Theoretical method for determining aerodynamic drag coefficient based on similar bullet samples

Phan Ba Binh*, Vu Quang Anh

Institute of Weapons, Vietnam Defense Industry, 51 Phu Dien, Bac Tu Liem, Hanoi, Vietnam.
*Corresponding author: panboping1976@gmail.com

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ABSTRACT

When a bullet moves in the air, due to the mutual impact between the bullet and air elements, aerodynamic drag is generated, reducing the bullet's range. The drag value depends on many factors and is determined through testing. In calculating exterior ballistics, the drag value is often estimated through the bullet shape coefficient. That leads to many errors compared to reality. With the desire to determine exterior ballistics problem to optimize the design, the paper presents a theoretical method to determine the drag value using the finite element method through a similar bullet type that has been tested. The paper has calculated and determined the exterior ballistic parameters of the newly designed bullet for design calculations. The calculation results deviate no more than 5% from the test results.

Keywords: Aerodynamic drag; Drag coefficient; Air blowing method; Exterior ballistics.

1. INTRODUCTION

When a bullet moves in the air, due to the mutual impact between the bullet and air elements, aerodynamic drag is generated. The drag is generated by friction between the bullet surface and air elements or by the imbalance of pressure acting on the bullet, creating drag, or the drag is generated by compressed air layers expanding to form pressure waves on the surface of the bullet nose. Air drag depends on factors such as: the ballistics characteristics of the bullet, the physical characteristics of the air, and the relative motion parameters of the bullet compared to the air. To determine this drag, people use testing methods such as: (1) wind tunnel testing method, (2) test firing method, and (3) optical method [1-4]. These methods are only performed when there is a bullet sample. That leads to difficulties in the bullet design process.

Nowadays, with the development of the finite element method, the drag coefficient can be determined [5-7]. However, this method only gives the drag coefficient when not taking into account the bullet's oscillation movements (The precession motion is the small oscillatory motion around the axis of symmetry of the bullet; the nutation motion is the motion of the bullet's axis rotating around the trajectory of the center of mass). The air drag generated by the influence of these movements is random, complex, and depends on many factors of both the gun and the bullet. These influencing factors can only be determined by testing [8].

From the above analysis, the paper has proposed a method of combining simulation and test firing results of similar bullet type (sample bullet) to calculate the preliminary drag coefficient for newly designed bullets to serve the bullet design calculation process.

2. DETERMINATION OF DRAG COEFFICIENT

2.1. Aerodynamic force and drag coefficient

For a bullet of a certain shape, air drag depends on factors such as: characteristics of the bullet (usually the bullet diameter d); physical characteristics of the air (pressure p, density ρ, viscosity properties μ, elasticity: speed of sound a and temperature); parameters characterizing the relative motion between the bullet and the air (speed of motion of the center of mass v, angle of oscillation *δ*, angular velocity of oscillation of the bullet axis *ω*, angular velocity of rotation around the symmetry axis of the bullet *r*). Thus, the drag is a function of the above factors, written in the general form [1, 2]:

$$R = f(\mathrm{d}, \mathrm{p}, \rho, \mu, \mathrm{a}, \mathrm{v}, \delta, \omega, \mathrm{r})$$

In the calculation of the exterior ballistics, based on the theory of similarity and dimension, the above function is transformed so that the quantities become dimensionless and the number of remaining variables of the function is minimal. At the same time, from the experiment, it shows that the influence of some insignificant factors can be ignored. Finally, we get the expression for calculating the aerodynamic drag R_x (also known as the drag force) as follows [1-2]:

$$R_x = \frac{1}{2} s \rho v^2 c_x$$

Where:

s - Maximum cross-sectional area of the bullet.

c_x - Aerodynamic drag coefficient (also called drag coefficient).

The drag coefficient c_x depends on the Mach number *M*, the angle of the oscillation *δ*, the shape of the bullet and is determined experimentally. Next, the paper will present a method for determining this coefficient by theoretical method through the drag coefficient of a similar type of bullet that has been tested before (sample bullet).

