be observed:

+ Transient Time: The original system G(s) exhibits the longest transient time (831.639 seconds). Among the 7th-order reduced systems, the transient times are, in descending order, G_BT(s) (762.186 seconds), G_MBT(s) (734.209 seconds), G_PRBT(s) (697.785 seconds), with G_MRLBT(s) showing the shortest transient time (551.537 seconds).

+ Settling Time: G_MRLBT(s) reaches a stable state in the shortest time (83.764 seconds), indicating that it stabilizes much faster than the other methods-G_BT(s) (678.93 seconds), G_MBT(s) (655.13 seconds), and G_PRBT(s) (623.77 seconds)-and especially compared to the original system (734.794 seconds).

+ SettlingMin and SettlingMax: G_MRLBT(s) has a significantly higher SettlingMin (4.3882) compared to the other systems (approximately 1.139), meaning that the lower bound of the settling value is close to the maximum value of 5. This indicates that G_MRLBT(s) achieves the desired stability with minimal oscillation amplitude.

+ Overshoot (%): G_MRLBT(s) exhibits the lowest overshoot at 0.964%, while G_PRBT(s) shows the highest at 10.118%. The other systems have comparable overshoot values-G(s) at 9.434%, G_BT(s) at 9.720%, and G_MBT(s) at 9.877%-indicating that G_MRLBT(s) stabilizes the fastest with the least oscillatory behavior

5. CONCLUSIONS

This paper proposed and implemented two novel model reduction algorithms-Mixed Balanced Truncation (MBT) and Mixed Riccati–Lyapunov Balanced Truncation (MRLBT)-to address the limitations of traditional methods such as Balanced Truncation (BT) and Positive-Real Balanced Truncation (PRBT). Both MBT and MRLBT are designed to preserve the stability and passivity of the system while significantly reducing computational cost.

The MBT algorithm improves computational efficiency compared to PRBT while maintaining the passivity and stability of the reduced system, with reduction errors that fall between those of BT and PRBT. MRLBT, an enhancement of MBT, has demonstrated superior performance in reducing complex continuous-time linear time-invariant systems by achieving the smallest reduction error among all the methods. In doing so, MRLBT overcomes the accuracy limitations of BT, PRBT, and MBT, yielding a reduced-order model that most closely approximates the original system.

Simulation and validation results on the RLC network show that both the MBT and MRLBT algorithms achieve effective model order reduction. While MBT produces a reduced model whose quality is intermediate between BT and PRBT, MRLBT yields the smallest reduction error according to the H_∞ and H_2 norms, along with superior time-domain and frequency-domain responses compared to the other techniques. These improvements open up new potential applications in model reduction for high-order electrical and electronic systems.

Acknowledgment: *The authors thank the Ministry of Education and Training for the funding by the Ministry of Education and Training Program for the project code B2023-TNA-17.*

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TÓM TẮT

Xây dựng thuật toán giảm bậc mới để nâng cao hiệu quả mô phỏng mạch điện, điện tử cỡ lớn: Cắt ngắn cân bằng hỗn hợp và Cắt ngắn cân bằng hỗn hợp Riccati-Lyapunov

Bài báo này nghiên cứu các phương pháp giảm bậc mô hình (MOR) trong mô phỏng hệ thống điện, điện tử tử cỡ lớn, nhằm giảm chi phí tính toán và tối ưu hoá hiệu suất mà vẫn duy trì được các đặc tính vật lý quan trọng. Đặc biệt, hai thuật toán giảm bậc mới, Cắt ngắn cân bằng hỗn hợp (MBT) và Cắt ngắn cân bằng hỗn hợp Riccati-Lyapunov (MRLBT), được phát triển để cải thiện hiệu quả so với các phương pháp Cắt ngắn cân bằng (BT) và Cắt ngắn cân bằng thực dương (PRBT). Cả hai thuật toán MBT và MRLBT đều bảo toàn tính ổn định và tính thụ động của hệ gốc. Bài báo trình bày chi tiết các bước triển khai thuật toán, so sánh hiệu quả của chúng trên mạng điện RLC, thông qua các mô phỏng tiến hành phân tích sai số và mô phỏng các đáp ứng trên miền thời gian và tần số. Kết quả cho thấy MBT đạt được sự cân bằng giữa độ chính xác và chi phí tính toán, sai số giảm bậc nằm giữa BT và PRBT, còn MRLBT có hiệu suất và đáp ứng các yêu cầu giảm bậc tốt nhất trong số bốn thuật toán được xét.

Từ khoá: Giảm bậc mô hình; Cắt ngắn cân bằng; Cắt ngắn cân bằng thực dương; Cắt ngắn cân bằng hỗn hợp; Cắt ngắn cân bằng hỗn hợp Riccati-Lyapunov; Mạch điện cỡ lớn.