

Synthesis of adaptive control algorithms based on output feedback with an implicit reference model for application on aerial vehicles

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ABSTRACT

This paper addresses adaptive control for an aircraft's longitudinal motion under parameter uncertainty. A mathematical model is developed, and simulations validate the proposed approach. A linear modal controller is designed for nominal parameters, while an adaptive controller with an implicit reference model ensures stability under uncertainties. MATLAB/Simulink simulations on the UAV-70V model show that the linear modal controller performs well with known parameters but becomes unstable with variations. In contrast, the adaptive controller maintains robust stability, rapid response, and precise trajectory tracking. These findings confirm its effectiveness in uncertain environments.

Keywords: Aerial vehicle; Adaptive control; Implicit reference model; Longitudinal motion; Output feedback; UAV; Parameter uncertainty.

1. INTRODUCTION

In the context of the rapid development of aviation technology, particularly Unmanned Aerial Systems (UAVs), ensuring the stability and efficiency of control algorithms plays a critical role. UAVs often operate in dynamic environments influenced by various factors such as wind, weather conditions, and changing payloads. This requires the control system to have high adaptability and the ability to respond quickly to these complex fluctuations.

To address the challenges of stability and adaptability for UAVs, several control methods have been researched and developed. The Model Reference Adaptive Control (MRAC) method is one of the most commonly discussed approaches, as introduced by Narendra K. S. and Fradkov A. L. [1, 2]. This method relies on the use of a reference model to define the desired behavior of the UAV system, and then adjusts control parameters to make the UAV follow the reference model. The strength of MRAC lies in its high accuracy and the ability to ensure the UAV operates stably according to the reference model's requirements. However, one of the challenges of this method is that the reference model needs to be precisely defined and match the actual operating conditions, which can become difficult when the UAV faces a nonlinear environment or unpredictable disturbances. Authors such as Zang, C., Pan, T., and colleagues [3, 4] have explored the Direct Adaptive Control method, in which the control parameters are directly adjusted based on feedback from the system without the need for a specific reference model. The advantage of this method is its ability to quickly adapt to changing conditions without relying on a complex mathematical model. However, direct adaptive control also has limitations in maintaining stability in nonlinear systems and environments with large fluctuations, which require better handling techniques. The Self-Tuning Control (STC) method has been applied to UAVs by authors like De Luca and Khosravi, M. A. [5, 6]. This method allows the system to automatically adjust control parameters in real-time based on feedback received from the UAV, improving its ability to respond flexibly to environmental changes. STC has the advantage of flexibility and rapid adaptability, but still

faces challenges when UAVs operate in rapidly changing environments or in situations involving complex nonlinear behaviors.

In prior works [7, 8], the authors proposed an adaptive control algorithm utilizing an implicit reference model, as well as a reinforcement learning algorithm, to address the control of UAV systems under parameter uncertainties. Although these approaches successfully ensured system stability, the position tracking accuracy in certain scenarios remained suboptimal. In [9], an adaptive control strategy for a nonlinear UAV model with uncertain parameters was developed using the speed-gradient algorithm in conjunction with a reference model. The results demonstrated the asymptotic stability and high control performance of the adaptive system. However, the control synthesis process was relatively complex, highlighting a need for further refinement in the methodology.

To address these limitations, this paper proposes an output feedback adaptive control method with an implicit reference model [10]. Instead of relying on a complex and explicit mathematical model of the UAV, this approach uses implicit models to enable the control system to flexibly adjust parameters based on input and output signals. This method not only improves the UAV's flight performance but also enhances its stability and accuracy during control, even in complex and fluctuating operational environments.

The structure of the paper is organized as follows: Section 2 presents the mathematical modeling of the UAV's longitudinal dynamics and the synthesis of both the linear modal controller and the adaptive controller with an implicit reference model. Section 3 provides a comparative analysis of the two controllers through simulations conducted in MATLAB/Simulink, highlighting the superior performance of the proposed adaptive controller over the modal controller. Finally, section 4 concludes the paper with a summary of the findings and key insights.

