Research on the influence of cutting parameters and optimization of spherical surface form error in ultra-precision turning using Box-Behnken and Genetic Algorithm

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ABSTRACT

Ultra-precision diamond turning is a crucial method in manufacturing optical components and precision mechanical parts, especially for machining spherical surfaces. However, the form error of the machined surface is influenced by various factors such as spindle speed, depth of cut and feed rate. This study analyzes the effects of cutting parameters on form error when machining spherical surfaces on an ultra-precision lathe. Fifteen experiments were conducted with machining parameters, including spindle speed, feed rate, and depth of cut, within the recommended range of the lathe. A form error model was developed based on the Box-Behnken model to simulate and evaluate the accuracy of the machined surface. Experimental analysis shows that all machining parameters significantly affect form error. Increasing or decreasing both spindle speed and depth of cut reduces form error, while increasing feed rate generally leads to higher form error. The Genetic Algorithm (GA) was applied to optimize cutting parameters, achieving a minimum form error of 0.846 µm with an optimal spindle speed of 2000 rpm, feed rate of 5 µm/min, and depth of cut of 8 µm. This study develops a predictive model for form error in diamond turning of spherical surfaces and optimizes cutting parameters. It also improves understanding of how machining parameters affect form error, aiding process prediction and optimization for better machining quality in single-point diamond turning (SPDT).

Keywords: SPDT; Lens; Spherical; Form error; Genetic algorithm.

1. INTRODUCTION

Form error is the deviation between a machined surface's actual and theoretical profiles, affecting component performance, especially in high-precision industries like optics. Ultra-precision machining (UPM) requires strict control of form error, often at the nanometer scale, to ensure quality. Study [1] analyzed tool misalignment's effect on the geometric accuracy of convex spherical surfaces and cutting forces in diamond turning, while study [2] investigated spindle imbalance's impact on form error in single-point diamond turning (SPDT).

UPM is an advanced technology that achieves extremely high accuracy, producing components with tight tolerances, smooth surfaces, and high-quality finishes. It is widely used in industries such as optics, aerospace, automotive, telecommunications, and biomedical fields [3]. UPM is classified into four main categories: cutting, grinding, polishing, and non-traditional machining (e.g., electron beam and ion beam figuring) [4]. Ultra-precision cutting uses ultra-hard tools like diamonds to achieve nanometer-level surface roughness and includes turning, milling, boring, and compound machining [4]. Among these, SPDT is widely used for machining optical surfaces, achieving micrometer-level form accuracy and nanometer-level roughness.

In study [5], an aspheric lens made from polymethylmethacrylate (PMMA) was fabricated using the Slow Tool Servo technique, where the cutting tool moves slowly in sync with the spindle to shape complex surfaces. This method enables precise machining of aspheric and freeform optics, achieving a form accuracy of 0.233 μm and a surface roughness of 14.2 nm. While the study demonstrates promising results, it is limited to PMMA, making its applicability to other materials such as glass or metal uncertain. In contrast, the Fast Tool Servo method, which uses a high-frequency actuator for rapid tool oscillation, was combined with a closed-loop toolpath optimization algorithm [6] to machine sinusoidal microstructures and microlenses, achieving form errors below 70 nm and 50 nm, respectively. Despite these achievements, the study primarily focuses on hexagonal lens arrays and sinusoidal microstructures, leaving its effectiveness for more complex geometries yet to be explored. Yuetian Huang et al. investigated the application of diamond turning and ion beam figuring for machining spherical aluminum components in [7]. Their results showed that the form error was nearly halved, decreasing from 226 nm to 113 nm, while the surface roughness improved from 3.02 nm to 2.86 nm. The study on SPDT machining of Ni-P plated CuNi and the use of precision glass molding to evaluate the quality of aspheric lenses in [8] resulted in lenses with low surface roughness (less than 4.5 nm) and high form accuracy (form error less than 0.5 μm). Form accuracy was also investigated in [9] using the response surface methodology (RSM) with two machining parameters: spindle speed and depth of cut. The study concluded that increasing spindle speed and depth of cut leads to a reduction in form error. However, other key factors like feed rate, tool wear, and vibrations, which also affect form accuracy, were not analyzed. Study [10] develops a cutting force prediction model for diamond turning of FSP-treated polycrystalline copper, considering the Hall-Petch effect and residual stress. FSP refines grain size, increases hardness, and improves surface finish (Sz from 23.399 nm to 18.667 nm), but process optimization is needed for varying cutting conditions.

