The trajectory simulation of an anti-submarine missile motion

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ABSTRACT

This paper analyses an anti-submarine missile's aerial motion, water entry motion and underwater motion. It establishes equations of motion for each phase and presents the ballistic characteristics of each section. According to the model, a trajectory is simulated. The method and the result provide an efficacious guarantee for the operational employment of an anti-submarine missile. It thus provides a basis for the design and calculation of an anti-submarine missile.

Keywords: Anti-submarine missile; Torpedo; Air trajectory; Underwater trajectory.

1. INTRODUCTION

The anti-submarine missile is an anti-submarine weapon that combines a missile and torpedo (or deep water bomb). According to different launch platforms, it can be divided into shipborne type, submarine launched type and airborne type [5, 6].

Reference [1] discusses equations of motion for flight simulation of the anti-submarine rocket. Equations of motion are presented for a three dimensional trajectory simulation under the assumption that, during the thrust phase, the configuration is a rigid body and has 90-degree rotational symmetry. However the model does not take into account the processes of rocket stage separation, shell separation, and parachute deployment. Reference [2] investigates simulation of torpedo air trajectory based on the dual-euler method. This work may provide a theoretical reference for the study of torpedo air trajectory. In the papers [3, 12], the aerodynamic layout and motion characteristics of the anti-submarine missile in parachute-free section are analyzed, and an aerodynamic parameters identification method based on real flight test data is proposed. Then the identification results are used in the simulation of large attitude change of the torpedo in parachute-free section in real navigation test, and the change law of attitude angle consistent with that from real flight is reproduced, verifying the correctness of the proposed method. This method can also be applied to the study of motion characteristics of other air-dropped torpedoes in parachute-free sections. Reference [4] explores a model of the rocket-assisted torpedo carrier with six degrees of free dom was built based on the momentum and momentum moment theorem, the control rule of the underwater trajectory was put forward, and a trajectory simulation with MATLAB/Simulink was conducted.

This paper analyses an anti-submarine missile's aerial motion, water entry motion and underwater motion. It establishes equations of motion for each phase and presents the ballistic characteristics of each section. It thus provides a basis for the design and calculation of an anti-submarine missile.

2. EQUATIONS OF MOTION

2.1. Trajectory overview

The aerial trajectory of an anti-submarine missile could be divided into a powered flight section, an inertial flight section and a torpedo parachute section. Figure 1 is the schematic diagram of anti-submarine missile aerial trajectory which adopts the inclinel launching. The anti-submarine missile aerial trajectory would be rounded and analyze in the three-dimensional space, which is determined by the ground coordinate system, velocity coordinate system, and torpedo body coordinate system.

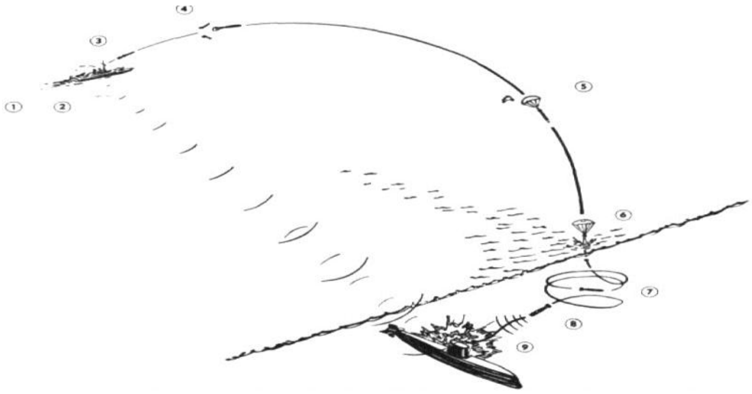


Figure 1. *Operational diagram of an anti-submarine missile.*

The paper idealizes anti-submarine missile aerial trajectory. Lateral motion parameters are assumed to be zero, the torpedo air longitudinal motion equations are achieved. Different from the former study, the paper advances a new model construction method, on the foundation of particularity of torpedo stressed in the torpedo parachute section and similarity of torpedo stressed in the non-torpedo parachute section. The model constructed by the new method divides the aerial trajectory into non-torpedo parachute section and torpedo parachute section.