2.2. Determination of the drag coefficient of the sample bullet

The drag component includes the drag component when the effect of the osicillation angle is not taken into account and the drag component caused by the osicillation angle. When calculating for a new type of bullet of the same type (the same artillery, mortar or grenade bullet and with the same bullet diameter), assuming that the effect of the drag components caused by the osicillation angle is the same, then we have the formula to calculate the drag coefficient for the newly designed bullet [3, 4]:

$$\frac{c_{xN}}{c_{x0N}} = \frac{c_x}{c_{x0}} \Rightarrow c_{xN} = c_{x0N} \frac{c_x}{c_{x0}} \tag{1}$$

Where:

- c_{xN} is the drag coefficient of the newly designed bullet when the oscillation angle component is taken into account;
- c_{x0N} is the drag coefficient of the newly designed bullet when the angular component of the thrust is not taken into account ($\delta = 0$);
- c_x is the drag coefficient of the bullet in the equipment when taking into account the component of the oscillation angle;
- c_{x0} is the drag coefficient of the bullet in the equipment when the component of the oscillation angle is not taken into account ($\delta = 0$).

In the formula (1), c_{x0N}, c_{x0} are parameters that can be determined by the finite element method, c_x is a parameter determined by testing.

To determine the drag coefficient c_x, rely on the exterior ballistics calculation equation system and the test results to determine.

With the following assumptions:

- Meteorological conditions are standard;

- Bullet conditions are standard, in which the drag coefficient c_x is a constant value in the velocity domain of the bullet;

- Movement conditions: ignore the asymmetry of the longitudinal axis and consider the bullet motion as the motion of a point mass with all mass located at the center of mass. During the motion, the oscillation angle is 0, so the direction of the reaction coincides with the direction of the velocity vector, and the total force of the air is the drag force R_x.

According to the hypotheses, apply the force system acting on the bullet (figure 1).

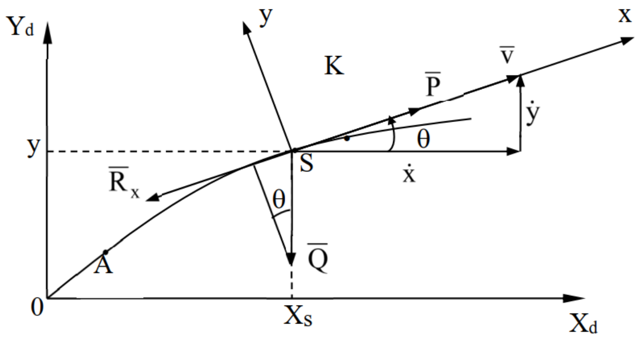


Figure 1. *Forces acting on the bullet.*

The system of equations describing the motion of the center of mass of the bullet [3-4]:

$$\begin{cases} m\dot{v} = -\dfrac{\rho v^2}{2} s c_x - \text{mg}\sin\theta \\ \dfrac{d\theta}{dt} = -\dfrac{g}{v}.\cos\theta \\ \dfrac{dy}{dt} = v.\sin\theta \\ \dfrac{dx}{dt} = v.\cos\theta \end{cases} \tag{2}$$

From this system of equations, if given input parameters such as: mass, initial velocity, launch angle, drag coefficient and environmental parameters, the value of the launch range will be determined. This means that if the test determines the firing range, the drag coefficient will be determined c_x.

Test firing with 40 mm grenade launcher, mass $m = 192 \pm 4g$, launch angle θ=15.61^0, ambient temperature $t_C^0 = 25\,^0C$, muzzle pressure p=1 atm, group of 07 shots (table 1).

