# Study on the effect of pressure on the thrust force of a solid propellant engine operating in a water environment

Nguyen Truong Thanh[1], Hoang The Dung[1*], Bui Dinh Tan[1],
Bui Ngoc Lam[2], Nguyen Van Hung[3], Nguyen Huy Thanh[3]

[1]Institute of Missile, Academy of Military Science and Technology, 17 Hoang Sam, Cau Giay, Hanoi, Vietnam;
[2]Factory X28, Navy Technical Department, Lien Khe, Thuy Nguyen, Hai Phong, Vietnam;
[3]Le Quy Don university, 236 Hoang Quoc Viet, Bac Tu Liem, Hanoi, Vietnam.
[*]Corresponding author: hnpanh@gmail.com

*Received 30 Nov. 2024; Revised 15 Jan. 2025; Accepted 04 Apr. 2025; Published 15 Apr. 2025.*
**DOI:** https://doi.org/10.54939/1859-1043.j.mst.102.2025.141-146

## ABSTRACT

*The paper presents the results of research on the impact of ambient pressure on the thrust of solid rocket engines when operating underwater at different depths. The study establishes a system of equations for rocket propulsion, considering the variations in ambient pressure while functioning in an aquatic environment. It develops algorithms and constructs a computational program for rocket propulsion in solid rocket engines operating underwater. The paper further explores how water pressure affects the engine's pressure and thrust.*

**Keywords:** Solid fuel rocket motor; Effect of pressure on thrust; Operation of solid propellant rocket engine in underwater environment.

## 1. INTRODUCTION

Solid rocket engines, with their notable advantages such as high combat readiness, ease of use in operations, and convenient storage and handling, have been extensively studied by scientists both domestically and internationally. In Vietnam, prominent studies include: Reference [1] discusses the dynamics of gas flow, drag parameters, and methods for calculating the thrust of solid rocket engines. It also covers the fundamentals of launch dynamics calculations, accounting for the heat generated by various propellants and thermal losses through the engine walls. This work proposes designs for nozzles and solid rocket engines. Reference [2] outlines the principles behind designing solid-propellant rocket engines for unguided projectiles. It includes discussions on packing density factors, the elements that influence these coefficients, and optimal engine structure design. Reference [4] investigates inverse launch dynamics to determine the parameters of the propellant charge. Reference [5] explores the impact of combustion chamber parameters on the operational characteristics of solid rocket engines, particularly focusing on the influence of multiphase flow, such as the presence of aluminum powder in the gas flow. The study highlights the reduction in thrust caused by the inclusion of solid phases in the gas flow. Despite the significant interest from scientists studying solid rocket engines, research on their operation in underwater environments remains limited.

Globally, there are various weapons and equipment that use solid rocket engines to operate in multi-environment conditions, such as the KPM, PM-1, PM-2, and PM-2G naval mines. However, due to their classified nature, accessing detailed documentation on these systems is challenging. Therefore, studying the effects of pressure on the thrust of solid rocket engines in underwater environments is both a pressing and scientifically significant issue.

## 2. ESTABLISHING THE LAUNCH DYNAMICS EQUATIONS FOR SOLID ROCKET ENGINES OPERATING IN UNDERWATER ENVIRONMENTS

When formulating the system of equations Solid Rocket Motor Internal Ballistics in studying the effects of environmental pressure on engine force, the following assumptions are adopted:

- The ignition charge undergoes complete combustion, ensuring reliable ignition of all combustible surfaces of the propellant charge;

- The static pressure along the combustion chamber is assumed to be constant and equal to the stagnation pressure at the trailing end of the propellant charge (adjacent to the nozzle inlet);

- The propellant charge burns according to a geometric combustion model;

- The gas flow within the nozzle is considered adiabatic;

- The temperature within the combustion chamber varies over time and is governed by a heat loss coefficient, $\chi < 1$, reflecting heat transfer to the chamber walls and the incomplete nature of chemical reactions.