2.2. Equation of motion

2.2.1. Non-torpedo parachute trajectory

The equations (1) are the torpedo air longitudinal motion equations in the non-torpedo parachute section [1].

$$\begin{cases} \dfrac{dV}{dt} = \dfrac{F_t - R_x}{m_t} - gSin\theta \\ \dfrac{d\theta}{dt} = \left(\dfrac{R_y}{mV} - \dfrac{g}{V} Cos\theta \right) \\ J_{zz} \dfrac{d\varphi}{dt} = M_z \\ \dfrac{d\varphi}{dt} = \omega_z \\ \dfrac{d\alpha}{dt} = \dfrac{d\varphi}{dt} - \dfrac{d\theta}{dt} \\ \dfrac{dx}{dt} = VCos\theta \\ \dfrac{dy}{dt} = V\sin\theta \end{cases} \tag{1}$$

Parameters in the equations represent:

m_t – The mass of missile at time; F_t– The thrust of missile at time; R_x, R_y– Air resistance x, airlift y; J_{zz} – Vertical rotational inertia z; M_z– Vertical moment z, Unknown quantities in the equations are: x– Range, y – Reach, V– Velocity, α – Angle of attack; θ – Angle of pitch, φ – Trajectory inclination angle, ω_z – Vertical angular velocity z.

Solving methods of parameters above are as follows: $m_t = m_0 - \dot{m}_t\, t$ (2)

m_o– The initial mass of the missile; $\dot{m}_t$ – The mass flow rate of solid fuel rocket engine.

The general expression of air resistance is shown as follows

$$R_x = \frac{1}{2}\rho S V^2 C_{x0}(\frac{V}{a}) \tag{3}$$

Parameters in the equations represent: ρ – Air density, Empirical formula is:

$$\rho = \rho_0 e^{-1,05.10^{-4} y},\ \rho_0 = 1,20574\ kg/m^3 \tag{4}$$

S – Sectional area of missile

$$S = 0,25\pi d^2 \tag{5}$$

d – Diameter of missile; a – Velocity of sound

The general expression of airlift is shown as follow

$$R_y = \frac{1}{2}\rho S C_y^{\alpha}\alpha V^2 + \frac{1}{2}\rho S L C_y^{\omega}\omega V \tag{6}$$

Where: $C_y^{\alpha}, C_y^{\omega}$ – Aerodynamic parameters, L – The distance between the center of mass and center of pressure of missile.

The general expression of vertical moment is shown as follows

$$M_z = \frac{1}{2}\rho S L m_z^{\alpha}\alpha V^2 + \frac{1}{2}\rho S L m_z^{\omega}\omega V \tag{7}$$

Parameters in the equations represent: $m_z^{\alpha}, m_z^{\omega}$ - Aerodynamic parameters.

2.2.2. Torpedo parachute trajectory

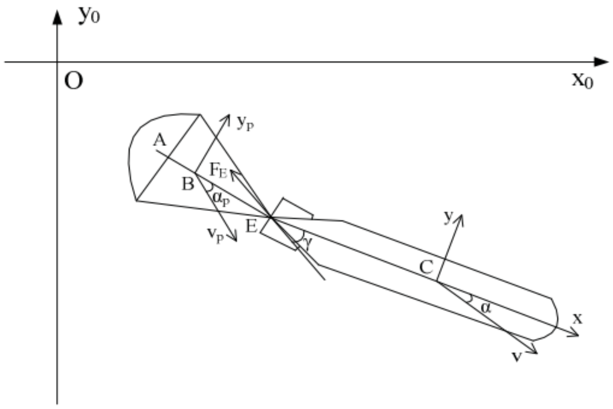


Figure 2. *The torpedo parachute model.*

The torpedo parachute section is a complex motion process. Establish its motion equations, conditions the following should be assumed.The radial dimension of connecting device between torpedo and the parachute is ignored. The connection between parachute and the end tail of torpedo is point-to-point connection. The acting force is transmitted, while the moment is not.The parachute is opened completely in the section. The torpedo and the parachute are considered as rigid body. The torpedo parachute system can move only in the vertical plane.

