Multi-period power flow analysis in distribution systems with distributed generation using the nonlinear programming model

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ABSTRACT

The traditional power flow analysis of power systems is usually conducted over a single time period. However, many problems in the power systems, such as determining energy loss and optimizing the location and capacity of shunt capacitors, require performing power flow analysis over multiple time periods. This paper presents a method based on nonlinear programming (NLP) to calculate multi-period power flow (MPF) for a distribution grid with distributed generation (DG). This nonlinear optimization model features a constant objective function, with constraints defined by a system of power balance equations and predetermined values for the voltage magnitude and phase angle at the slack bus. The NLP model is programmed using the GAMS language, and the solution is computed using the KNITRO optimization solver. The proposed multi-time period power flow analysis method was evaluated on the distribution grid in Luc Ngan district, Bac Giang province, with 54 nodes and 96 time periods. The calculation results show that the solution of the proposed approach has a very small error compared to the traditional Newton-Raphson technique. At the same time, distributed generation units have a significant impact on energy loss and voltage profile in the power distribution network.

Keywords: Multi-period power flow (MPF); Nonlinear programming (NLP); Distributed generation (DG).

1. INTRODUCTION

Power flow analysis is an important problem in the operation and planning of electrical systems [1]. This problem aims to determine the steady-state parameters such as the voltage magnitude at nodes, the phase angle of voltages, branch power flows, etc [2]. The solution to the steady-state power flow problem is used to evaluate the operational status of the electrical system. Therefore, various power flow analysis methods have been studied in several papers.

The system of power flow equations is nonlinear, so power flow analysis is often performed using traditional iterative methods such as Gauss-Seidel [3] or Newton-Raphson [4]. The Gauss-Seidel method is simple and suitable for small electrical systems. However, this method uses complex numbers and has a slow convergence rate, especially when calculating for large electrical systems. The Newton-Raphson (NR) method applies a Taylor expansion around the operating point and solves nonlinear power flow equations through successive solutions of multiple linear equations. However, this method may fail to converge when used for steady-state calculations in distribution networks where high R/X ratios and very short-line segments are very popular [5]. Additionally, while distribution networks are often designed with a meshed topology, they generally operate in a radial configuration. As a result, iterative techniques such as the power summation method and the current summation method are commonly employed [6-7]. These methods are simpler and offer faster computation speeds than traditional iterative approaches. However, their applicability is limited to networks with a radial structure.

In distribution networks, due to the increasing integration of distributed generation (DG), effective control and operation of the distribution network are required. Key operational challenges, including the optimal placement of reactive power sources, efficient power distribution, and the selection of optimal switching points, are all subject to the constraints of power flow equations. However, these equations are inherently nonlinear and non-convex, making their direct solution complex. To overcome this issue, two popular techniques are applied in research: (1) linearizing the power flow equations [8] and (2) relaxing their constraints to transform the optimization problem into a convex formulation. Paper [9] uses a modified linear power flow model (MD) to compute the steady state of distribution networks. However, this linearization technique has significant errors in many cases. Therefore, the technique of relaxing the constraints of the power flow equations has been studied in [10, 11]. Paper [10] uses a second-order cone programming (SOCP) model to analyze the steady state for distribution networks. However, study [10] is only applicable to networks with a radial structure; thus, study [11] suggested the SOCP model to analyze power flow for weakly meshed networks.

Studies [3-11] typically perform traditional power flow analysis of electrical systems at a single time period. However, many problems in electrical systems, such as determining energy losses and selecting the location and capacity of shunt capacitors to minimize energy losses, require power flow analysis over multiple periods. Therefore, the aim of this research is to perform steady-state analysis over multiple periods using a nonlinear optimization model. The results of steady-state calculations include node voltages, branch power flows, power losses, and energy losses. The main contributions of this paper include: (1) Proposing a method for calculating power flow over multiple periods using a nonlinear optimization model; (2) Evaluating the accuracy of the proposed method on an actual distribution network with 54 nodes in Luc Ngan district, Bac Giang province, in the year 2021, using day-night load curves and generation output of distributed generation sources (DG) with 96 time periods; (3) Assessing the impact of DG on voltage quality and energy losses in the network using the proposed method.