Table 1. *Test firing results.*

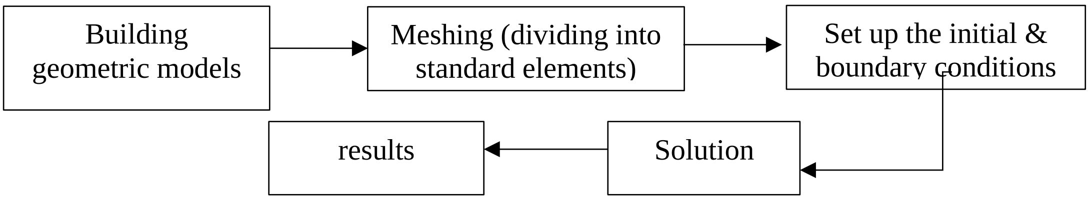
Order	1	2	3	4	5	6	7	Medium
Launch angle (^{0})	$\theta_0 = 15.61^0$							
Mass (g)	192.2	193.3	190.6	191.7	194.4	195.0	195.7	**193.3**
Initial velocity (m/s)	70.1	70.0	69.6	71.4	70.4	69.7	71.9	**70.4**
Range (m)	212.6	213.6	215.5	217.5	220.5	223.3	236.8	**220.0**
Coefficient c_x								**0.2789**

From the firing results and the exterior ballistics calculation program, we find the drag coefficient $c_x = 0.2789$.

2.3. Determination of drag coefficient by finite element method

To determine the drag coefficient for the newly designed bullet c_{xN} , it's required to find the coefficients c_{x0} and c_{x0N} . These coefficients will be found by finite element method when the bullet is stationary, the air flow is parallel to the bullet axis ($\delta = 0$).

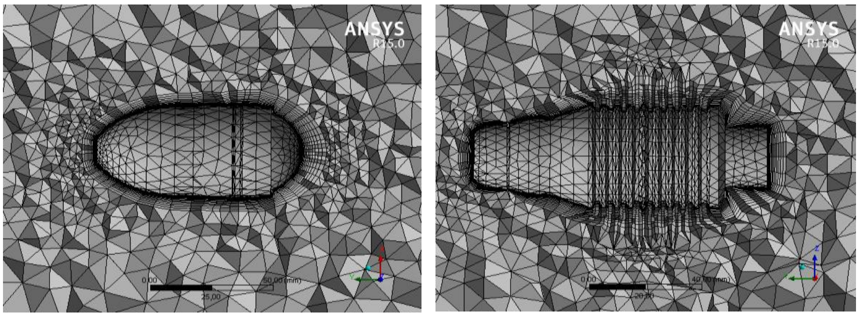
Using Ansys software, Fluent module, follow these steps:



- Geometric modeling and meshing:

+ Building geometric model: The bullet is a detail with the profile of the sample bullet (figure 1.a) and the newly designed bullet (figure 1.b). These are all 40 mm grenade launchers. The 3D geometric model was built using Inventer software and then transferred to Ansys software to build the air blowing zone. The air blowing zone has dimensions of 0.5m×0.5m×1.5m. The front of the bullet is 0.5 m long, the back of the bullet is 1 m long. The size of the air blowing zone is large enough to ensure that the velocity at the edge is zero (not affect the air blowing result).

+ Meshing: to balance the processing speed of the computer and the required accuracy, the mesh is divided to ensure that the results converge. At the edge of the bullets, the mesh is divided into 15 layers with incremental size (increase coefficient 1.2), the edge closing layer has a thickness of 0.2 mm (figure 2).



a. Sample bullet; *b. New design bullet.*

Figure 2. *Meshing model.*

- Set up the initial & boundary conditions:

+ The problem uses the k-omega model, SST model (this model is suitable for marginal flow);