2. SYNTHESIS OF AERIAL VEHICLE CONTROL SYSTEM

Consider the longitudinal motion model of the aerial vehicle as shown in figure 1. In this case, the following assumptions are made:

The motion problem of the aerial vehicle (AV) is treated as a rigid body dynamics problem with given aerodynamic characteristics, neglecting the effects of elasticity and structural deformation of the AV.

The AV is equipped with ideal sensors that measure coordinates, velocity, altitude, spatial orientation, angular velocities, and overload factors. This means that the sensors are assumed to have no static or dynamic errors, and all the sensors measuring the UAV's motion parameters are modeled with ideal amplifying components.

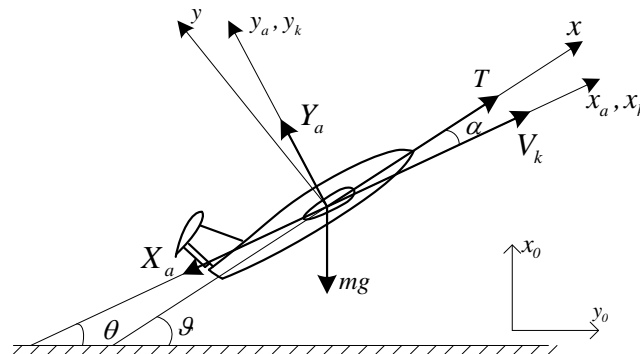


Figure 1. Longitudinal motion model of the aerial vehicle.

The actuators (servos) are considered as first-order inertial elements with a transfer function

representing an amplifier with a gain of 1 and a given time constant. In figure 1, the following coordinate systems are used: x_0Oy_0 : Ground coordinate system; x_aOy_a : Velocity coordinate system; x_kOy_k : Link coordinate system [11].

The mathematical model of the longitudinal motion of the aerial vehicle is expressed as [11]:

$$\begin{cases} m\dot{V}(t) = T \cos \alpha - X_a - mg \sin \theta; \\ mV\dot{\theta}(t) = T \sin \alpha + Y_a - mg \cos \theta; \\ J_z \dot{\omega}_z = qSbm_z; \dot{\vartheta} = \omega_z; \\ \dot{x}(t) = V \cos \theta; H(t) = V \sin \theta, \end{cases} \quad (1)$$

Here: $\vartheta, \theta, \alpha$ - Pitch angle, trajectory inclination angle, and angle of attack of the aerial vehicle. V - Velocity of the aerial vehicle; mg - Gravitational force of the aerial vehicle; T - Engine thrust force; S - Characteristic area of the aerial vehicle; J_z - Moment of inertia about the z-axis; q - Air density; x, H - Coordinates of the center of mass of the aerial vehicle in the ground coordinate system; X_a, Y_a - Components of the drag and lift forces, respectively; m_z - Moment coefficient about the z-axis; b - Average aerodynamic chord length.

Linearizing system (1): In this problem, only short-period (angular) motions are considered, neglecting variations in speed, altitude, and engine operating conditions. Furthermore, it is assumed that the lift force of the elevator is neglected. The system of differential equations based on small deviations is obtained:

$$\begin{cases} \dot{\alpha}(t) = \omega_z(t) + a_y^\alpha \alpha(t) - a_y^{\delta_c} \delta_c; \\ \dot{\omega}_z(t) = -a_{m_z}^\alpha \alpha(t) - a_{m_z}^{\omega_z} \omega_z(t) - a_{m_z}^{\delta_c} \delta_c; \\ \dot{\vartheta}(t) = \omega_z(t); \end{cases} \quad (2)$$

Here: δ_c - Deflection angle of the aerial vehicle's control surface; $a_y^\alpha, a_y^{\delta_c}, a_{m_z}^\alpha, a_{m_z}^{\omega_z}, a_{m_z}^{\delta_c}$ are the flight dynamics coefficients.

