Adaptive-based hierarchical sliding mode control for ball-balancing robots moving on an inclined plane

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ABSTRACT

In contrast to managing a robot on a flat surface, controlling a balanced robot on a spherical wheel on an inclined surface is more challenging since the robot travelling up and down a slope has a higher chance of falling over. In this paper, the kinematic equation of the ball balancing robot (ballbot) is built as the learning method according to a 2D mathematical model. The authors use an adaptive-based hierarchical sliding mode control (AHSMC) based on Lyapunov theory to handle the target of a system whose parameters are undefined or change over time. The proposed sliding surfaces are demonstrated to be asymptotically stable and are validated through a simulation model implemented in Simscape software. The simulation results of the closed control design work correctly to control the motion of a ballbot when moving up the inclined surface.

Keywords: Ballbot; Hierarchical sliding mode control; Lyapunov; Adaptive control; Simscape.

1. INTRODUCTION

A ballbot is a type of self-balancing robot designed based on the concept of an inverted pendulum. Ballbots are designed with various drive mechanisms, such as using two slip rollers [1], [2], or three omnidirectional wheels [3, 4], but generally, the movements of a ballbot rely on altering the position of a sphere in all directions while maintaining the body of a ballbot balance above the sphere. Thanks to the design of its drive mechanism, a ballbot moves using a single point of contact with the ground, making it more agile than other types of autonomous robots. Some notable strengths include its ability to perform complex movements (turning left, right, backwards, etc.) in confined spaces and its capability to adjust its body angle to maintain balance even when carrying objects or moving on an inclined surface.

The problem of controlling the motion of a ballbot on a plane has been extensively studied in recent years. This includes simple controllers based on linearized models such as PID [2, 3], LQR [4], LMI [5], and more advanced nonlinear controllers such as sliding mode control (SMC) [6–9], and hierarchical sliding mode control (HSMC) [10, 11]. However, these studies only consider the motion of the ballbot on a horizontal plane, meaning the potential energy of the ball remains constant and the balance point of the body is always aligned with the vertical axis. As mentioned above, the ballbot's agility and self-balancing capability on inclined surfaces are significant advantages. Therefore, in some previous studies, authors have modelled the motion of ballbot on slopes, and designed controllers such as 2-loop PID [12] and LQR [13]. However, these controllers still rely on linearised models, which introduce certain errors in calculating control signals, especially when the ballbot operates far from the working point used for linearization. Thus, applying nonlinear control to solve this problem allows for directly constructing control signals through the mathematical model, minimising linearization errors.

In this study, the authors propose using an HSMC with a Lyapunov-based adaptive rule to maintain the position of ballbot and tilt angle convergence based on a single control signal. The HSMC is the main controller to balance and track the ballbot. However, using the HSMC controller with a fixed sliding surface in all working states will not provide the highest quality. Therefore, this paper used HSMC-based adaptive control to ensure that the ballbot is accurately positioned and well-balanced, and that the tilt angle of a ballbot is estimated.

Moreover, the movement of a ballbot on a slope requires a more complex computational model than on a plane. Previous studies have primarily focused on the design of controllers, which prompted the authors to propose using Simscape to model the ballbot. Simscape will physically simulate the interactions between components of the ballbot, and verify the correctness of the model and the designed controller. Subsequently, the calculated torque signal from the AHSMC controller is fed directly into the Simscape model to monitor the output responses.

The contributions of this work are two-fold: i) Implementing AHSMC to efficiently control the movement of a ballbot on both uphill and downhill terrains while ensuring balance. ii) The proposed sliding surfaces are asymptotically stable and validated through a simulation model implemented in Simscape software.