### 2.1. Establishing the system of equations for calculating launch dynamics in rocket engines operating underwater

According to reference [3], the system of equations for the internal ballistics of the engine is expressed in the form:

$$\begin{cases} \dfrac{dW_{td}}{d\tau} = Su; \dfrac{de}{d\tau} = u = u_1 p^{v}; \dot{m} = \dfrac{\varphi_2 . K_0(k) . F_{th} . p}{\sqrt{RT}}; \\ \dfrac{dT}{d\tau} = \dfrac{T}{W_{td}\, p}\left[ S\rho_T uR\left(\chi T_v - T\right) - \left(k-1\right)\dot{m}RT \right]; \\ \dfrac{dp}{d\tau} = \dfrac{1}{W_{td}}\left( S\rho_{tp} uR\chi T_v - k\dot{m}RT - Spu \right). \end{cases} \tag{1}$$

With the initial conditions when:

$$t=0:\ W_{td} = W_{td\,0};\ e = e_0;\ m = m_0;\ T = T_m;\ p = p_{0m}. \tag{2}$$

When the propellant is fully combusted, the system of differential equations for the free gas expulsion stage takes the following form:

$$\begin{cases} \dfrac{dT}{d\tau} = -\left(k-1\right).\varphi_2 . A_k . F_{th} . \sqrt{RT} . \dfrac{T}{W_{td}}; \\ \dfrac{dp}{d\tau} = -\dfrac{k.\dot{m}.R.T}{W_{td}}; \dot{m} = \dfrac{\varphi_2 . K_k(k) . F_{th}\, p}{\sqrt{RT}} \end{cases} \tag{3}$$

With the initial conditions when $t=t_1$:

$$W_{td} = W_{Max};\ e = e_0;\ m = m_0;\ T = T_m;\ p = p_{k1}. \tag{4}$$

where:

+ $\dot{m}$ : The mass flow rate of the gas at any cross-sectional area;

+ $u_1, v$: The burn rate coefficients of the ballistic propellant;

+ S: The burning surface area of the ballistic propellant;

+ $F_{th}$: Critical area;

+ $W_{td}, W_{td0}, W_{max}$ : The free volume of the combustion chamber at time ttt, the initial time, and the maximum;

+ $\varphi_2$ : Considering flow losses characterized by the loss coefficient;

+ $k$: Adiabatic exponent;

+ $T$, $T_m$: Represent the temperature in the combustion chamber at time t and the ambient temperature;

+ m, $m_0$: Represent the ballistic mass at time ttt and the initial ballistic mass;

+ $e$, $e_0$: Represent the burn thickness at time t and the initial burn thickness;

+ $R$ is the specific gas constant;

+ $\chi$: The thermal loss through the engine combustion chamber walls;

+ $\rho_{tp}$: The density of the ballistic propellant;

+ $p_{k1}$: The combustion chamber pressure at complete ballistic propellant burnout;

+ $K_0(k) = \left(\dfrac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\dfrac{2k}{k+1}}; A = \dfrac{K_0(k)}{\sqrt{R.T_0}}$: is the gas flow coefficient, theo [1].

According to reference [1], the thrust of the rocket engine is determined by the formula:

$$P = \varphi_2.K_0(k).F_w\left(\xi_a, k\right).F_{th}.p_0 + F_a.\left(p_a - p_n\right) \tag{5}$$

where:

$$+F_w\left(\xi_a, k\right) = \sqrt{\frac{2k}{k-1}.\left(1 - x^{\frac{k-1}{k}}\right)}; \xi_a = \frac{d_a}{d_{th}}; x = \frac{p_a}{p_0} \tag{6}$$

+ $p_0$, $p_n$ and $p_a$: represent the pressure at the nozzle exit and the ambient pressure;

+ $d_{th}$ and $d_a$: represent the critical diameter and the nozzle exit diameter;

+ $F_a$: the area of the nozzle exit.