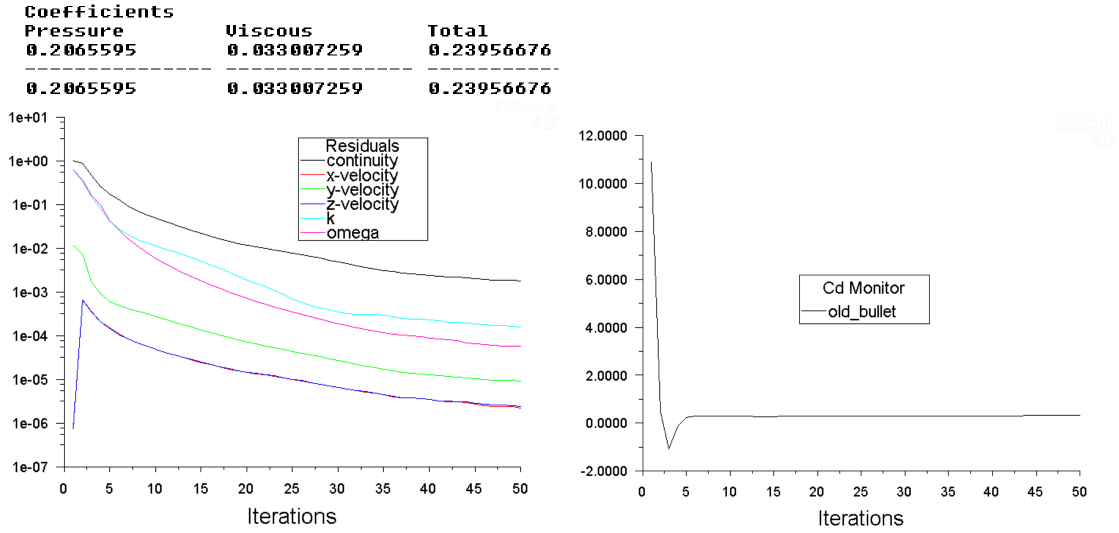
+ The fluid in the air blowing zone is air at standard conditions: ρ = 1.225 kg/m^3, T^{0}K = 288.16 K, pressure p = 1 atm, speed of sound a = 341 m/s;

+ Air inlet velocity: 76 m/s;

+ Air blowing zone: The marginal zone includes a 0.5m×0.5m×1.5m rectangular box and the bullet profile. The motion at the marginal surface is non-slip motion (no slip).

- Solve the problem and process the results:

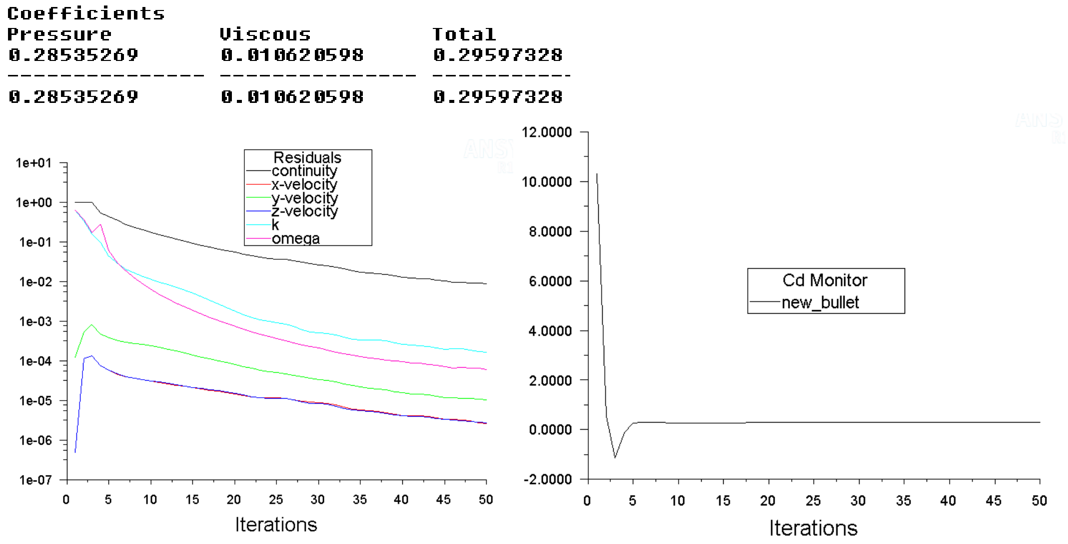
After running with 50 loops, the velocity difference in x, y, z directions between loops is only about 10^{-3} (figure 3.a), shows that the results quickly reach converge. The drag coefficient C_d=0.2396 (figure 3.b) (This is the drag coefficient c_{x0}) achieve stable value after only 05 loops.