For this reason, the torpedo parachutes coordinate system is established as shown in figure 2. In the coordinate system, parameters with subscript are physical quantities of the parachute, correspondingly, parameters without subscripts are physical quantities of the torpedo.

Motion equations of torpedo parachute section are established as follows [2, 7, 12].

$$\begin{cases} \dfrac{dV}{dt} = \dfrac{-R_x - F_E Cos(\gamma - \theta)}{m_{NL}} - g\,\mathrm{Sin}\,\theta \\ \dfrac{d\theta}{dt} = \dfrac{R_y + F_E\,\mathrm{Sin}(\gamma - \theta)}{m_{NL} V} - \dfrac{g}{V} Cos\theta \\ J_{zz}\dfrac{d\omega_z}{dt} = M_z - F_E L_{CE}\,\mathrm{Sin}\,\gamma \\ \dfrac{d\varphi}{dt} = \omega_z \\ \dfrac{d\alpha}{dt} = \dfrac{d\varphi}{dt} - \dfrac{d\theta}{dt} \\ \dfrac{dx}{dt} = VCos\theta \\ \dfrac{dy}{dt} = V\sin\theta \end{cases} \tag{8}$$

Parameters in the equations represent: m_{NL} – The torpedo mass of the torpedo parachute section; L_{CE} – The range from point C to point E, point C is the center of mass of the torpedo; γ - The included angle of the force F_E và x-axis in the torpedo coordinate system; F_E – The interaction force between torpedo and parachute.

$$F_E = \frac{1}{2}\rho C_{DS} V_E{}^2 = K_D V_E{}^2 \tag{9}$$

K_D - Parachute-opening load parameter, V_E - The velocity at the point E.

$$\begin{cases} V_E = \sqrt{V^2 + \left(\omega L_{CE}\right)^2 - 2V\omega L_{EC}\sin\alpha} \\ \alpha_E = \arctan\dfrac{V\sin\alpha + \omega L_{CE}}{V\mathrm{Cos}\,\alpha} \end{cases} \tag{10}$$

Other parameters are the same as parameters in the non-torpedo parachute section equations.

2.2.3. Underwater trajectory of torpedo

Model of the unpowered phase of water entry. After the torpedo enters the water at a certain angle and speed, it shatters the head protection cover and separates the parachute through a release mechanism. During this period, the torpedo has no power supply and the engine is powered. The torpedo has not started yet.

Motion equations of torpedo are established as follows.

$$\begin{cases} x_t = x_{t_0} + V_r \cos\tau \sin\alpha_T (t - t_0) \\ y_t = y_{t_0} + V_r \cos\tau \cos\alpha_T (t - t_0) \end{cases} \quad (11)$$

Parameters in the equations represent: x_{t_0}, y_{t_0} – The coordinates of the entry point of the rocket-assisted torpedo; t_0 - The starting time of the unpowered phase of entry c; V_r - The average speed of the torpedo during the entry phase;, τ - The initial entry angle; α_T - The torpedo heading.

Diving depth search model. After entering the water, the seawater battery is activated, the engine starts, and the generator is driven to work, supplying power to the entire torpedo. The torpedo dives at a certain diving angle. When it reaches a certain depth above the search depth, the diving angle command is canceled, and the diving angle gradually decreases. After the initial super-deep dive, it is finally leveled to the search depth. Since the time it takes for the torpedo engine to start supplying power and accelerate the torpedo to reach normal speed is very short, and the torpedo entry speed is not much different from the normal speed of the torpedo, it can be assumed that the torpedo moves at normal speed from the diving depth search stage.