## 2.2. Solution method for the system of equations in thrust computation

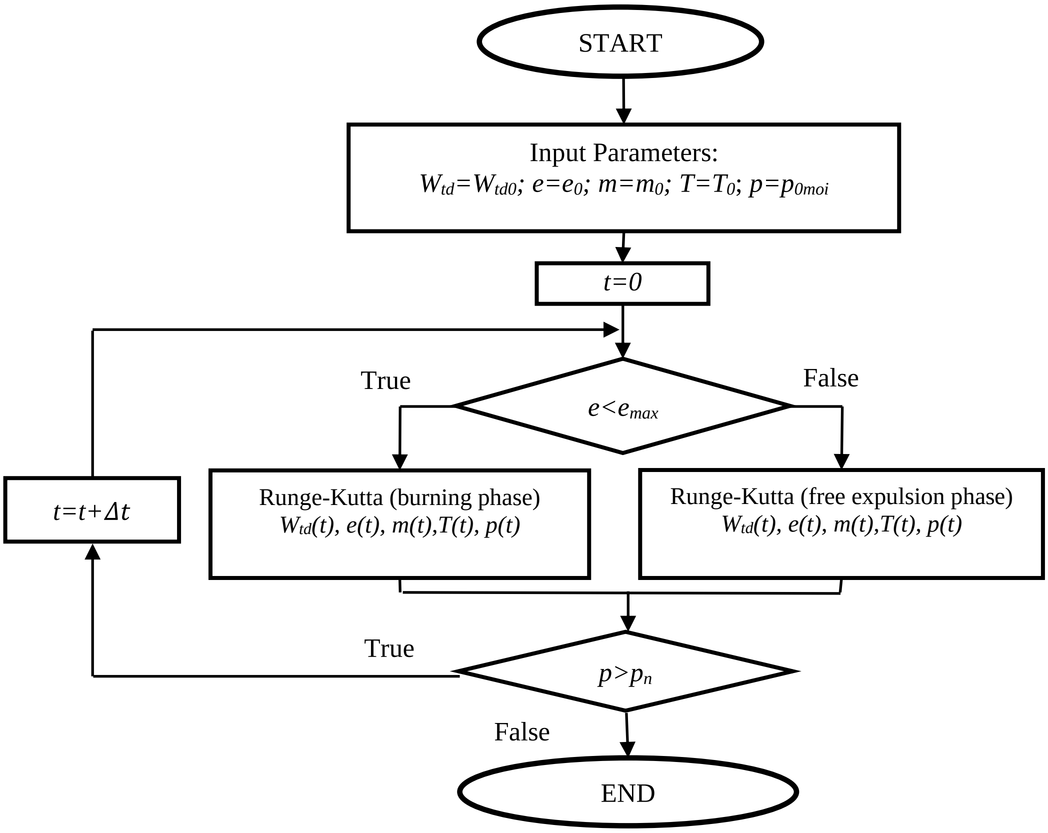


***Figure 1.*** *Block diagram of the algorithm for solving the thrust calculation problem in ballistic solid rocket engines.*

The Runge-Kutta method is employed to solve the differential equations (1) and (3). The algorithmic flowchart for solving the thrust computation problem of the engine is presented in figure 1.

## 3. INVESTIGATION OF THE INFLUENCE OF PRESSURE ON THE THRUST OF SOLID PROPELLANT ENGINES OPERATING IN WATER ENVIRONMENTS

### 3.1. Input data

Study of solid rocket engines (with a structure as shown in figure 2, table 1, and fuel charge parameters according to figure 3, table 2), operating in water environments at different depths.

***Table 1.*** *Structural parameters and fuel charge parameters of the rocket engine.*

| TT | Parameters | Symbol | Value | Unit |
|---|---|---|---|---|
| 1 | Inner diameter of the engine | $D_k$ | 0,116 | m |
| 2 | Engine length | $L_k$ | 0,880 | m |
| 3 | Critical diameter of the nozzle | $d_{th}$ | 0,037 | m |
| 4 | Nozzle exit diameter | $d_a$ | 0,066 | m |

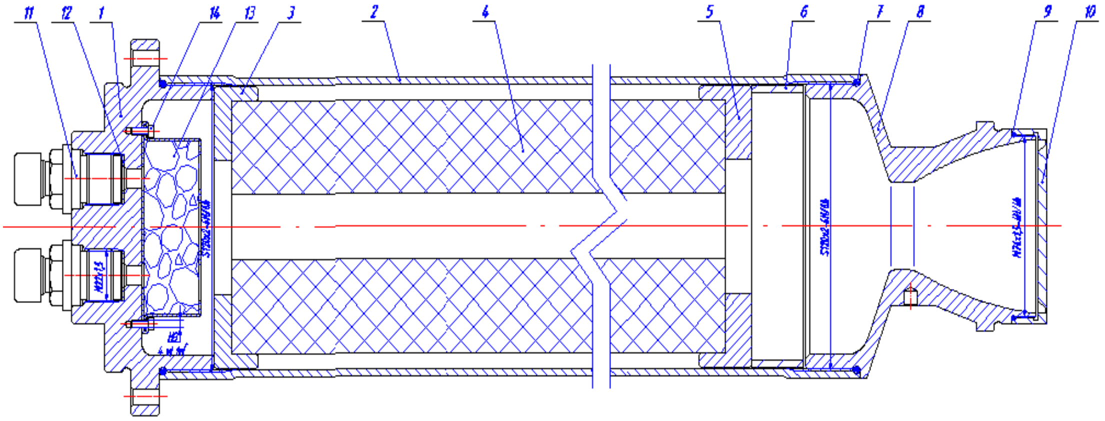


***Figure 2.*** *Structural diagram of a solid propellant engine operating in water environment:*
*1. Front bottom; 2. Engine cover; 3. Front cigarette lighter; 4. Fuel dose;*
*5. Rear cigarette lighter 1; 6. Bushing; 7. Gasket; 8. Rear bottom; 9. Gasket;*
*10. Nozzle cover; 11. Ignition point; 12. Copper gasket; 13. Primer box; 14. Screw.*