Optimization algorithms have been proven to be more powerful and flexible than traditional optimization methods [11]. According to study [12], various optimization algorithms were compared, with Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) identified as the most effective population-based search techniques. Due to its strong global search capability, GA is highly effective in exploring vast search spaces and finding global optimal solutions [13]. GA has been successfully applied in numerous studies for surface roughness optimization in both conventional and modern machining processes, including alloy steel [14], ductile iron [15], and Inconel [16]. Furthermore, GA has also been used for multi-objective optimization in machining processes. Study [17] used GA as a method for optimizing machining parameters in the turning of composite materials.

Based on the studies, the objective of this paper is to predict the form error of spherical copper surfaces in diamond turning, identify the key machining parameters affecting form error, and determine the optimal machining parameters to minimize form error. The form error optimization process is conducted within the range of machining parameters, including spindle speed, feed rate, and depth of cut. A regression model is developed to predict form error by varying these cutting parameters. Copper is selected for its excellent machinability, high thermal conductivity, and ability to achieve ultra-smooth surfaces, making it ideal for precision optical components. The Box-Behnken design (BBD) is chosen due to its efficiency in minimizing the number of required experiments while ensuring the accuracy of the regression model. The response surface methodology (RSM) is constructed based on the BBD and is then used as the objective function in the GA to efficiently explore the design space and determine the optimal machining parameters.

2. EXPERIMENT

2.1. Experimental methodology

A total of 15 experiments were conducted based on three cutting parameters: spindle speed n (rpm), feed rate F (mm/min), and depth of cut t (μm), all within the machine's recommended range and suitable for the cutting tool. The specific steps are described as follows:

Step 1: Constructing the experimental model using the Box-Behnken method;

Step 2: Conducting experimental machining;

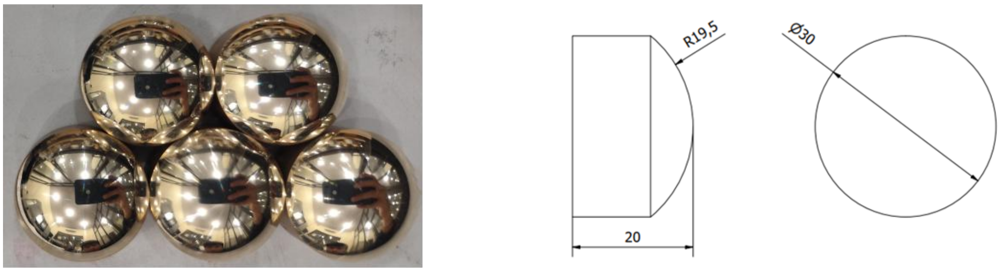
Step 3: Developing a regression model based on ANOVA;

Step 4: Optimizing by GA.

The experiment utilized a copper workpiece (figure 1) with a diameter of 30 mm, a height of h = 20 mm, and a spherical radius of R = 19.5 mm. The chemical composition of the workpiece material is presented in table 1.

Table 1. *Chemical composition of copper alloy.*

	Cu	**Zn**	**Pb**	**Sn**	**Fe**	**Ni**	**Al**	**Si**	**Mn**	**Cr**	**P**
%	54.12	40.94	2.69	0.97	0.75	0.38	0.11	0.02	<0.01	<0.01	<0.01