The paper consists of four parts. Section 1 provides an overview of the research content. Section 2 develops the NLP model for analyzing the steady state over multiple time periods for the distribution network considering DG. Section 3 presents the calculation results and discussion when the model is applied to the actual distribution network with 54 nodes in Luc Ngan district, Bac Giang province, in 2021. Finally, section 4 presents the conclusions and future research directions.

2. METHODOLOGY

2.1. Newton-Raphson method

The purpose of power flow analysis is to determine the voltage magnitude, phase angle of voltages at nodes, active power, and reactive power of power sources. The input data for power flow analysis includes: (1) Complex voltage at the reference node; (2) Active and reactive powers of load nodes; (3) Active power and voltage magnitude for voltage-controlled nodes; (4) Parameters in the equivalent diagram for lines and transformers in the electrical network.

Let N be the total number of nodes in the electrical system. The nodal admittance matrix $\mathbf{Y}_{\text{bus}}$ is expressed as follows:

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} \dot{Y}_{11} & \dot{Y}_{12} & \cdots & \dot{Y}_{1k} & \cdots & \dot{Y}_{1N} \\ \dot{Y}_{21} & \dot{Y}_{22} & \cdots & \dot{Y}_{2k} & \cdots & \dot{Y}_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \dot{Y}_{i1} & \dot{Y}_{i2} & \cdots & \dot{Y}_{ik} & \cdots & \dot{Y}_{iN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \dot{Y}_{N1} & \dot{Y}_{N2} & \cdots & \dot{Y}_{Nk} & \cdots & \dot{Y}_{NN} \end{bmatrix} \tag{1}$$

where, $\dot{Y}_{ii}\left(\forall i=\overline{1,N}\right)$ denotes the self-admittance of node i; $\dot{Y}_{ik}\left(\forall i, \forall k, i \neq k\right)$ stands for the mutual admittance between nodes i and k.

The system of nonlinear equations describing the nodal power at steady state is presented:

$$\begin{cases} P_i = U_i \sum_{k=1}^{N} U_k \left(G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik} \right); i = 1,...,N \\ Q_i = U_i \sum_{k=1}^{N} U_k \left(G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik} \right); i = 1,...,N \end{cases} \tag{2}$$

in which:

- P_i and Q_i denote the active and reactive power injection at node i;
- U_i and U_k represent the voltage magnitudes at nodes i and k, respectively;
- G_{ik} and B_{ik} are the real and imaginary parts of the ik element in the nodal admittance matrix;
- δ_{ik} is the phase angle difference between adjacent nodes i and k ($\delta_{ik} = \delta_i - \delta_k$);
- δ_i and δ_k represent the phase angles at nodes i and k, respectively.

Expressions (2) show that there are two nonlinear equations and four unknowns for each node i. Therefore, to solve the power flow equations, we need to know two variables for each node. Thus, the nodes in the power flow analysis problem are classified as follows:

PQ Node: The active and reactive powers injected at node i are known:

$$P_i = P_i^{\mathrm{sp}} = -P_{\mathrm{D}i}^{\mathrm{sp}}; \quad Q_i = Q_i^{\mathrm{sp}} = -Q_{\mathrm{D}i}^{\mathrm{sp}} \tag{3}$$

where $P_{\mathrm{D}i}^{\mathrm{sp}}$ and $Q_{\mathrm{D}i}^{\mathrm{sp}}$ represent the active and reactive powers consumed by the load at node i.

The two variables to be found are the voltage magnitude at the node (U_i) and the phase angle at the node (δ_i).

PV Node: The voltage magnitude at node i and the active power injected at node i are known:

$$U_i = U_i^{\mathrm{sp}}; \quad P_i = P_i^{\mathrm{sp}} = P_{\mathrm{G}i}^{\mathrm{sp}} - P_{\mathrm{D}i}^{\mathrm{sp}} \tag{4}$$

where $P_{\mathrm{G}i}^{\mathrm{sp}}$ is the active power generated at node i.

The two variables to be found are the reactive power injected at node i (Q_i) and the phase angle at the node (δ_i).

Reference Node: The voltage magnitude (U_i) and phase angle (δ_i) are known. The variables to be determined are the active power (P_i) and reactive power (Q_i) injected at node i.