The dive angle command cancels the previous model.

$$\begin{cases} x_t = x_{t_{11}} + V_t \cos\beta_1 \sin\alpha_T (t - t_{11}) \\ y_t = y_{t_{11}} + V_t \cos\beta_1 \cos\alpha_T (t - t_{11}) \end{cases} \quad (12)$$

Where: $x_{t_{11}}, y_{t_{11}}$ – The coordinates of the starting point of the diving depth search phase; t_{11} – The starting time of the diving depth search; V_t - The torpedo speed, β_1 - The diving angle before the command is canceled.

The dive angle command cancels the rear model

$$\begin{cases} x_t = x_{t_{12}} + V_t \cos\beta_2 \sin\alpha_T (t - t_{12}) \\ y_t = y_{t_{12}} + V_t \cos\beta_1 \cos\alpha_T (t - t_{12}) \end{cases} \quad (13)$$

Parameters in the equations represent: $x_{t_{12}}, y_{t_{12}}$ is the coordinate of the diving angle command cancellation point; t_{12} - The diving angle command cancellation time, β_2 - The diving angle after the command is canceled.

Circular search segment model. After reaching the search depth, the torpedo begins a circular search. The torpedo's circular search angular velocity changes $\omega_1, \omega_2, \omega_3$ in each rotation. After 3 turns, the torpedo's circular search angular velocity remains unchanged at ω_3

$$\begin{cases} x_t = x_{t_2} + X\cos(2\pi\text{-}\alpha_T)\text{-Y}\sin(2\pi\text{-}\alpha_T) \\ y_t = y_{t_{12}} + X\sin(2\pi\text{-}\alpha_T)\text{+Y}\cos(2\pi\text{-}\alpha_T) \\ \text{X=R}(1\text{-}\cos\beta_3) \\ \text{Y=R}\sin\beta_3 \\ \beta_3 = \omega_T (t - t_3) \\ R = \left| \dfrac{V_T}{\omega_T} \right| \end{cases} \quad (14)$$

Where: x_{t_2}, y_{t_2} – The coordinates of the starting point of the circular search; t_3 – The time when the diving angle command is canceled; ω_T - The angular velocity during the circular search; R -

The turning radius of the torpedo.

Tracking segment model: The control system adjusts the yaw and pitch angles according to the instructions of the homing system to track and attack the target

$$\begin{cases} x_{i+1} = x_i + V_T \sin\alpha_{T_{i+1}}(t_{i+1} - t_i) \\ y_t = y_{t_{12}} + V_T \cos\alpha_{T_{i+1}}(t_{i+1} - t_i) \\ \alpha_{T_{i+1}} = \alpha_{T_i} + \Delta\alpha_{T_i} \end{cases} \quad (15)$$

Where: x_i, y_i - The position coordinates of the torpedo at the moment t_i; $\alpha_{T_{i+1}}$ - The heading of the torpedo at the moment.

Self-guided detection model. The acoustic homing torpedo detection model theoretically calculates the homing range based on the sonar equation. This model will be presented in future studies.

2.3. Solution method for the system of equations

The Runge-Kutta method is employed to solve the differential equations (1), (9), (11)-(15). A trajectory simulation with MATLAB/Simulink was conducted. The initial conditions for solving x=x_0=0, y=y_0=0, $\theta = \theta_0$, V=V_0 and the initial parameters of missiles. The output parameters of the previous motion phases are the input conditions of the next motion phase.

3. RESULTS AND DISCUSSION

3.1. Input data

The initial parameters of missiles are shown in table 1.