2.1. Modal linear controller

Modal control is a linear state feedback, given in the following form:

$$u_{\alpha} = g(t) \cdot \beta + \mathbf{k}^T \mathbf{x} = g(t) \cdot \beta + k_1 \alpha + k_2 \omega_z + k_3 \vartheta, \quad (3)$$

Here: $g(t)$ - Desired reference signal, β - Gain coefficient, $\mathbf{k} = [k_1 \quad k_2 \quad k_3]^T$ - A real vector of the feedback coefficients for the state variables, calculated based on the desired location of the poles of the characteristic equation of a closed-loop system. The characteristic polynomial of the closed-loop system (2), (3):

$$\varphi_{dt}(\lambda) = \det(\mathbf{A} + \mathbf{B}\mathbf{k}^T - \lambda\mathbf{E}) \quad (4)$$

Here: \mathbf{A}, \mathbf{B} are the state matrix and control matrix, respectively, when representing system (2) in the form of a matrix equation; \mathbf{E} - Identity matrix.

By substituting the matrices \mathbf{A} and \mathbf{B} , equation (4) is rewritten as follows:

$$\begin{aligned} \varphi_{dt}(\lambda) = & -\lambda^3 + (a_y^\alpha - a_{m_z}^{\omega_z} - a_y^{\delta_c} k_1 - a_{m_z}^{\delta_c} k_2) \lambda^2 + \\ & \left[-(a_{m_z}^{\delta_c} + a_{m_z}^{\omega_z} a_y^{\delta_c}) k_1 + (a_y^\alpha a_{m_z}^{\delta_c} + a_{m_z}^\alpha a_y^{\delta_c}) k_2 - a_{m_z}^{\delta_c} k_3 - a_{m_z}^\alpha + a_y^\alpha a_{m_z}^{\omega_z} \right] \lambda + \\ & + (a_y^\alpha a_{m_z}^{\delta_c} + a_{m_z}^\alpha a_y^{\delta_c}) k_3. \end{aligned} \quad (5)$$

Choose the desired characteristic polynomial of the linearized closed-loop system (2), (3) in the form:

$$\begin{aligned}\varphi_{mm}(\lambda) &= -(\lambda + \omega_1)(\lambda + \omega_2)(\lambda + \omega_3) \\ &= -\lambda^3 + (\omega_1 + \omega_2 + \omega_3)\lambda^2 + (-\omega_1\omega_2 - \omega_1\omega_3 - \omega_2\omega_3)\lambda + \omega_1\omega_2\omega_3\end{aligned}\quad (6)$$

The roots of it are multiples of negative real numbers in the form:

$$\lambda_1 = -\omega_1, \lambda_2 = -\omega_2, \lambda_3 = -\omega_3$$

By equating the coefficients of the same degree of the variable in the polynomials (5) and (6), the algebraic equation system to calculate the feedback coefficients $\mathbf{k} = [k_1 \quad k_2 \quad k_3]^T$ is obtained as follows:

$$\begin{cases} a_y^\alpha - a_{m_z}^{\omega_z} - a_y^{\delta_c} k_1 - a_{m_z}^{\delta_c} k_2 = \omega_1 + \omega_2 + \omega_3; \\ -(a_{m_z}^{\delta_c} + a_{m_z}^{\omega_z} a_y^{\delta_c}) k_1 + (a_y^\alpha a_{m_z}^{\delta_c} + a_{m_z}^\alpha a_y^{\delta_c}) k_2 + \\ -a_{m_z}^{\delta_c} k_3 - a_{m_z}^\alpha + a_y^\alpha a_{m_z}^{\omega_z} = -\omega_1\omega_2 - \omega_1\omega_3 - \omega_2\omega_3; \\ (a_y^\alpha a_{m_z}^{\delta_c} + a_{m_z}^\alpha a_y^{\delta_c}) k_3 = \omega_1\omega_2\omega_3 \end{cases}\quad (7)$$