2. PROBLEM FORMULATION

2.1. Modeling of the ballbot moving in an inclined plane

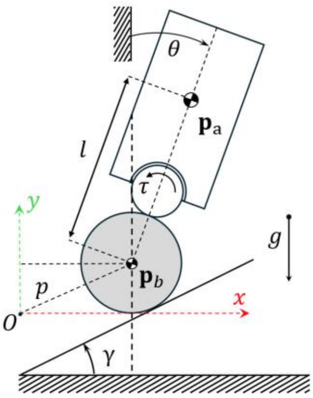


Figure 1. *Ballbot architecture analysis.*

The equation of motion of the ballbot moving in the inclined plane can be found by solving the Euler-Lagrange equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{J}\tau \qquad (1)$$

Where the Jacobian matrix of input force was calculated:

$$\mathbf{J} = \frac{\partial \dot{\varphi}}{\partial \dot{\mathbf{q}}} \qquad (2)$$

Where $\dot{\varphi} = -\left(2 + \frac{r_c}{r_b}\right)\dot{\psi} + \frac{r_b}{r_c}\dot{\theta}$, r_b and r_c are the radius of the ball and omniwheel, respectively.

The total kinetic and potential energy of the ball, body, and omni-wheel calculates in the Lagrange equation:

$$\begin{aligned} L &= T_a + T_b + T_c - V_a - V_b \\ &= \left(\frac{1}{2}\dot{\mathbf{p}}_a^{b\top} m_a \dot{\mathbf{p}}_a^b + \frac{1}{2}J_a\theta^2\right) + \left(\frac{1}{2}\mathbf{p}_b^\top m_b \dot{\mathbf{p}}_b + \frac{1}{2}J_b\dot{\psi}^2\right) \\ &\quad + g^\top \mathbf{p}_b m_b - \mathbf{g}^\top \mathbf{p}_a^b m_a - g^\top \mathbf{p}_b m_b \end{aligned} \qquad (3)$$

Where $\dot{\psi} = \frac{\dot{p}}{r_b}$ m_a and m_b are the total mass of the rigid system body-Omniwheel and the ball, respectively.

Finally, the matrix of motion of the ballbot can be written:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{J}\tau \tag{4}$$

Where,

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & c_{12} \\ 0 & 0 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}, \ m_{11} = \frac{J_b}{r_b^2} + 4\frac{J_c}{r_b^2} + \frac{J_c r_c^2}{r_b^4} + 4\frac{J_c r_b r_c}{r_b^4} + m_a + m_a.$$

$$m_{12} = m_{21} = -2\frac{J_c}{r_c} - \frac{J_c}{r_b} + l m_a \cos(\gamma + \theta), \ \ m_{22} = m_a l^2 + J_a + J_c \frac{r_b^2}{r_c^2}.$$

$$c_{12} = -l m_a \dot{\theta} \sin(\gamma + \theta), \ g_1 = g\sin(\gamma)(m_a + m_b), \ \ g_2 = -g l m_a \sin(\theta)$$

Remark 1: Difference from the ballbot moving in the non-inclined plane, if the ballbot is stable in the inclined plane, i.e. $\gamma \neq 0$ the equilibrium point $\tilde{\theta}$ and $\tilde{\tau}$ do not equal 0, it can be determined by solving the equation of motion (4) with $\dot{\mathbf{q}} = 0$ và $\ddot{\mathbf{q}} = 0$, yield.

$$\sin(\tilde{\theta}) = \frac{r_b^3 \sin(\gamma)(m_a + m_b)}{l m_a r_c (2r_b + r_c)}; \tilde{\tau} = -\frac{g r_b^2 \sin(\gamma)(m_a + m_b)}{2r_b + r_c} \tag{5}$$

2.2. Ballbot adaptive control

2.2.1. HSMC-based balancing control

Our control objectives are designing position p tracks to the reference position p_r and ensuring the tilt angle θ converges to $\tilde{\theta} + \theta_r$. The tracking error of ball position and the angle error of body are defined respectively as:

$$e_1 = p - k\int_0^t (p_r - p(\tau))d\tau \ ; \qquad e_2 = \theta - \tilde{\theta} - \theta_r \tag{6}$$

Where k is a positive constant and taking the derivative of both errors yields:

$$\dot{e}_1 = \dot{p} - k(p_r - p); \qquad \dot{e}_2 = \dot{\theta} - \dot{\theta}_r \tag{7}$$

We establish a pair of competent sliding mode surfaces for the ball and body subsystems s_1 and s_2 with the positive parameters k_1 and k_2 respectively as:

$$s_i = \dot{e}_i + k_i e_i, \quad i = 1,2 \tag{8}$$

Then, the total sliding surface is defined as:

$$s = \alpha\left(\dot{e}_1 + k_1 e_1\right) + \beta\left(\dot{e}_2 + k_2 e_2\right) \tag{9}$$