Table 1. *The initial parameters of missiles.*

Parameters	Mass, kg	Propellant mass *(kg)*	Center of gravity *(m)*	Moment of inertia $(kg*m)^2$
t=0s	723	245	X_G = 0 Y_G = 0 Z_G = 3,14	I_{xx0} = 18,59 I_{yy0} = 1586 I_{zz0} = 1586
t=1s	685	207	X_G = 0 Y_G = 0 Z_G = 3,06	I_{xx0} = 18,1 I_{yy0} = 1502 I_{zz0} = 1502
t=2s	637	159	X_G = 0 Y_G = 0 Z_G = 2,95	I_{xx0} = 17,13 I_{yy0} = 1384 I_{zz0} = 1384
t=3s	591	113	X_G = 0 Y_G = 0 Z_G = 2,827	I_{xx0} = 16 I_{yy0} = 1253 I_{zz0} = 1253
t=4s	552	74	X_G = 0 Y_G = 0 Z_G = 2,707	I_{xx0} = 14,9 I_{yy0} = 1125 I_{zz0} = 1125
t=5s	521,5	43,5	X_G = 0 Y_G = 0 Z_G = 2,60	I_{xx0} = 13,97 I_{yy0} = 1013 I_{zz0} = 1013
t=6s	498,5	20,5	X_G = 0 Y_G = 0 Z_G = 2,51	I_{xx0} = 13,19 I_{yy0} = 919 I_{zz0} = 919

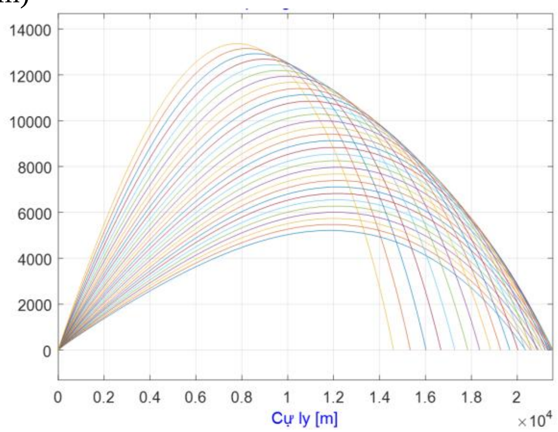
t=7s	481,2	3,2	$X_G = 0$ $Y_G = 0$ $Z_G = 2{,}437$	$I_{xx0} = 12{,}53$ $I_{yy0} = 844$ $I_{zz0} = 844$
t=8s	478	0	$X_G = 0$ $Y_G = 0$ $Z_G = 2{,}423$	$I_{xx} = 12{,}4$ $I_{yy} = 829$ $I_{zz} = 829$
T>8s	416	0	$X_G = 0$ $Y_G = 0$ $Z_G = 2{,}057$	$I_{xx} = 8{,}28$ $I_{yy} = 381$ $I_{zz} = 381$

+ The initial mass of missile, kg: 723;
+ Diameter of missile, mm: 430;
+ Propellant mass, kg: 245;
+ Total impulse, kN.s: 550
+ Working time of engine, s: 8;
+ Length of torpedo, mm: 26500;
+ Diameter of torpedo, mm: 380
+ Mass torpedo, kg: 300;
+ Total length of missile, mm: 5350

3.2. Simulation results and comments

The calculation program with input parameters from section 3.1 and the simulation diagram of aerial trajectory in the typical reach is shown in figure 3.

Reach(m)



Range(m) with launch angle 45^0-75^0

Figure 3. *The simulation diagram of aerial trajectory in the typical reach.*

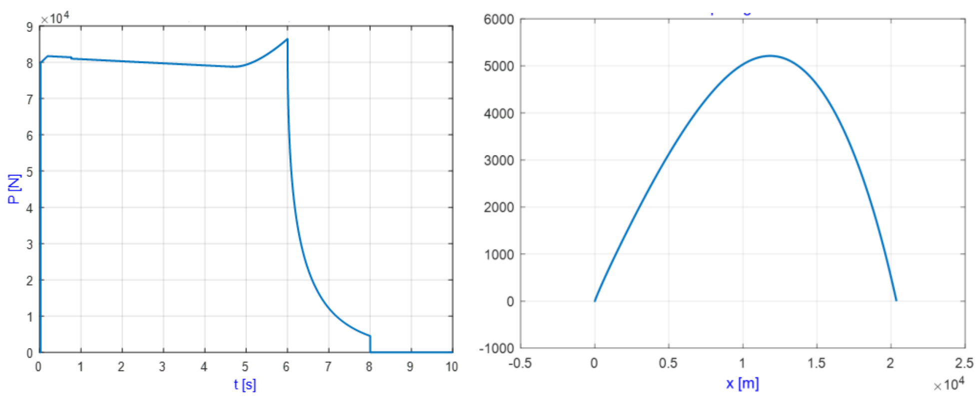


Figure 4. *Maximum flight range, engine thrust of missile with 45-degree launch angle.*