From (2) the following transfer function is obtained:

$$W_g^{\delta_c} = \frac{a_{m_z}^{\delta_c} s - (a_{m_z}^{\delta_c} a_y^\alpha + a_y^{\delta_c} a_{m_z}^\alpha)}{s^3 + (a_{m_z}^{\omega_z} - a_y^\alpha) s^2 + (a_{m_z}^\alpha - a_y^\alpha a_{m_z}^{\omega_z}) s}$$

Since the transfer function $W_g^{\delta_c}$ has a single zero at $s = -1.06$, the desired poles of the control system are chosen such that $\lambda_1 = -\omega_1 = -1.06$; $\lambda_2 = -\omega_2 = -40$; $\lambda_3 = -\omega_3 = -40$. From this, by using formulas (6) and (7) the value of the feedback coefficient vector of modal controller can be obtained as $\mathbf{k} = [-0.5684 \quad 2.3882 \quad 48.589]^T$.

The gain coefficient is:
$$\beta = \frac{1}{-\mathbf{C}(\mathbf{A} + \mathbf{Bk})^{-1} \mathbf{B}} = -48.589$$

2.2. Output-feedback control algorithm with implicit reference model

Now, let's consider the application of the passivation theorem to the synthesis problem of the controller for a system with a variable structure (CIPC) [10] and a parameter-signal adaptive controller (CIIAP) [10]. Consider the following system:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}; \mathbf{y} = \mathbf{Cx} \quad (8)$$

Consider the linear system (8), where the control objective is $\lim_{t \rightarrow \infty} \mathbf{x}(t) = 0$. The following control algorithm is used:

$$u = -\gamma \text{sign} \sigma, \sigma = \mathbf{Gy}, \gamma > 0 \quad (9)$$

As shown in [10], the objectives for the system (8), (9) are achieved if there exist a matrix $\mathbf{P} = \mathbf{P}^T > 0$ and a vector \mathbf{K} such that $\mathbf{PA} + \mathbf{A}^T \mathbf{P} < 0$, $\mathbf{PB} = \mathbf{GC}$, $\mathbf{A}_* = \mathbf{A} + \mathbf{BK}^T \mathbf{C}$. As derived from Theorem 4, the specified conditions are satisfied only when the transfer function is $\mathbf{GW}(\lambda)$, where $W(\lambda) = \mathbf{C}(\lambda \mathbf{E}_n - \mathbf{A})^{-1} \mathbf{B}$. To eliminate the dependency of the system's stability on the initial conditions and parameters of the object, the parameter-signal adaptive control algorithm or its combination is used as follows:

$$u = \mathbf{K}^T(t)\mathbf{y}(t) - \gamma \text{sign}\sigma, \sigma(\mathbf{y}) = \mathbf{G}\mathbf{y}, \dot{\mathbf{K}}(t) = -\sigma(\mathbf{y})\mathbf{\Gamma}\mathbf{y}(t) \quad (10)$$

where $\mathbf{\Gamma} = \mathbf{\Gamma}^T > 0, \gamma > 0$ is a matrix and a scalar gain coefficient.

At steady-state, systems (8) and (10) are described as follows:

$$0 = (\mathbf{A} + \mathbf{B}\mathbf{K}_0^T\mathbf{C})\mathbf{x}_* - \gamma\mathbf{B}\text{sign}(\sigma_*); \sigma_* = \mathbf{G}\mathbf{y}_* = 0; \mathbf{y}_* = \mathbf{C}\mathbf{x}_* \quad (11)$$

Thus, equation (11) represents the implicit reference model, which must be stable, meaning $(\mathbf{A} + \mathbf{B}\mathbf{K}_0^T\mathbf{C})$ is a Hurwitz matrix (* indicates that the parameters in (11) belong to the implicit reference model, and \mathbf{K}_0 is the value of \mathbf{K} at steady-state). This is the fundamental difference between the implicit reference model and the classical reference model. Additionally, the use of an implicit reference model simplifies the control structure and reduces the requirements for the completeness of the observed data.