The Newton-Raphson method is considered the most widely used method for solving nonlinear equations (2). In this approach, at the r-th iteration, we need to solve the following linear equations:

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix}^{(r)} = \begin{bmatrix} \mathbf{H} & \mathbf{N} \\ \mathbf{M} & \mathbf{L} \end{bmatrix}^{(r)} \begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\mathbf{U}/\mathbf{U} \end{bmatrix}^{(r)} \tag{5}$$

where **H, N, M**, and **L** are four submatrices of the Jacobian matrix.

$$\left[\Delta\mathbf{P} \middle| \Delta\mathbf{Q} \right]^{\mathrm{T}} = \left[\Delta P_1, \Delta P_2, \ldots, \Delta P_{N-1} \,|\, \Delta Q_1, \Delta Q_2, \ldots, \Delta Q_{N_{\mathrm{D}}} \right]^{\mathrm{T}} \tag{6}$$

$$\begin{aligned} \Delta P_i &= P_i^{\mathrm{sp}} - U_i \sum_{k=1}^{N} U_k (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik}); i = \overline{1, N-1} \\ \Delta Q_i &= Q_i^{\mathrm{sp}} - U_i \sum_{k=1}^{N} U_k (G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik}); i = \overline{1, N_{\mathrm{D}}} \end{aligned} \tag{7}$$

The Newton-Raphson method [12] is considered the most popular method for solving nonlinear systems of equations. According to this method, the values of the voltage magnitude and phase angle at the (r + 1)-th iteration are calculated as follows:

$$\begin{bmatrix}\boldsymbol{\delta}\\ \mathbf{U}\end{bmatrix}^{(r+1)}=\begin{bmatrix}\boldsymbol{\delta}\\ \mathbf{U}\end{bmatrix}^{(r)}+\begin{bmatrix}\boldsymbol{\Delta\delta}\\ \mathbf{\Delta U}\end{bmatrix}^{(r)} \tag{8}$$

2.2. Nonlinear optimization-based method

The equivalent diagram of a two-node distribution network is described in figure 1, where r_{ik} và x_{ik} are the resistance and series reactance of branch *ik*, respectively.

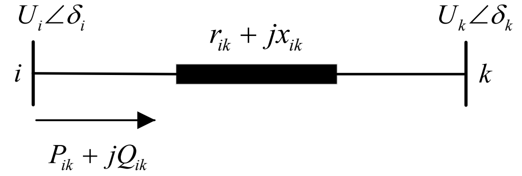

Figure 1. *Equivalent diagram of a two-node distribution network.*

The active power flow (P_{ik}) and reactive power flow (Q_{ik}) on branch *ik* at node *i* are calculated:

$$P_{ik}=g_{ik}U_i^2-U_iU_k\left(g_{ik}\cos\delta_{ik}+b_{ik}\sin\delta_{ik}\right) \tag{9}$$

$$Q_{ik}=-b_{ik}U_i^2-U_iU_k\left(g_{ik}\sin\delta_{ik}-b_{ik}\cos\delta_{ik}\right) \tag{10}$$

where g_{ik} and b_{ik} are defined by:

$$g_{ik}=\frac{r_{ik}}{r_{ik}^2+x_{ik}^2};\quad b_{ik}=-\frac{x_{ik}}{r_{ik}^2+x_{ik}^2} \tag{11}$$

Consider a distribution network with N nodes (where node 1 is the reference node). The nonlinear optimization (NLP) model for the analysis of the steady state with T time periods of the distribution network integrated with DG is described as follows:

$$\min \mathbf{0} \tag{12}$$

subject to the constraints:

$$\sum_{k\in\Omega_i}^{N}P_{ik,t}=P_{\mathrm{G}i,t}-P_{\mathrm{D}i,t};\quad \forall i\in\Omega_{\mathrm{D}}\cup\Omega_{\mathrm{G}};t=1,2,...T \tag{13}$$

$$\sum_{k\in\Omega_i}^{N}Q_{ik,t}=Q_{\mathrm{G}i,t}-Q_{\mathrm{D}i,t};\quad \forall i\in\Omega_{\mathrm{D}};\ t=1,2,...T \tag{14}$$

$$P_{ik,t}=g_{ik}U_{i,t}^2-U_{i,t}U_{k,t}\left(g_{ik}\cos\delta_{ik,t}+b_{ik}\sin\delta_{ik,t}\right);\ t=1,2,...T \tag{15}$$