(a) *(b)*

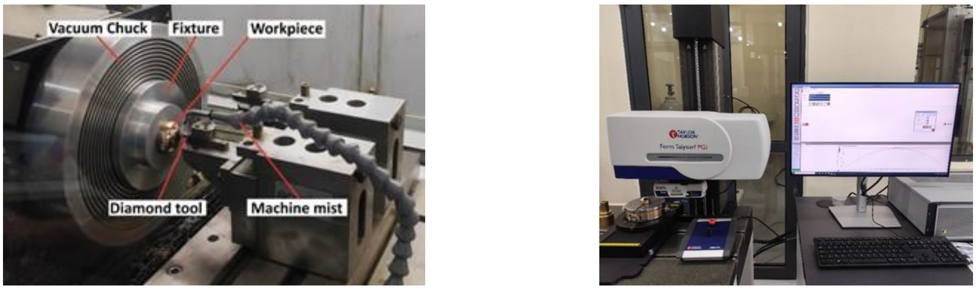
Figure 1. *Workpieces (a); Workpiece drawing (b).*

The cutting tool used is a diamond turning tool NN60R0635mWGC-MS0454, with the specifications shown in table 2.

Table 2. *Cutting tool parameters.*

	Radius	**Rake Angle**	**Cutting Height**	**Conical Clearance**
Value	0.684 mm	-25°	7.475 mm	12°

Machining was performed on the Nanoform® X ultra-precision lathe. The workpiece was mounted on the lathe using a specialized fixture. The runout of the chuck was adjusted to ensure that the P-V value was less than 0.005 μm to minimize system error. The machining process was carried out under laboratory environmental conditions.



(a) *(b)*

Figure 2. *Machining system (a); Form Talysurf® i-Series PRO profiler (b).*

The machining system (figure 2a) consists of a vacuum chuck, the copper workpiece, a fixture, the diamond turning tool, and a misting cooling system. The form error of the workpiece was measured along a center crossing curve using the Form Talysurf® i-Series PRO profiler (figure 2b), which offers high-precision evaluation of optical surfaces with a 20 mm measurement range and 0.2 nm resolution.

2.2. Response surface methodology

Box-Behnken Design (BBD) is an experimental design method within RSM, developed by Box and Behnken in 1960 [18]. It helps model and optimize processes with a minimal number of experiments. A key feature of the Box-Behnken design is that it does not include points at the corners of the design space, reducing the risks associated with extreme experimental conditions. It also utilizes multiple center points, which are suitable for assessing the repeatability and stability of the model. The machining parameters are coded into 3 levels, as shown in table 3.

Table 3. *The factors and levels of RSM based on BBD.*

Factor	Parameter	Unit	Level		
			-1	0	1
A	Spindle speed (n)	rpm	1000	1500	2000
B	Feed rate (F)	mm/min	5	15	25
C	Depth of cut (t)	µm	2	5	8

The regression equation was developed using Design Expert software with a Box-Behnken experimental model in RSM, where the input consists of 3 factors. A total of 15 experiments were conducted, with 3 center-point experiments.

Table 4. *Box-Behnken design and results.*

No.	n (rpm)	F (mm/min)	t (µm)	Form Error (µm)
1	1000	5	5	0.8937
2	2000	5	5	0.8635
3	1000	25	5	1.0815
4	2000	25	5	0.9726
5	1000	15	2	0.9878
6	2000	15	2	1.1060
7	1000	15	8	1.0931
8	2000	15	8	0.9906
9	1500	5	2	0.9180
10	1500	25	2	0.9964
11	1500	5	8	0.9325
12	1500	25	8	1.1650
13	1500	15	5	1.0488
14	1500	15	5	1.0285
15	1500	15	5	1.0530