Where α and β are the positive numbers. The derivative of the sliding surface is:

$$\begin{aligned} \dot{s} \quad &= \alpha(\mathbf{\Gamma}\mathbf{M}^{-1}(-\mathbf{C}\dot{\mathbf{q}} - \mathbf{G}) + \mathbf{\Gamma}\mathbf{M}^{-1}\mathbf{J}u - \ddot{p}_r + k_1\dot{e}_1) \\ &+\beta(\mathbf{\Theta}\mathbf{M}^{-1}(-\mathbf{C}\dot{\mathbf{q}} - \mathbf{G}) + \mathbf{\Theta}\mathbf{M}^{-1}\mathbf{J}u - \ddot{\theta}_r + k_2\dot{e}_2) \end{aligned} \tag{10}$$

The control signal is the solution of

$$\begin{aligned} -(\alpha\mathbf{\Gamma}\mathbf{M}^{-1}\mathbf{J} + \beta\mathbf{\Theta}\mathbf{M}^{-1}\mathbf{J})u \quad &= \alpha(\mathbf{\Gamma}\mathbf{M}^{-1}(-\mathbf{C}\dot{\mathbf{q}} - \mathbf{G}) - \ddot{p}_r + k_1\dot{e}_1) \\ &+\beta(\mathbf{\Theta}\mathbf{M}^{-1}(-\mathbf{C}\dot{\mathbf{q}} - \mathbf{G}) - \ddot{\theta}_r + k_2\dot{e}_2) + \lambda s + \eta\,\text{sign}(s) \end{aligned} \tag{11}$$

where $\mathbf{\Gamma}$ = [1 0] and $\mathbf{\Theta}$ = [0 1]

Theorem 1. Consider the ballbot moving in the inclined plane, which is described in equation (4) if the control drives it as the solution of the (11) with the positive chosen numbers k_1, k_2, λ, η, then the sliding surface s, s_1 and s_2 are convergence to zero.

Proof: The Lyapunov candidate is chosen as:

$$\mathcal{V} = \frac{1}{2}s^2 \tag{12}$$

Thus, the derivative of Lyapunov $\mathcal{V}$ can be determined:

$$\begin{aligned}\dot{\mathcal{V}} = \dot{s}s = \Big[&\alpha(\mathbf{\Gamma M}^{-1}(-\mathbf{C\dot{q}} - \mathbf{G}) + \mathbf{\Gamma M}^{-1}\mathbf{J}u - \ddot{p}_r + k_1\dot{e}_1) \\ &+\beta(\mathbf{\Theta M}^{-1}(-\mathbf{C\dot{q}} - \mathbf{G}) + \mathbf{\Theta M}^{-1}\mathbf{J}u - \ddot{\theta}_r + k_2\dot{e}_2)\Big]s\end{aligned} \tag{13}$$

Substituting (10) to (12), we achieve:

$$\dot{\mathcal{V}} = -\lambda s^2 - \eta|s| < 0 \tag{14}$$

Then, based on Barbalat's lemma and the control parameters α, β are chosen as positive numbers, the Theorem is proven.

2.2.2. Adaptive plane inclination

Motivated by the control signal in (11) with the parameter γ defined, the control signal for the estimated parameter $\hat{\gamma}$ can be defined by us as follows

$$\begin{aligned}(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\hat{\mathbf{M}}^{-1}\mathbf{J}u = &\alpha(\mathbf{\Gamma}\hat{\mathbf{M}}^{-1}(\hat{\mathbf{C}}\dot{\mathbf{q}} + \mathbf{G}) + \ddot{p}_r - k_1\dot{e}_1) \\ &+ \beta(\mathbf{\Theta}\hat{\mathbf{M}}^{-1}(\hat{\mathbf{C}}\dot{\mathbf{q}} + \hat{\mathbf{G}}) + \ddot{\theta}_r - k_2\dot{e}_2) - \lambda s - \eta\,\text{sign}(s)\end{aligned} \tag{15}$$