a. Residuals after loops; *b. Drag coefficient.*

Figure 3. *Result of running sample bullet.*

Figure 4 shows the running results for the new design bullet. The calculated drag coefficient is C_d=0.2960.



a. Deviation after loops; *b. Drag coefficient.*

Figure 4. *Result of running New design bullet.*

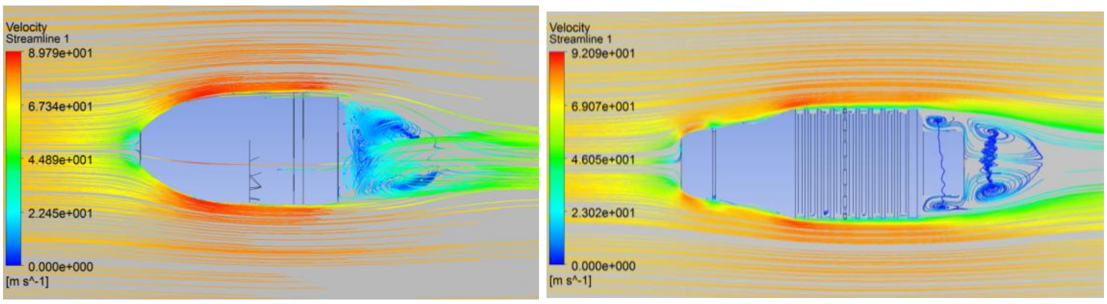


Figure 5. *Image of flow surrounding the bullet body.*

Figure 5 shows the air flow around the sample bullet and the newly designed bullet. The simulation shows that the drag force mainly comes from the following components: the drag directly acting on the bullet nose, the friction force on both sides of the bullet body, and the vortex drag force behind the bottom of the bullet. The wave drag component does not appear for bullets moving at low speeds (subsonic speeds).

When the bullet moves on the trajectory, due to air resistance, the velocity gradually decreases. According to preliminary calculations, the above bullets have an initial velocity of 76 m/s, the minimum impact velocity is 50 m/s. Therefore, to have the drag coefficient on the entire trajectory, the simulation is performed on the velocity range from 50 m/s ÷ 76 m/s. The simulation results are shown in table 2.

Table 2. *Simulation results determine the coefficient c_x at different speed ranges.*

Velocity (m/s)	50	55	60	65	70	76	Average
Drag coefficient c_{x0}	0.2392	0.2390	0.2394	0.2392	0.2395	0.2396	**0.2393**
Drag coefficient c_{x0N}	0.2969	0.2963	0.2960	0.2962	0.2961	0.2960	**0.2963**

The simulation results show that the drag coefficient c_{x0} does not change when the velocity changes (the error in the result is due to the error of the simulation method). This is also consistent with the actual test results: the drag coefficient changes in the subsonic and supersonic velocity regions [1-2].

From formula (1), we find the drag coefficient of the new designed bullet:

$$c_{xN} = c_{x0N}\frac{c_x}{c_{x0}} = 0.2963\frac{0.2789}{0.2393} = 0.3453$$

The results show that the drag coefficient of the new bullet (with a pointed nose) is larger than that of the sample bullet (with a spherical nose) when the bullet velocity is below the speed of sound. When designing a new bullet, many factors need to be considered to ensure harmony between the aerodynamic problem and other structural factors.