3. RESULTS AND DISCUSSION

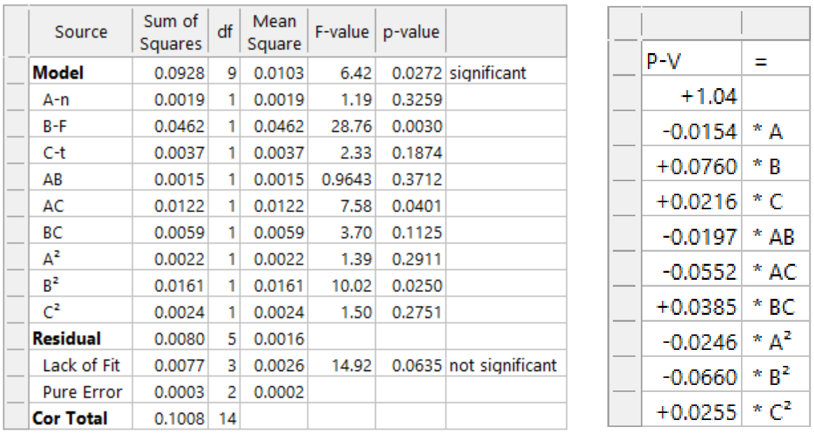
3.1. Results of the ANOVA analysis

Figure 3-a presents the results of the ANOVA analysis and the correlation coefficients. The F-value column reflects the goodness-of-fit of the regression model, indicating the difference in variability between groups of mean values. A high F-value suggests that the model is better at explaining the variation in the data compared to a model without influential factors. If the F-value is large and the P-value is less than 0.05, the model is considered statistically significant. With an F-value of 6.42 and a P-value of 0.0272 (less than 0.05), the model demonstrates high statistical significance and is consistent with the experimental data.

Figure 3-b shows the coefficients of the regression equation. The coefficients, marked in Figure 3-a, have a significant impact on the value of the regression equation due to their high reliability

(p-value < 0.05), while other coefficients can be excluded due to their negligible impact. The regression equation for the form error obtained is as follows:

$$PV = 1.043 + 0.076B - 0.055AC - 0.066B^2 \quad (1)$$



Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	0.0928	9	0.0103	6.42	0.0272	significant
A-n	0.0019	1	0.0019	1.19	0.3259	
B-F	0.0462	1	0.0462	28.76	0.0030	
C-t	0.0037	1	0.0037	2.33	0.1874	
AB	0.0015	1	0.0015	0.9643	0.3712	
AC	0.0122	1	0.0122	7.58	0.0401	
BC	0.0059	1	0.0059	3.70	0.1125	
A²	0.0022	1	0.0022	1.39	0.2911	
B²	0.0161	1	0.0161	10.02	0.0250	
C²	0.0024	1	0.0024	1.50	0.2751	
Residual	0.0080	5	0.0016			
Lack of Fit	0.0077	3	0.0026	14.92	0.0635	not significant
Pure Error	0.0003	2	0.0002			
Cor Total	0.1008	14				

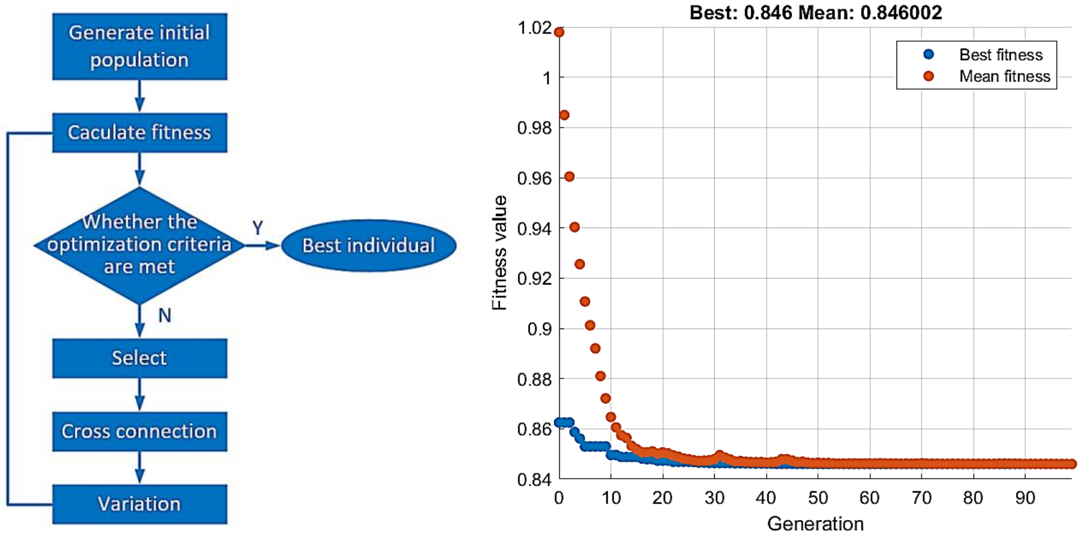
P-V	=
+1.04	
-0.0154	* A
+0.0760	* B
+0.0216	* C
-0.0197	* AB
-0.0552	* AC
+0.0385	* BC
-0.0246	* A²
-0.0660	* B²
+0.0255	* C²