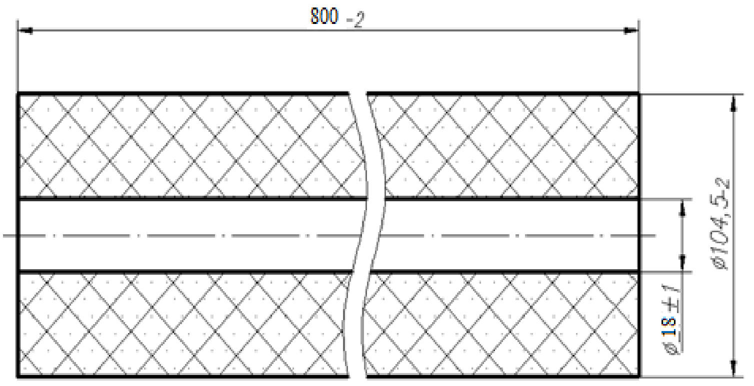


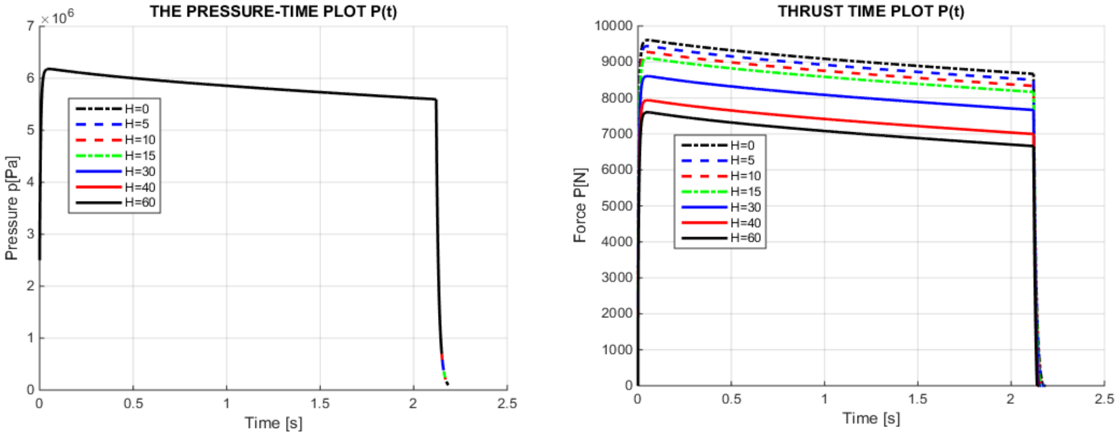
***Figure 3.*** *Structural diagram of the fuel charge.*

***Table 2.*** *Characteristic parameters of the single-hole rocket propellant charge.*

| N$^0$ | **Parameters** | **Symbol** | **Value** | **Unit** |
|---|---|---|---|---|
| 1 | Outer diameter | $D_n$ | 104,5 | m |
| 2 | Inner diameter | $d_{tr}$ | 0,0180 | m |
| 3 | Length | $L_{tp}$ | 0,80 | m |
| 4 | Specific energy | $Q_e$ | 3590 | *KJ/kg* |
| 5 | Exponential index of the temperature segment | $k$ | 1,25 | |
| 6 | Gas constant of the combustion products | $R$ | 362 | *J/kg.K* |
| 7 | Burning rate coefficient | $u_1$ | $42.64*10^{-6}$ | *m/s* |
| 8 | Exponential index of the burning rate | $\nu$ | 0,3456 | |
| 9 | Environmental temperature influence coefficient | $K_T$ | 0,0034 | |
| 10 | Fuel density | $\rho_T$ | 1600 | *kg/m$^3$* |

### 3.2. Study on the impact of depth on the thrust of solid rocket engines operating in aquatic environments

The calculation program with input parameters from table 1, table 2, and figures 2, 3 follows the algorithm diagram in figure 1, considering the variation of ambient pressure (which depends on the operating depth of the engine). The range of environmental pressure changes from a depth of 0m to 60m, corresponding to an environmental pressure from $10^5$ Pa to $7.10^5$ Pa. The calculation results are shown in figure 4 and table 3.