It should be noted that convergence to zero in a finite time is a fundamental property of the variable structure system (CIIC) with forced sliding modes. It can be said that this property is satisfied for any bounded range of the initial states of the system (2), (9). Based on the control method above, the tracking control algorithm for system (2) is given as follows:

$$u = \mathbf{K}^T(t)\mathbf{y} - \gamma \cdot \text{sign}(\sigma),$$

here: $\dot{\mathbf{K}}(t) = -\sigma \cdot \mathbf{\Gamma} \cdot \mathbf{y}; \sigma = \mathbf{G}\mathbf{y}; \mathbf{y} = \begin{bmatrix} e & 0.05 \cdot \frac{p}{10^{-3}p+1} e \end{bmatrix}; e = g_m - g; \mathbf{\Gamma} = \begin{bmatrix} 10^3 & 0 \\ 0 & 10^3 \end{bmatrix};$

$\mathbf{G} = [1 \ 1]; \gamma = 2$. The matrix $\mathbf{\Gamma}$ and the coefficient γ are chosen to be as large as possible for faster adaptation speed; However, they are selected to be sufficiently large so that the control signal does not become excessively large.

3. RESULTS AND DISCUSSION

In this section, the controlled object chosen for the simulation is the UAV-70V. The UAV has the following basic parameters: a length of 2707 mm, a weight of 56.3 kg, a wing area of 1.05 m², and a wingspan of 3000 mm. The average aerodynamic chord is 350 mm, and the cruising speed is 40 m/s. Additionally, the moment of inertia of the UAV is 31.3 kg·m². The simulation of the UAV's reference signal tracking is investigated with the aerodynamic coefficients [12] in the 3 parameter scenarios (tentatively called scenarios a₁, a₂, a₃ shown in table 1) when using different controllers.

Table 1. Aerodynamic parameters of the aerial vehicle.

Parameter Scenario	$a_y^\alpha (s^{-1})$	$a_{m_z}^\alpha (s^{-2})$	$a_{m_z}^{\omega_z} (s^{-1})$	$a_y^{\delta_c} (s^{-1})$	$a_{m_z}^{\delta_c} (s^{-2})$
a ₁	-1.10	15.5	1.20	0.09	33.0
a ₂	-0.86	5.81	0.18	0.06	9.15
a ₃	-1.34	-12.5	0.45	0.07	15.2

The simulation results are shown in figures 2 – 4.

Observations:

- The results in figure 2 demonstrate that the adaptive controller adjusts the output to closely follow the reference signal with zero steady-state error, whereas the modal controller exhibits a persistent output error relative to the reference signal. The results in figures 3 and 4 indicate that the adaptive controller meets stability criteria, adapts quickly, and achieves high control quality under parameter variations: maximum transient time < 0.2 s, overshoot approximately 0%, and zero steady-state error. In contrast, the modal controller performs well only when the parameter

values are known in advance. When parameters vary, the system experiences high overshoots (10–20%), maximum transient times around 0.4 s, and nonzero steady-state errors. Furthermore, the system may become unstable under significant parameter changes.