$$Q_{ik,t}=-b_{ik}U_{i,t}^2-U_{i,t}U_{k,t}\left(g_{ik}\sin\delta_{ik,t}-b_{ik}\cos\delta_{ik,t}\right)\ ;t=1,2,...T \tag{16}$$

$$U_{1,t}=U_{ref,t};\delta_{1,t}=0\ ;t=1,2,...T \tag{17}$$

$$U_{i,t}=U_{i,t}^{sp},\forall i\in\Omega_{\mathrm{G}};\ ;t=1,2,...T \tag{18}$$

where Ω_{D} và Ω_{G} denote the sets of PQ buses and PV buses, respectively; $P_{ik,t}$ và $Q_{ik,t}$ are the active and reactive power flows on branch *ik* at time *t*; $P_{\mathrm{G}i,t}$ và $Q_{\mathrm{G}i,t}$ are the active and reactive powers of generating units at bus *i* at time *t*; $P_{\mathrm{D}i,t}$ và $Q_{\mathrm{D}i,t}$ are the active and reactive power demand at bus *i* at time *t*; $U_{i,t}$ and $U_{k,t}$ are the voltage magnitudes at buses *i* and *k* at time *t*; $\delta_{ik,t}$ is the phase angle difference between buses *i* and *k* at time t; $U_{ref,t}$ is the specified voltage magnitude at the slack bus; $U_{i,t}^{sp}$ is the specified voltage magnitude for PV buses.

The nonlinear optimization model has an objective function (12) set as a constant (chosen as 0), which means that the feasible solution found for the problem only needs to satisfy all the constraints. Constraints (13)-(14) describe the nodal power balance equations in the electrical system. The branch power flow is represented through equations (15)-(16). Expression (17) specifies the voltage magnitude and phase angle at the known reference node. Equation (18) describes the voltage magnitude at the PV nodes.

3. RESULTS AND DISCUSSION

3.1. Description of test system

The proposed NLP model is deployed to analyze the steady-state operation of a 54-node distribution grid in the Luc Ngan district, Bac Giang province (see figure 2). The grid consists of 54 nodes and 53 branches, with its parameters provided in the appendix. The rated voltage of the system is 22 kV. This study examines the steady-state operation over a 24-hour period, divided into 96 time intervals. Various types of loads are considered, including industrial load (IL), commercial load (CL), and domestic load (DL). Table 1 presents the load types at each node and their respective maximum power demands. Additionally, all loads are assumed to have a power factor of 0.9. Figure 3 illustrates the percentage of power consumption relative to the maximum power demand over the 96 time intervals.

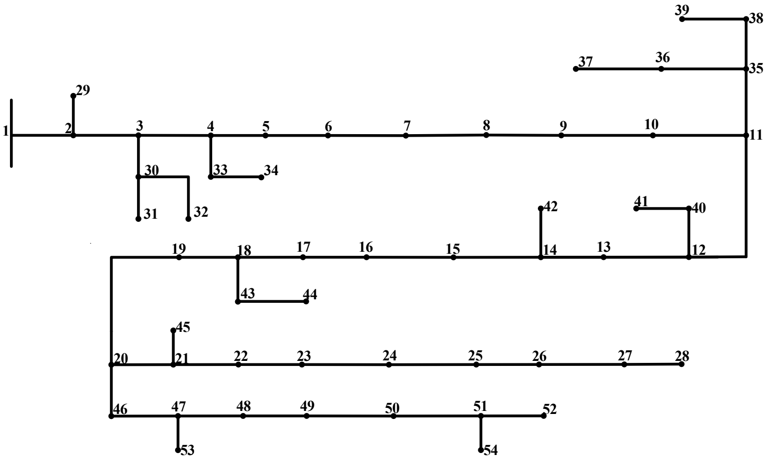

Figure 2. *Schematic diagram of the 54-node distribution grid in Luc Ngan, Bac Giang.*