(a) *(b)*

Figure 3. *The ANOVA and coded coefficients for the prediction model (a); Final Equation in Terms of Coded Factors (b).*

3.2. Optimization using genetic algorithm

GA was proposed by Professor John Holland from the University of Michigan in 1975. It is an optimization algorithm that simulates the mechanisms of genetic inheritance and natural selection, based on Darwin's theory of biological evolution. The Genetic Algorithm is one of the most practical, highly effective, and powerful optimization techniques. It provides a general framework for solving complex problems, such as nonlinear, multimodal, and multi-objective problems. It can find global optimal solutions more effectively than traditional optimization methods. The algorithm flow is illustrated in figure 4a.



(a) *(b)*

Figure 4. *GA process (a); Variation of best fitness with number of generations (b) .*

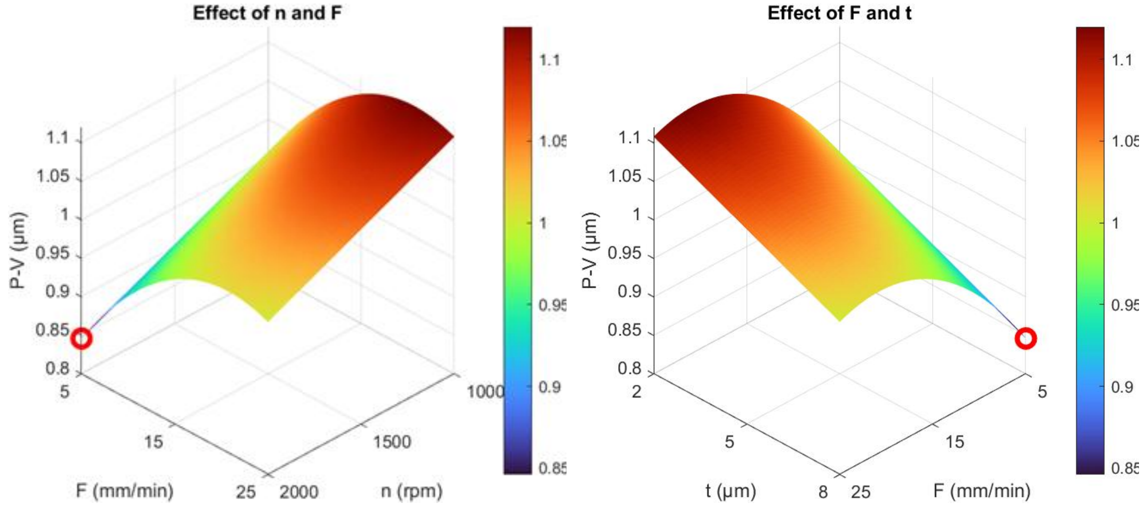
In this study, the Matlab Optimization Toolbox was used to implement GA. Equation (1) was used as the fitness function, with a population size of 100, a crossover fraction of 0.8, and a stopping condition set at 300 generations. The parameters of the algorithm are shown in table 5.

Table 5. *Genetic algorithm parameters.*

Number	Parameter	Value
1	Population Size	100
2	Max Generations	300
3	Crossover Fraction	0.8

The optimal results are shown in figure 4b. The optimal form error value is 0.846 µm, with the encoded technological parameters A = 1, B = -1, and C = 1, corresponding to the cutting parameters: spindle speed n = 2000 rpm, feed rate F = 5 µm/min, and depth of cut t = 8 µm.