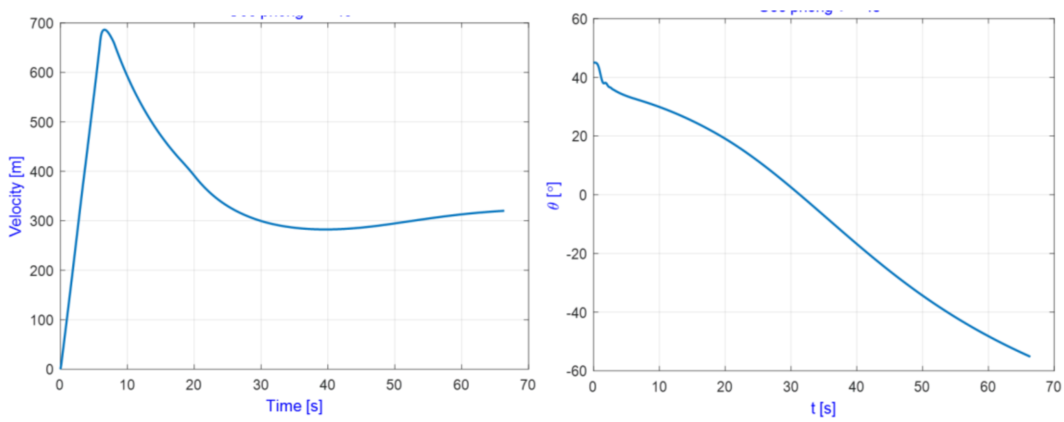


Figure 5. *Velocity and angle of pitch of missile with 45-degree launch angle.*

Thus, with the expected configuration, the missile achieves a maximum range of 25.2 km at an initial launch angle of 56-degrees. To achieve the desired range of 20 km, the initial launch angle only needs to be launched at a 45-degree angle. The results of the maximum firing range, thrust, and elevation angle of the missile at an initial launch angle of 45-degrees are shown in figure 4 and figure 5.

With the proposed configuration of the missile as above, the preliminary simulation results show that the maximum flight distance without control is nearly 20 km. This result is quite similar to the features of some anti-submarine missile complexes in the world such as K-asroc (Korea), Yu-8 (China), Asroc VLA (USA) and Medveka (Russia) [6, 8, 13, 14].

4. CONCLUSIONS

The paper successfully establishes a system of equations of motion for each phase of an anti-submarine missile trajectory. Initial use of mathematical models to survey and evaluate the maximum flight distance without control for anti-submarine missile models. Evaluation of the accuracy when giving out the problem for preliminary design.

The method and the result provide an efficacious guarantee for the operational employment of anti-submarine missiles. In addition, this method can be extended to the application of other weapon aerial trajectory research.

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TÓM TẮT

Xây dựng mô hình chuyển động của thiết bị bay chống ngầm

Bài báo này trình bày mô hình chuyển động trên không, chuyển động khi vào nước và chuyển động dưới nước của thiết bị bay chống tàu ngầm. Thiết lập các phương trình chuyển động cho từng giai đoạn và trình bày các đặc điểm đạn đạo tương ứng. Mô phỏng quỹ đạo sơ bộ trên mô hình vừa xây dựng. Phương pháp và kết quả bài báo hướng đến sử dụng hiệu quả thiết bị bay chống ngầm. Nghiên cứu này có thể làm cơ sở cho việc thiết kế và tính toán sơ bộ các tổ hợp thiết bị bay chống tàu ngầm.

Từ khóa: Thiết bị bay chống ngầm; Ngư lôi; Quỹ đạo trên không; Quỹ đạo dưới nước.