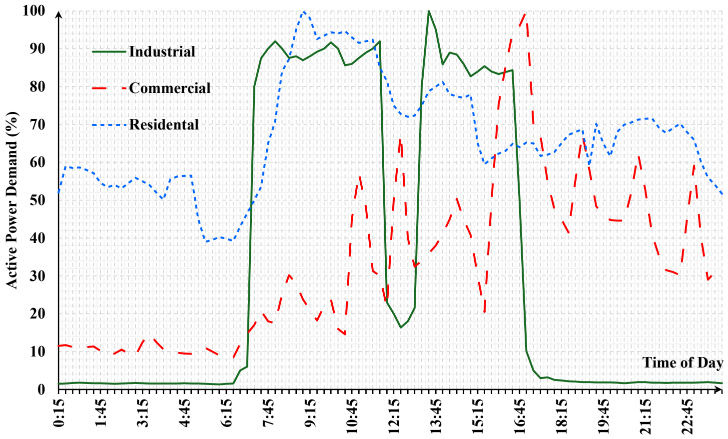

Figure 3. *Percentage of power consumption over 96-time intervals relative to load's maximum power.*

Table 1. *Maximum power consumption of different load types at each node.*

Node	**Load type**	P_{Dmax} **(kW)**	**Node**	**Load type**	P_{Dmax} **(kW)**	**Node**	**Load type**	P_{Dmax} **(kW)**	**Node**	**Load type**	P_{Dmax} **(kW)**
2		0	**16**	DL	145.8	**30**		0	**44**	IL	202.5
3		0	**17**	DL	81	**31**	DL	145.8	**45**	IL	202.5
4		0	**18**		0	**32**	DL	81	**46**	IL	145.8
5	IL	510.3	**19**	IL	145.8	**33**	IL	453.6	**47**		0
6	IL	324	**20**		0	**34**	DL	129.6	**48**	CL	202.5
7	CL	648	**21**		0	**35**		0	**49**	CL	202.5
8	CL	453.6	**22**	CL	405	**36**	DL	81	**50**	CL	145.8
9	CL	453.6	**23**	CL	145.8	**37**	DL	145.8	**51**		0
10	CL	145.8	**24**	CL	145.8	**38**	DL	453.6	**52**	IL	259.2
11		0	**25**	IL	145.8	**39**	IL	1012.5	**53**	IL	145.8
12		0	**26**	IL	81	**40**	IL	145.8	**54**	DL	202.5
13	IL	145.8	**27**	CL	1012.5	**41**	DL	145.8			
14		0	**28**	CL	202.5	**42**	DL	81			
15	DL	81	**29**	IL	145.8	**43**	IL	81			

In this study, distributed generation (DG) sources, including wind and solar power, are considered. Table 2 provides details on the DG sources at each node and their corresponding maximum generation output. All DG sources operate with a power factor of 0.95. Figure 4 depicts the percentage of power generation at each time interval relative to the maximum generation output of the DG sources. When performing the calculations, the voltage at node 1 (slack node) is chosen to be 1.05 p.u., and all parameters are calculated in a per-unit system with a base power of 1 MVA.

Table 2. *Maximum generation output of distributed generation (DG) sources at each node.*

Node	**10**	**22**	**37**	**50**
DG type	WT	WT	PV	PV
P_{DGmax} **(kW)**	1000	1000	1200	800

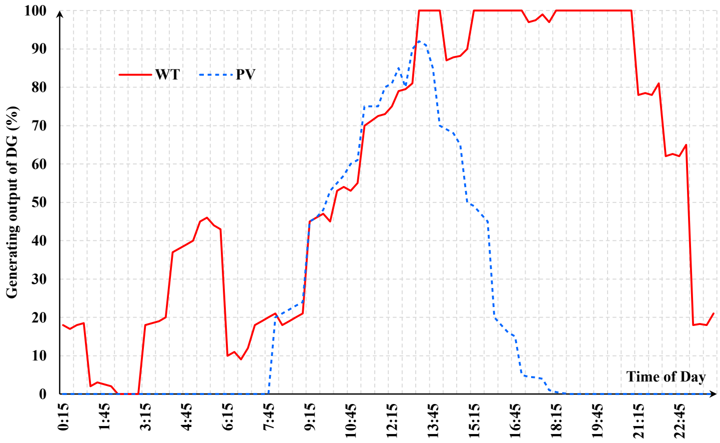

Figure 4. *Percentage of power output over 96-time intervals relative to maximum DG output.*

3.2. Calculation results and discussion

This section presents the results obtained from the suggested NLP model for power flow analysis in the 54-node distribution grid of Luc Ngan district, Bac Giang province. The NLP model is programmed with the GAMS language [13] and solved using the commercial solver KNITRO.