*The Pressure-Time plot* *Thrust-Time plot*

***Figure 4.*** *Pressure and thrust plots of the rocket engine at different depths.*

***Table 3.*** *Operating parameters of the rocket engine at different depths.*

| N$^0$ | Depth (m) | Working time (s) | Maximum pressure (Mpa) | Average pressure (Mpa) | Maximum thrust (N) | Average thrust (N) | Total Impulse (N.s) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 2,361 | 6,216 | 5,676 | 9549,4 | 8688,2 | 20512,7 |
| 2 | 5 | 2,353 | 6,216 | 5,695 | 9348,5 | 8518,0 | 20042,9 |
| 3 | 10 | 2,347 | 6,216 | 5,709 | 9147,6 | 8340,2 | 19574,3 |
| 4 | 15 | 2,343 | 6,216 | 5,718 | 8946,7 | 8154,8 | 19106,6 |
| 5 | 30 | 2,335 | 6,216 | 5,737 | 8344,0 | 7583,3 | 17706,9 |
| 6 | 50 | 2,328 | 6,216 | 5,753 | 7540,4 | 6806,8 | 15846,1 |
| 7 | 60 | 2,326 | 6,216 | 5,757 | 7138,6 | 6413,3 | 14917,4 |

Remarks:

+ As the rocket engine at greater depths, its operational time becomes shorter; however, this effect is minimal.

+ When the ambient pressure is lower than the critical pressure, the ambient pressure has little impact on the pressure inside the combustion chamber.

+ Depth has a significant effect on the rocket engine's thrust: The deeper the rocket engine, the lower the thrust. The thrust decreases by 33,6% when operating at a depth of 60 meters below the water surface compared to operation on land. As the depth increases, the total thrust of the engine decreases.

## 4. CONCLUSIONS

The paper successfully establishes a system of differential equations to calculate the thrust of a rocket engine operating underwater at various depths by solving the gas flow motion problem under varying environmental conditions. Based on this, the study examines the effects of structural parameters and ambient pressure on engine thrust. The greater the operating depth of the engine, the lower the thrust efficiency. The computational results provide a theoretical foundation for the design and analysis of rocket engines operating in underwater environments (e.g., torpedoes, naval mines, underwater-launched bombs, etc.)

## REFERENCES

[1]. P. T. Phiet, *"Theory of Rocket Engines"*, Military Technical Academy Publishing House (2006).

[2]. P. T. Phiet, *"Fundamentals of Calculating Unguided Rocket Projectiles"*, Military Technical Academy Publishing House (2006).

[3]. L. S. Tung, N. C. Hoe, H. T. Dung, B. D. Tan, *"Fundamentals of Calculation and Design of Solid Propellant Engines"*, Military Science and Technology Institute Publishing House (2015).

[4]. D. H. Trien, N. T. Thanh, *"Application of the Inverse Problem of Launch Dynamics for Calculating and Designing Replacement Charges in Solid Rocket Engines"*, Journal of Military Science and Technology Research, Weapon Science Conference, pp. 137-144, (2007).

[5]. H. T. Dung, *"Study on the Effects of Certain Combustion Chamber Parameters on the Characteristic Operating Parameters of Solid Propellant Rocket Engines"*, Ph.D. Thesis, Military Science and Technology Institute, pp. 40-60, (2018).

[6]. Q. Wu, Q. Ren, *"Tuning the Ballistic Performance of a Single-Burning-Rate Grain Solid Rocket Motor via New Discontinuous Embedded Metal Wires"*, Aerospace, vol. 11, no. 4, pp. 308, (2024).

[7]. T. Thomas, L. Simitz *"Experimental characterization of thrust for ≤ 20 N-s impulse solid rocket motors"*, Journal of Emerging Investigators, vol. 7, pp.1-7, (2024).

## TÓM TẮT

### Nghiên cứu ảnh hưởng của áp suất đến lực đẩy của động cơ nhiên liệu rắn làm việc trong môi trường nước

*Bài báo trình bày kết quả nghiên cứu ảnh hưởng của áp suất môi trường đến lực đẩy của động cơ nhiên liệu rắn khi làm việc trong môi trường nước ở các độ sâu khác nhau. Đã thực chỉ ra sự phụ thuộc lực đẩy vào áp suất môi trường khi hoạt động trong môi trường nước ở các độ sâu khác nhau. Thiết lập thuật giải và xây dựng chương trình tính toán thuật phóng trong động cơ nhiên liệu rắn khi hoạt động trong môi trường nước. Từ đó, nghiên cứu ảnh hưởng của áp suất nước đến áp suất và lực đẩy của động cơ.*

**Từ khoá:** Động cơ nhiên liệu rắn; Ảnh hưởng của áp suất đến lực đẩy; Động cơ nhiên liệu rắn hoạt động trong môi trường nước.