The optimal results show that, when the depth of cut remains constant, the form error increases as the spindle speed decreases. When the spindle speed remains constant, the form error increases as the depth of cut decreases. In general, an increase in feed rate leads to a rise in the form error, but when the feed rate exceeds 20 mm/min, the form error slightly decreases (figures 5).



(a) *(b)*

Figure 5. *Effect of feed rate (F) and spindle speed (n) on form error (a); Effect of feed rate (F) and deep of cut (t) on form error (b).*

When the feed rate remains constant, the form error is influenced by both the spindle speed and the depth of cut (figure 6). As the spindle speed and depth of cut increase, the form error decreases, as the cutting process becomes more stable. A higher spindle speed helps reduce the impact of vibrations, particularly low-frequency vibrations, by shortening the contact time between the tool and the workpiece surface. Additionally, a larger depth of cut helps maintain a sufficiently large cutting force to stabilize the tool, minimizing the effects of material elasticity. On the other hand, when both spindle speed and depth of cut decrease, the form error increases. In this case, both the cutting force and the mechanical impact on the workpiece are reduced, leading to lower elastic deformation and residual stress on the surface. Due to the lower spindle speed, the heat generated in the cutting zone is less, which helps mitigate the influence of thermal expansion on the form error.

In the case of a high spindle speed but a small depth of cut, the form error increases due to the small cutting force, which may cause the tool to be affected by microscopic vibrations, resulting in unwanted surface deviations. Furthermore, with a small depth of cut, the material tends to elastic rebound after the tool passes, increasing the form error. Additionally, high spindle speeds can raise the cutting zone temperature, causing uneven thermal expansion and surface deformation. On the other hand, if the spindle speed is low but the depth of cut is large, the cutting force increases significantly, potentially causing system vibrations and disrupting the cutting process stability. At the same time, the residual stresses on the surface increase due to the longer tool contact time with the workpiece, leading to possible unwanted deformation. Moreover, the higher cutting force can accelerate tool wear, negatively affecting the form accuracy.

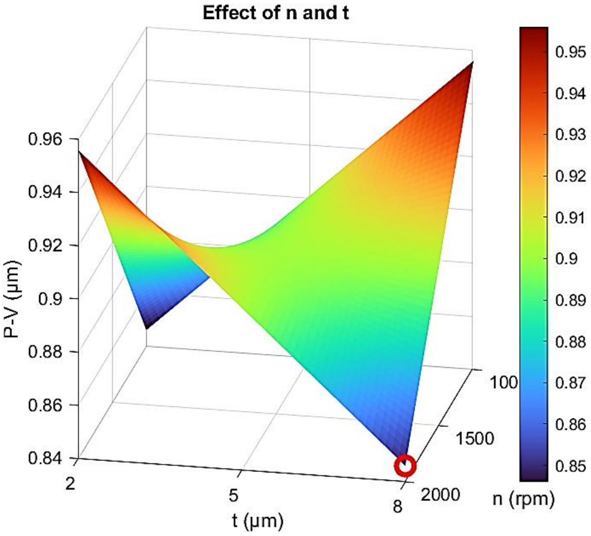


Figure 6. *Effect of the deep of cut (t) and spindle speed (n) on form error.*

From the above analysis, it is evident that both spindle speed and depth of cut need to be balanced to ensure high form accuracy in machining. When both parameters increase or decrease together, the cutting process becomes more stable, and the form error decreases. However, when one parameter increases while the other decreases, the system becomes unbalanced, leading to vibrations, residual stresses, or elastic deformation, which in turn increases the form error. Spindle speed and depth of cut are interdependent factors that significantly influence form error. Meanwhile, an increase in feed rate generally leads to higher form error. Therefore, optimizing these parameters is crucial in diamond turning to achieve the highest accuracy.

The study results indicate a trend of decreasing form error with increasing spindle speed and depth of cut, consistent with the findings of study [9]. However, a key distinction is that this research identifies feed rate as the most influential factor affecting form error, an aspect not thoroughly analyzed in previous studies. Additionally, some other studies have achieved higher accuracy through advanced, whereas this study focuses solely on optimizing cutting parameters.