The NLP model was implemented on a computer equipped with an Intel Core i5-8250U processor (1.6 GHz) and 8 GB of RAM. To ensure accuracy and effectiveness, the results from the NLP model are compared with those obtained from the conventional Newton-Raphson (NR) method using MATPOWER [14].

Table 3. *Comparison of voltage magnitudes between the NLP model and the Newton-Raphson method.*

Node	***U* (pu)**		**Error (%)**	**Node**	***U* (pu)**		**Error (%)**
	NLP	**NR**			**NLP**	**NR**	
1	1.050000	1.050000	0.00000	**28**	**1.013074**	**1.013070**	0.00041
2	1.029048	1.029046	0.00021	**29**	1.028987	1.028985	0.00019
3	1.028617	1.028615	0.00017	**30**	1.028461	1.028459	0.00022
4	1.027719	1.027717	0.00016	**31**	1.028403	1.028401	0.00018
5	1.025980	1.025978	0.00024	**32**	1.028429	1.028427	0.00022
6	1.025626	1.025624	0.00020	**33**	1.027659	1.027657	0.00018
7	1.024062	1.024060	0.00023	**34**	1.027646	1.027644	0.00016
8	1.023492	1.023489	0.00028	**35**	1.022065	1.022063	0.00023
9	1.023122	1.023119	0.00025	**36**	1.022346	1.022344	0.00024
10	1.022757	1.022754	0.00028	**37**	1.022464	1.022461	0.00031
11	1.022471	1.022469	0.00024	**38**	1.021914	1.021911	0.00026
12	1.021510	1.021507	0.00025	**39**	1.021595	1.021592	0.00027
13	1.020637	1.020634	0.00029	**40**	1.021361	1.021358	0.00025
14	1.019490	1.019487	0.00033	**41**	1.021303	1.021301	0.00024
15	1.018854	1.018851	0.00027	**42**	1.019466	1.019463	0.00034
16	1.018138	1.018135	0.00028	**43**	1.016029	1.016025	0.00035
17	1.016903	1.016900	0.00033	**44**	1.015837	1.015833	0.00040
18	1.016059	1.016055	0.00036	**45**	1.014829	1.014825	0.00037
19	1.015561	1.015557	0.00043	**46**	1.015006	1.015002	0.00042
20	1.015094	1.015090	0.00039	**47**	1.014776	1.014772	0.00039
21	1.015001	1.014997	**0.00044**	**48**	1.014710	1.014706	0.00043
22	1.014962	1.014958	0.00043	**49**	1.014694	1.014690	0.00035
23	1.014359	1.014355	0.00038	**50**	1.014722	1.014718	0.00041
24	1.013894	1.013890	0.00040	**51**	1.014321	1.014317	0.00036
25	1.013637	1.013633	0.00039	**52**	1.014206	1.014202	0.00037
26	1.013567	1.013563	0.00041	**53**	1.014730	1.014726	0.00041
27	1.013259	1.013255	0.00042	**54**	1.014200	1.014196	0.00041

Table 3 presents the voltage errors at 14:00 (the 56th time interval) between the proposed NLP model and the Newton-Raphson method. The observed differences are negligible for practical operation, with the maximum error recorded at only 0.00044% (at node 21 at 14:00). Furthermore, the lowest voltage magnitude is observed at node 28, with values of 1.013074 pu for the proposed NLP model and 1.013070 pu for the NR method.

Table 4 provides a comparison of active and reactive power losses at 14:00 (56th time interval) between the two methods. The differences in power loss calculations are small enough, with an error of 0.0171510% for active power loss and 0.0171501% for reactive power loss. These results confirm the accuracy and computational efficiency of the proposed NLP model.

To assess the impact of DG on power quality and energy losses, the NLP model is applied under two scenarios: (1) a power system without DG, and (2) a power system with DG (this study). Figure 5 illustrates the voltage patterns for both scenarios at 14:00. It is evident that the voltage magnitudes at all nodes (except the slack node) are consistently higher when DG is considered. This is attributed to lower voltage drops in the DG-integrated scenario compared to the non-DG case, leading to a significant improvement in power quality.