3. EXTERIOR BALLISTICS OF NEW DESIGN BULLET

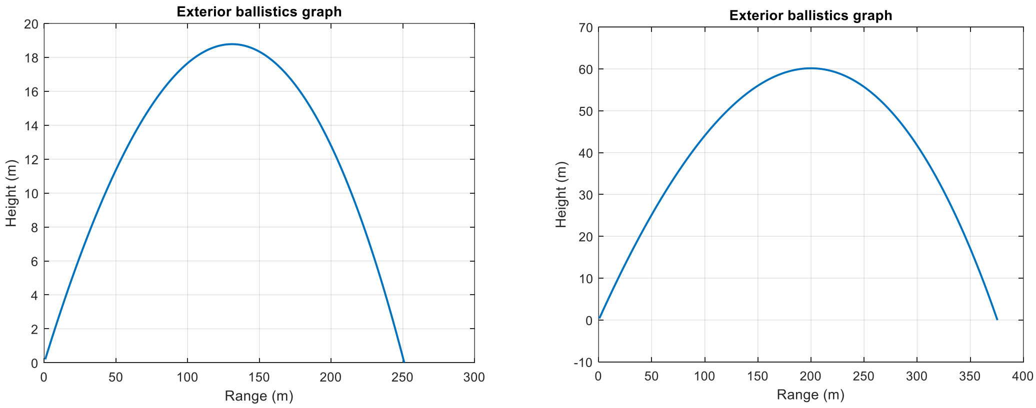


Figure 6. *Calculation results of exterior ballistics at 250 m and 375 m range.*

After determining the drag coefficient, the complete bullet algorithm problem is determined. Substitute the values: $m = 255g,\ v = 76m/s,\ c_{xN} = 0.3453,\ t_C^0 = 15\,^0C$ and the standard meteorological conditions in the exterior ballistics calculation program are established from the system of equations (2) to get the results of the exterior ballistics parameters. Figure 6 is the result of the bullet trajectory at a range of 250 m (effective range) and a range of 375 m (longest range).

Calculations for basic ranges give the launch angle results in table 3.

Table 3. *Table of Range corresponding to launch angles.*

Order	Launch angle θ_0 (^{0})	Range according to the sight (m)	Calculation results (m)	Deviation
1	2.38	50	52	4,0%
2	4.97	100	105	5,0%
3	8.15	150	157	4,7%
4	11.65	200	197	-1,5%
5	15.77	250	249	-0,4%
6	20.72	300	290	-3,3%
7	27.2	350	342	-2,3%
8	32.06	375	369	-1,6%

The newly designed bullet in this paper is actually based on an existing bullet model (Russian GP-25 bullet - figure 7). The results from actual firing were entered into the sight [9], the values are given in table 3.

Figure 7. *Russian GP-25 bullet.*

The simulation calculation results and actual shooting results (already entered into the sight) have a deviation of no more than 5%.

4. CONCLUSIONS

Through the results of firing test in the equipment and simulation results, the paper has proposed a theoretical method to determine the drag coefficient for the new type of bullet with the same system as the type of bullet in the equipment. The calculated results compared with the actual shooting results deviate by no more than 5%. This is the input parameter to solve the exterior ballistic equations for calculating the bullet design. The calculation results are the basis for building a test plan, correcting the test results and completing the design documents. The above method can be applied to calculate artillery shells and mortar shells to optimize the process of designing new bullet models.

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TÓM TẮT

Phương pháp lý thuyết xác định hệ số lực cản khí động dựa trên mẫu đạn tương tự

Khi đạn chuyển động trong không khí do có sự tác động tương hỗ giữa đạn và các phần tử không khí, sinh ra lực cản khí động, làm giảm tầm bắn của đạn. Giá trị lực cản phụ thuộc vào nhiều yếu tố và được xác định thông qua thử nghiệm. Trong tính toán thuật phóng ngoài, giá trị lực cản thường được ước lượng thông qua hệ số hình dạng đạn. Điều đó dẫn đến nhiều sai số so với thực tế. Với mong muốn xác định được bài toán thuật phóng ngoài nhằm tối ưu hóa thiết kế, bài báo trình bày một phương pháp lý thuyết xác định giá trị lực cản bằng phương pháp phần tử hữu hạn thông qua mẫu đạn tương tự đã được thử nghiệm. Bài báo đã tính toán xác định được tham số thuật phóng ngoài của đạn thiết kế mới phục vụ tính toán thiết kế đạn. Kết quả tính toán có sai lệch không quá 5% so với kết quả thử nghiệm.

Từ khoá: Lực cản khí động; Hệ số lực cản; Phương pháp thổi khí; Thuật phóng ngoài.