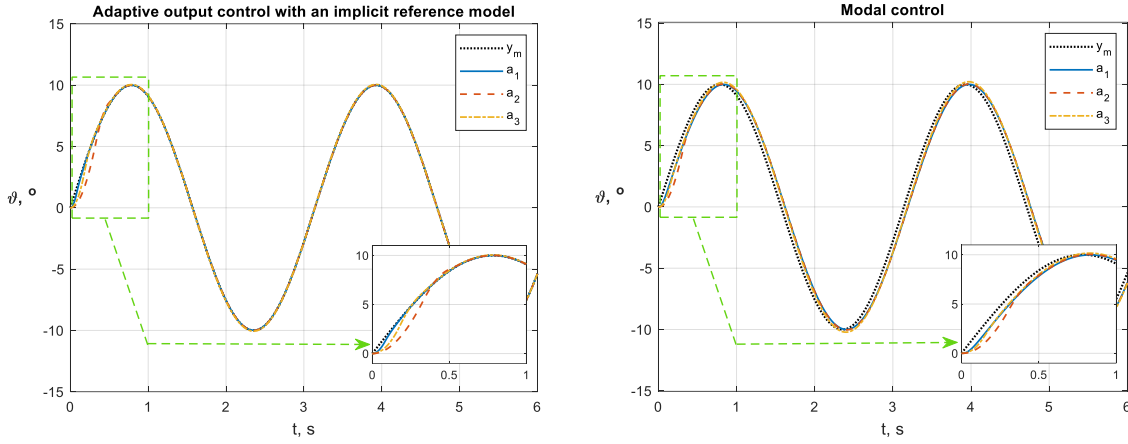


Figure 2. Response of the pitch angle ϑ to a sinusoidal reference signal.

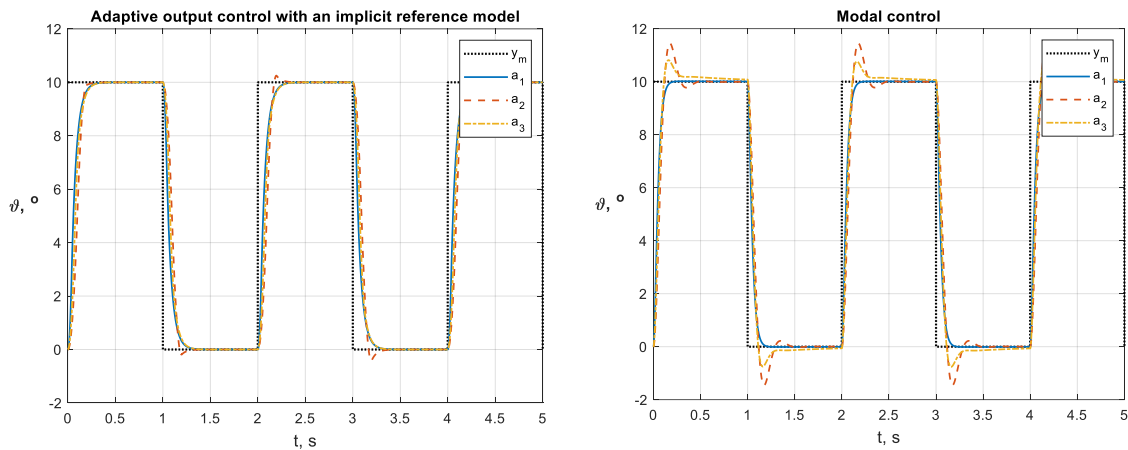


Figure 3. Response of the inclination angle ϑ to a square wave reference signal.