Experimental testing was conducted to verify the optimal cutting parameters, and the resulting form error was found to be 0.8853 μm, with an error of 4.65%, demonstrating the reliability of the model and the genetic algorithm in predicting and optimizing the process of ultra-precision diamond turning.

4. CONCLUSIONS

In this study, RSM was used to design experiments for investigating the ultra-precision turning process of copper alloy spherical surfaces. GA was applied to find the optimal cutting parameters for achieving the minimum form error. Based on the results obtained, the following conclusions can be drawn:

1. The prediction model for form error was developed, where the linear, quadratic, and interaction components all significantly impact the form error. ANOVA showed that the most significant factor influencing form error was the feed rate, followed by spindle speed and cut depth.

2. Based on the genetic algorithm, when optimizing for reduced form error, the optimal value obtained was 0.846 μm, corresponding to the cutting parameters n = 2000 rpm, F = 5 μm/min, and t = 8 μm. These optimal cutting parameters were verified experimentally, confirming the reliability of both the model and the genetic algorithm.

3. The influence of cutting parameters on form error was analyzed in detail, providing a basis for selecting appropriate cutting parameters. This result helps improve the quality of spherical surface machining through ultra-precision turning.

Additionally, this study only examines a limited range of parameters and has not considered other factors such as surface roughness or tool life. The model could be expanded to optimize over a broader range or combined with advanced algorithms such as artificial neural networks. The research results have potential applications in the manufacturing of optical components, precision molds, and high-accuracy products while also paving the way for more effective optimization in the field of mechanical manufacturing.

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TÓM TẮT

Nghiên cứu ảnh hưởng của chế độ cắt và tối ưu hóa sai số biên dạng mặt cầu khi tiện siêu chính xác sử dụng Box-Berken và thuật toán Di truyền

Tiện kim cương siêu chính xác là phương pháp gia công quan trọng trong chế tạo quang học và linh kiện cơ khí chính xác, đặc biệt trong việc tạo hình bề mặt cầu. Tuy nhiên, sai số biên dạng của bề mặt gia công bị ảnh hưởng bởi nhiều yếu tố như tốc độ trục chính, chiều sâu cắt và lượng chạy dao. Nghiên cứu này phân tích ảnh hưởng của các thông số cắt đến sai số hình dạng khi gia công bề mặt cầu trên máy tiện siêu chính xác. Mười lăm thí nghiệm đã được thực hiện với các thông số gia công, bao gồm tốc độ trục chính, lượng chạy dao và chiều sâu cắt trong phạm vi khuyến nghị của máy. Mô hình sai số biên dạng được xây dựng dựa trên Box-Berken để mô phỏng và đánh giá độ chính xác của bề mặt gia công. Phân tích kết quả thí nghiệm cho thấy các thông số gia công đều có ảnh hưởng đến sai số biên dạng. Tốc độ trục chính và chiều sâu cắt cùng tăng hoặc giảm sẽ làm giảm sai số biên dạng, trong khi lượng chạy dao tăng thường làm tăng sai số biên dạng. Thuật toán di truyền được áp dụng để tối ưu hóa các thông số cắt, đạt sai số biên dạng tối thiểu 0,846 µm với tốc độ trục chính 2000 vòng/phút, tốc độ tiến dao 5 µm/phút và độ sâu cắt 8 µm. Nghiên cứu này phát triển mô hình dự đoán sai số hình dạng trong gia công tiện kim cương bề mặt cầu và tối ưu hóa thông số cắt. Đồng thời, nghiên cứu giúp nâng cao hiểu biết về ảnh hưởng của các thông số gia công đến sai số hình dạng, hỗ trợ dự đoán và tối ưu hóa quy trình nhằm nâng cao chất lượng gia công trong tiện kim cương.

Từ khoá: Tiện kim cương; Thấu kính; Bề mặt cầu; Sai số biên dạng; Thuật toán di truyền.