Figure 6 illustrates the active power losses across 96 time intervals for both scenarios. In every time interval, the system without DG experiences greater power losses compared to the system with DG. The highest power loss occurs at 16:30 (66th time interval), where the losses are 0.6373 MW for the non-DG scenario and 0.3283 MW for the DG-integrated system.

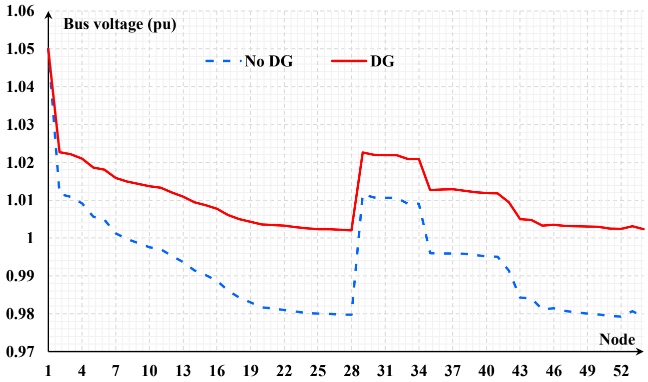

Figure 5. *Voltage pattern comparison between the two scenarios (with and without DG) at 14:00.*

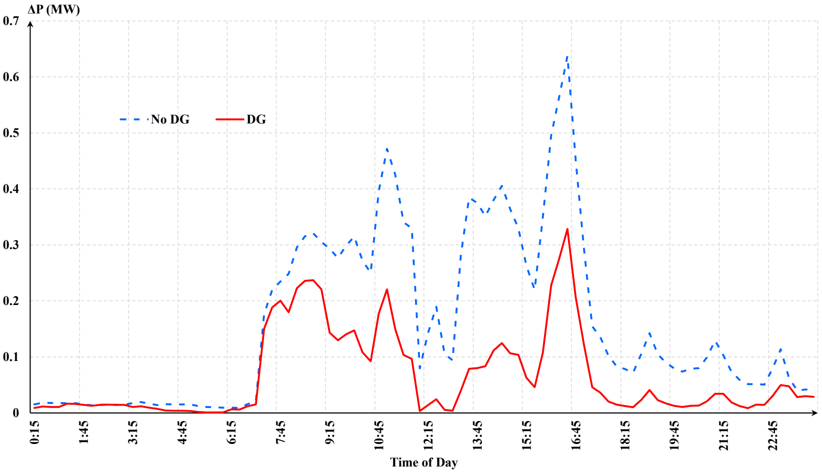

Figure 6. *Comparison of active power losses over 96 time intervals for the two scenarios (with and without DG).*

Table 4. *Comparison of power losses between the NLP model and the Newton-Raphson method at 14:00.*

	NLP	NR	Error (%)
$\Delta P(\mathrm{MW})$	0.0833331	0.0833188	0.0171510
$\Delta Q(\mathrm{MVAr})$	0.0743940	0.0743812	0.0171501

Table 5 presents a comparison of total energy losses between the two scenarios. The results indicate that the system without DG incurs energy losses approximately 2.5 times higher than the DG-integrated system, with losses of 3.8305 MWh and 1.5525 MWh, respectively. These findings demonstrate that integrating DG improves power quality while significantly reducing energy losses in the distribution network.

***Table 5.** Comparison of total energy losses between the two scenarios (with and without DG).*

$\Delta A(\text{MWh})$	**Without DG (Scenario 1)**	**With DG (Scenario 2)**
	3.8305	1.5525

4. CONCLUSIONS

This paper presents a nonlinear programming (NLP) optimization model for multi-period power flow analysis in distribution networks with DG integration. The NLP model features a constant objective function, with system constraints including power flow equations and predefined values for voltage magnitude and phase angle at the slack node. The proposed model is applied to a 54-node distribution network in Luc Ngan, Bac Giang, over 96 time intervals. The results demonstrate the high accuracy and efficiency of the NLP model, with negligible errors compared to the conventional Newton-Raphson method.