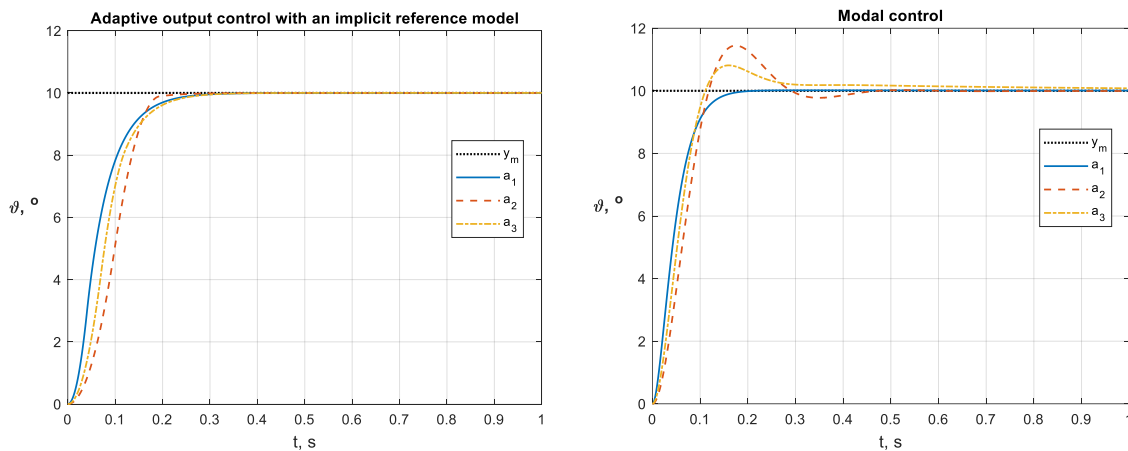


Figure 4. Response of the pitch angle ϑ to a step input signal.

- The adaptive controller tracks the reference signals such as sine wave, square wave, and step input with minimal error. The system demonstrates a short settling time, low overshoot, and fast convergence to the nominal value, particularly when responding quickly to sudden signal changes.

- The modal linear controller also tracks the reference signal, but its response time is slower and it has significant overshoot, especially leading to large errors when the signal changes abruptly, such as with square wave and step inputs. The system still reaches a stable state, but the convergence time is longer compared to the adaptive method.

- To implement the modal state controller, full state measurement of the system is required, or a state observer needs to be built. On the other hand, output-feedback controllers only require the measurement of the output state. Therefore, output-feedback controllers are more compact in design, saving costs on expensive sensors with measurement noise, and completely eliminating the need for observer design.

- Comparing the adaptive and modal linear controllers shows that the adaptive controller performs better based on the criteria of faster response, closer tracking to the reference signal with smaller tracking error, and no overshoot during the initial phase of the adaptation process.

4. CONCLUSIONS

This paper presents the synthesis and design of a modal state-variable linear controller and an output-feedback adaptive controller with an implicit reference model. Simulation results demonstrate that the adaptive controller, with its simple structure, makes the system more compact and particularly suitable for small-sized aerial vehicles (UAVs), optimizing weight and size. The adaptive method has a significant advantage in that it does not depend on the internal state variables of the system. When comparing control performance, the results show that the adaptive controller offers faster response times and better stability maintenance than the modal controller.

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TÓM TẮT

Tổng hợp thuật toán điều khiển thích nghi theo tín hiệu đầu ra với mô hình tham chiếu ẩn ứng dụng trên thiết bị bay

Bài báo này nghiên cứu về vấn đề điều khiển thích nghi kênh chuyển động dọc của thiết bị bay trong điều kiện tham số bất định. Trong đó, tiến hành xây dựng mô hình toán học và mô phỏng đối tượng điều khiển. Bộ điều khiển modal tuyến tính được tổng hợp sử dụng các giá trị danh định của tham số hệ thống. Để hệ thống điều khiển ổn định trong điều kiện tham số bất định, bộ điều khiển thích nghi theo tín hiệu đầu ra với mô hình tham chiếu ẩn được xây dựng. Kết quả mô phỏng so sánh hai bộ điều khiển bằng MATLAB/Simulink với mô hình UAV-70V chứng minh rằng, bộ điều khiển modal tuyến tính chỉ hoạt động tốt khi tham số đối tượng biết trước và hệ thống có thể mất ổn định khi các tham số thay đổi. Trong khi đó, bộ điều khiển thích nghi với mô hình tham chiếu ẩn tuy có cấu trúc đơn giản nhưng vẫn đảm bảo hệ thống điều khiển UAV ổn định tiệm cận với khả năng đáp ứng nhanh, bám sát theo mọi quỹ đạo bay trong trường hợp bất định các tham số.

Từ khóa: Thiết bị bay; Điều khiển thích nghi; Mô hình tham chiếu ẩn; Chuyển động dọc của UAV; Phản hồi đầu ra; UAV; Tham số bất định.