Where $\hat{\mathbf{M}}, \hat{\mathbf{C}}$ and $\hat{\mathbf{G}}$ are the values of the matrices $\mathbf{M}, \mathbf{C}$ and $\mathbf{G}$, respectively, with the estimated parameter $\hat{\gamma}$ of γ. By defining $\tilde{\gamma} = \hat{\gamma} - \gamma$, the adaptive rule of $\hat{\gamma}$ is defined as follows

$$\begin{aligned}\dot{\hat{\gamma}} = \frac{-\beta_\gamma}{\tilde{\gamma}}[&(-(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\mathbf{M}^{-1}(\mathbf{C\dot{q}} + \mathbf{G}) \\ &+ (\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})(\mathbf{M} + \mathbf{M})^{-1}(\tilde{\mathbf{C}}\dot{\mathbf{q}} + \tilde{\mathbf{G}})s(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\mathbf{M}(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})^\top \\ &+ \frac{1}{2}s^2(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\dot{\mathbf{M}}(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})^\top]\end{aligned} \tag{16}$$

Where $\tilde{\mathbf{M}} = \hat{\mathbf{M}} - \mathbf{M}$, $\tilde{\mathbf{C}} = \hat{\mathbf{C}} - \mathbf{C}$, *and* $\tilde{\mathbf{G}} = \hat{\mathbf{G}} - \mathbf{G}$. Then, it leads to the following Theorem 2

Theorem 2. Consider the ballbot moving in the inclined plane described in equation (4) if the ballbot is controlled by the control signal as the solution of (15) with the adaptive rule for plane inclination (16), then the sliding surfaces s, s_1 and s_2 is convergence to zero and the error of the estimated parameter is stable.

Proof. Consider the system (4), yields:

$$(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\ddot{\mathbf{q}} = -(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\mathbf{M}^{-1}\mathbf{C\dot{q}} + (\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\mathbf{M}^{-1}\mathbf{G} + (\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\mathbf{M}^{-1}\mathbf{J}u \tag{17}$$

Substituting (14) and (16), we achieve:

$$\begin{aligned}(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\ddot{\mathbf{q}} = &-(\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\mathbf{M}^{-1}\mathbf{C\dot{q}} - (\alpha\mathbf{\Gamma} + \beta\mathbf{\Theta})\mathbf{M}^{-1}\mathbf{G} \\ &+ \alpha(\mathbf{\Gamma}\hat{\mathbf{M}}^{-1}(\hat{\mathbf{C}}\dot{\mathbf{q}} + \hat{\mathbf{G}}) + \ddot{p}_r - k_1\dot{e}_1) \\ &+ \beta(\mathbf{\Theta}\hat{\mathbf{M}}^{-1}(\hat{\mathbf{C}}\dot{\mathbf{q}} + \hat{\mathbf{G}}) + \ddot{\theta}_r - k_2\dot{e}_2) - \lambda s - \eta\,\text{sign}(s)\end{aligned} \tag{18}$$

That means

$$(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\ddot{\mathbf{q}} = -(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}^{-1}(\mathbf{C}\dot{\mathbf{q}}+\mathbf{G}) + (\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\hat{\mathbf{M}}^{-1}(\hat{\mathbf{C}}\dot{\mathbf{q}}+\hat{\mathbf{G}}) \\ +\alpha(\ddot{p}_r - k_1\dot{e}_1) + \beta(\ddot{\theta}_r - k_2\dot{e}_2) - \lambda s - \eta \operatorname{sign}(s) \tag{19}$$

Then, based on the definition of sliding surface (10), yields:

$$\dot{s} + \lambda s + \eta \operatorname{sign}(s) = -(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}^{-1}(\mathbf{C}\dot{\mathbf{q}}+\mathbf{G}) + (\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})(\tilde{\mathbf{M}}+\mathbf{M})^{-1}(\hat{\mathbf{C}}\dot{\mathbf{q}}+\hat{\mathbf{G}}) \tag{20}$$

Lyapunov candidate is chosen as:

$$V = \frac{1}{2}s^2(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top + \frac{1}{2\beta_\gamma}\tilde{\gamma}^2 \tag{21}$$