Additionally, this study examines the impact of DG integration by evaluating steady-state operation under two scenarios: with and without DG. The findings reveal that integrating DG enhances voltage quality and significantly reduces energy losses. In future research, the proposed multi-period power flow model can be further extended to optimize the placement and sizing of DG units in distribution networks.

APPENDIX

***Table 6.** Line parameters of the 54-node distribution grid.*

Start Node	**End Node**	$r(\Omega)$	$x(\Omega)$	**Start Node**	**End Node**	$r(\Omega)$	$x(\Omega)$	**Start Node**	**End Node**	$r(\Omega)$	$x(\Omega)$
1	**2**	1.985	1.772	**14**	**15**	0.177	0.158	**30**	**31**	0.169	0.151
2	**3**	0.042	0.038	**14**	**42**	0.127	0.113	**30**	**32**	0.169	0.151
2	**29**	0.169	0.151	**15**	**16**	0.211	0.189	**33**	**34**	0.042	0.038
3	**4**	0.093	0.083	**16**	**17**	0.405	0.362	**35**	**36**	0.169	0.151
3	**30**	0.296	0.264	**17**	**18**	0.296	0.264	**35**	**38**	0.042	0.038
4	**5**	0.211	0.189	**18**	**19**	0.232	0.207	**36**	**37**	0.063	0.057
4	**33**	0.042	0.038	**18**	**43**	0.042	0.038	**38**	**39**	0.127	0.113
5	**6**	0.051	0.045	**19**	**20**	0.262	0.234	**40**	**41**	0.169	0.151
6	**7**	0.253	0.226	**20**	**21**	0.127	0.113	**43**	**44**	0.380	0.339
7	**8**	0.106	0.094	**20**	**46**	0.084	0.075	**46**	**47**	0.338	0.302
8	**9**	0.076	0.068	**21**	**22**	0.169	0.151	**47**	**48**	0.211	0.189
9	**10**	0.084	0.075	**21**	**45**	0.338	0.302	**47**	**53**	0.127	0.113
10	**11**	0.042	0.038	**22**	**23**	0.253	0.226	**48**	**49**	0.211	0.189
11	**12**	0.198	0.177	**23**	**24**	0.211	0.189	**49**	**50**	0.160	0.143
11	**35**	0.211	0.189	**24**	**25**	0.127	0.113	**50**	**51**	0.355	0.317
12	**13**	0.211	0.189	**25**	**26**	0.042	0.038	**51**	**52**	0.177	0.158
12	**40**	0.211	0.189	**26**	**27**	0.211	0.189	**51**	**54**	0.253	0.226
13	**14**	0.304	0.271	**27**	**28**	0.760	0.679				

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TÓM TẮT

Phân tích trào lưu công suất nhiều khoảng thời gian cho lưới điện phân phối có nguồn điện phân tán sử dụng mô hình tối ưu phi tuyến

Phân tích trào lưu công suất truyền thống thường chỉ được thực hiện trong một khoảng thời gian. Tuy nhiên, nhiều bài toán trong hệ thống điện như tính toán tổn thất điện năng, tối ưu vị trí và công suất tụ bù yêu cầu phân tích trào lưu công suất theo nhiều khoảng thời gian. Bài báo này đề xuất một phương pháp dựa trên tối ưu phi tuyến (NLP) để tính toán trào lưu công suất nhiều khoảng thời gian (MPF) cho lưới điện phân phối có nguồn điện phân tán (DG). Mô hình NLP có hàm mục tiêu hằng số và các ràng buộc là hệ phương trình cân bằng công suất với giá trị cố định của mô-đun và góc pha điện áp tại nút cân bằng. Mô hình được lập trình bằng GAMS và giải bằng bộ tối ưu KNITRO. Phương pháp được kiểm chứng trên lưới điện phân phối huyện Lục Ngạn, Bắc Giang với 54 nút, 96 khoảng thời gian. Kết quả tính toán cho thấy phương pháp đề xuất có sai số rất nhỏ so với phương pháp Newton-Raphson truyền thống. Đồng thời, DG có ảnh hưởng đáng kể đến tổn thất điện năng và chất lượng điện áp của lưới phân phối.

Từ khoá: Phân tích trào lưu công suất nhiều khoảng thời gian (MPF); Tối ưu phi tuyến (NLP); Nguồn điện phân tán (DG).