Then, the derivative of the Lyapunov function is:

$$\dot{V} = \dot{s}s(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top + \frac{1}{2}s^2(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\dot{\mathbf{M}}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top + \frac{1}{\beta_\gamma}\tilde{\gamma}\dot{\tilde{\gamma}} \tag{22}$$

Based on equations (16) and (20), $\dot{V}$ can be expanded as:

$$\begin{aligned} \dot{V} = \Big[& -\lambda s - \eta \operatorname{sign}(s) - (\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}^{-1}(\mathbf{C}\dot{\mathbf{q}}+\mathbf{G}) \\ & + (\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})(\tilde{\mathbf{M}}+\mathbf{M})^{-1}(\hat{\mathbf{C}}\dot{\mathbf{q}}+\hat{\mathbf{G}})\Big] s(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top \\ & + \frac{1}{2}s^2(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\dot{\mathbf{M}}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top - \Big[-(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}^{-1}(\mathbf{C}\dot{\mathbf{q}}+\mathbf{G}) \\ & - (\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})(\tilde{\mathbf{M}}+\mathbf{M})^{-1}(\tilde{\mathbf{C}}\dot{\mathbf{q}}+\tilde{\mathbf{G}})s(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top \\ & - \frac{1}{2}s^2(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\dot{\mathbf{M}}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top \Big] \end{aligned} \tag{23}$$

That implies

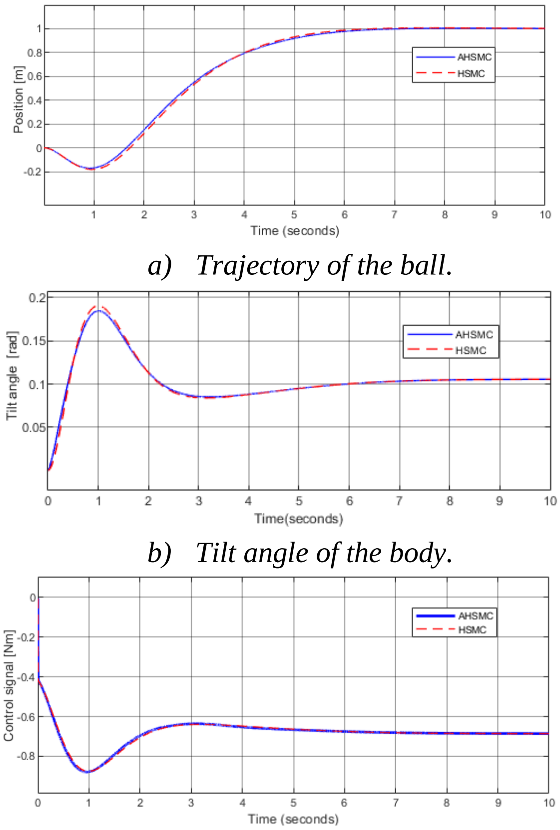
$$\dot{V} = (-\lambda s - \eta \operatorname{sign}(s))s(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})\mathbf{M}(\alpha\mathbf{\Gamma}+\beta\mathbf{\Theta})^\top \Rightarrow \dot{V} = -\lambda s^2 - \eta|s| < 0 \tag{24}$$

Thus, the theorem is proven based on Theorem 1 and the definition of the Lyapunov candidate (21).

3. RESULTS AND DISCUSSION

The authors used Matlab software to simulate the ballbot moving in the inclined plane with 15^0. System parameters were chosen: The mass of the body is $m_a = 4.59\,kg$, the mass of the ball is $m_b = 0.62\,kg$, the mass of the Omni-wheel is $m_w = 0.19\,kg$, the radius of the ball is $r_b = 0.12\ m$ the radius of the omni-wheel is $r_w = 0.05\ m$, the moment of inertial of the body is $J_a = 0.6075\,kg.m^2$, the moment of inertial of the ball is $J_b = 0.004464\,kg.m^2$, the moment of inertial of the omni-wheel is $J_c = 2.375*10^{-4}\,kg.m^2$, and the distance from the ball's CoM to the body's CoM is $l = 0.345\ m$.

Figures 2 and 3 depict the ballbot's performance when moving uphill and downhill on inclined planes, respectively, with comparisons between Adaptive Hierarchical Sliding Mode Control (AHSMC) and Hierarchical Sliding Mode Control (HSMC). Subplot (a) in each figure shows the ball's trajectory, where AHSMC demonstrates faster convergence to the desired position, achieving stability by approximately 8 seconds uphill and 6 seconds downhill. Subplot (b) displays the tilt angle of the body, where both control methods effectively manage the incline, though AHSMC provides smoother stabilization. Subplot (c) illustrates the torque input, with AHSMC showing reduced control effort over time, highlighting its efficiency in stabilizing the ballbot under varying incline conditions. These results collectively underscore AHSMC's superior performance in trajectory tracking, stability, and energy efficiency compared to HSMC.

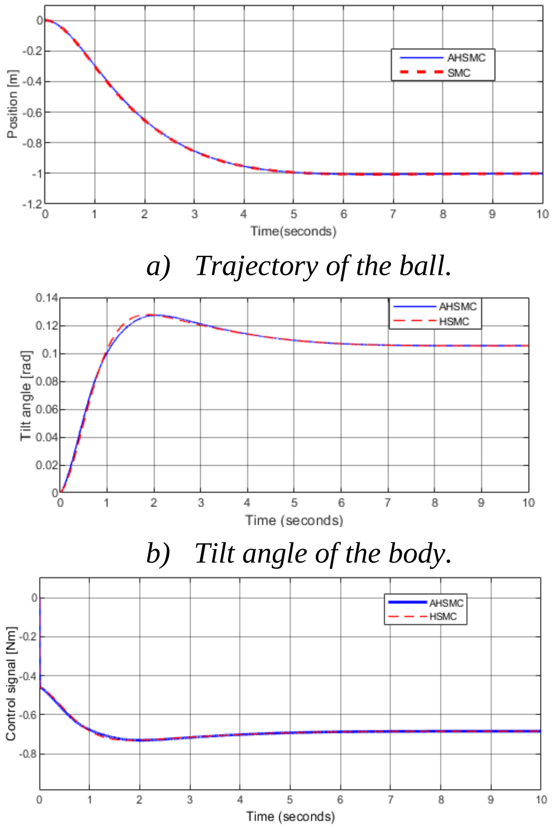


a) Trajectory of the ball.

b) Tilt angle of the body.

c) Torque input acting on the body.

Figure 2. *The ballbot moving uphill in a plane inclination.*



a) Trajectory of the ball.

b) Tilt angle of the body.

c) Torque input acting on the body.

Figure 3. *The ballbot moving downhill in a plane inclination.*

4. CONCLUSIONS

In this paper, the benefits of the adaptive hierarchical sliding mode controller (AHSMC) are illustrated by stabilizing the trajectory of the ballbot, and keeping the ballbot balanced when it travels both uphill and downhill. Additionally, AHSMC estimates the tilt angles of the inclined plane without having to measure them in advance. But when a load of ballbot increases, instability develops, and the ballbot could topple over. To get around this problem, the research team must create an appropriate controller in the upcoming effort.

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TÓM TẮT

Điều khiển robot cân bằng bóng di chuyển trên mặt phẳng nghiêng

Điều khiển một robot cân bằng trên một bánh xe hình cầu chuyển động trên một bề mặt nghiêng khó hơn điều khiển một robot khác trên một bề mặt nằm ngang do robot lên xuống dốc dễ bị đổ hơn. Trong bài báo này, phương trình động học của robot cân bằng bóng (Ballbot) được xây dựng theo mô hình toán học 2D. Các tác giả sử dụng bộ điều khiển trượt phân cấp thích nghi (AHSMC) dựa trên lý thuyết Lyapunov để xử lý hệ thống có các tham số không xác định hoặc thay đổi theo thời gian. Các bề mặt trượt đề xuất được chứng minh là ổn định tiệm cận và được xác thực thông qua một mô hình mô phỏng triển khai trong phần mềm Simscape. Kết quả mô phỏng của thiết kế cho thấy Ballbot hoạt động tốt khi di chuyển lên bề mặt nghiêng.

Từ khoá: Ballbot; Điều khiển trượt; Lyapunov; Điều khiển thích nghi; Simscape.