Research on optimization of surface roughness of aspherical aluminum during Single-Point Diamond Turning using BBK model and PSO algorithm

Dao Van Duong[1], Nguyen Kim Hung[2], Ngo Van Tuan[2],
Vu Thi Khanh Van[3], Duong Xuan Bien[2*]

[1]Ho Chi Minh City University of Industry and Trade, 140 Le Trong Tan, Tay Thanh, Ho Chi Minh City, Vietnam;
[2]Le Quy Don Technical University, 236 Hoang Quoc Viet, Dong Ngac, Hanoi, Vietnam;
[3]Viet Tri University of Industry, 9 Tien Son, Thanh Mieu, Phu Tho, Vietnam.
[*]Corresponding author: duongxuanbien@lqdtu.edu.vn
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ABSTRACT

The paper presents the results of optimizing the surface quality of aspheric aluminum during Single Point Diamond Turning (SPDT) based on the Box-Behnken Design (BBD) experimental method. The experimental dataset consists of 15 experiments established from the BBK model, supported by the ANOVA module in the specialized software DESIGN EXPERT. The objective function for roughness was established, resulting in a second-order multivariable regression equation that defines the roughness of the aspheric surface based on the data obtained after conducting all experiments. The objective function is formed based on the relationship between the roughness of the aspheric surface and the cutting parameters: spindle speed (n - RPM); feed rate (F - mm/min); and depth of cut (ap - mm). The modeling results, with a reliability of R^2 =0.9536, show a high correlation between the model data and the experimental values. By using the Particle Swarm Optimization (PSO) algorithm, the optimal surface roughness value achieved is 0.8 nm, obtained under the machining conditions of n = 2000 RPM, F = 8 mm/min, and ap = 4.2 µm. This study is significant for enhancing the optical surface quality in ultra-precision machining and provides a reliable foundation for building experimental models to optimize and accurately assess the machining process.

Keywords: SPDT; Aspheric surface; Surface roughness; Box-Behnken; PSO.

1. INTRODUCTION

Aspheric lenses play a very important role; they are designed to reduce optical aberrations, enhance the convergence of light beams, and improve the quality and performance of optical systems. Aspheric lenses redistribute light rays, while other lenses merely shape them [1]. Research by Kawamura and colleagues [2] has shown that aspheric lenses can effectively transform Gaussian beams into flat-top profiles. Aspheric surfaces are widely used in precision and ultra-precision measurement engineering, aerospace, biomedical engineering, and sensors. Surface roughness is a parameter that directly affects optical performance, reflectivity, energy loss, mechanical durability, and the applicability of lenses [3]. According to previous studies, the roughness of a surface depends on factors such as the tool nose radius, technological parameters, relative fit between the tool and the chip, material properties, microstructure, and the cutting edge quality of the tool [4]. Technological modes, including spindle speed, have a direct and significant impact on the quality of the machined surface [5].

Single Point Diamond Turning (SPDT) is a modern ultra-precision machining method [6], applied to produce rotational optical components with high surface quality and profile accuracy [7]. Products created using SPDT exhibit high surface quality with roughness in the nanometer range [8]. Therefore, SPDT is utilized to create high-quality surfaces without the need for polishing or grinding [9]. SPDT is increasingly playing a crucial role in the production of optical lenses, lens molding, optical plastic molds, and reflective mirrors.

There are many optimization algorithms, such as Ant Colony Optimization (ACO), Artificial Bee Colony (ABC), Grey Wolf Optimization (GWO), and Genetic Algorithm (GA), Particle Swarm Optimization (PSO). PSO is an algorithm that has advantages over other algorithms in terms of ease of implementation, low cost, simple structure, good scalability, and high accuracy of the results obtained [10]. The PSO algorithm can find the optimal solution faster than any other search algorithm. The PSO algorithm is widely used in manufacturing industries, such as optimizing tool paths, energy, and machining costs on 4-axis milling machines [11].

In this paper, a Box-Behnken experimental model was developed based on the relationship between technological parameters and the surface roughness of aspheric aluminum materials. With a minimal number of experiments, a mathematical surface was constructed. Analysis of variance (ANOVA) was conducted, along with which the influence of parameters on surface quality was evaluated and their impact was understood. A PSO algorithm based on the mathematical regression function (BBK) was implemented to optimize surface roughness and identify the corresponding parameter set.

2. RESEARCH CONTENT

2.1. Aspheric lenses and preparation of aspheric surfaces

Aspherical lenses have the ability to effectively correct aberrations and spherical aberrations. The equation representing the standard aspheric surface is:

$$z = \frac{Cr^2}{1+\sqrt{1-(1+K)C^2r^2}} + A_1r^2 + A_2r^4 + ... + A_7r^{14} + A_8r^{16} \tag{1}$$

Where: C is the curvature of the apex (mm^{-1}); K: The conic constant describes the shape of the conic surfaces. K < -1: Hyperbolic shape, K = -1: Parabolic shape, -1 < K < 1: Stretched elliptical shape, K > 0: Flattened elliptical shape, K = 0: Spherical shape; A_1, A_2...A_8: Aspheric coefficients; r: Radial distance (mm); For a sphere, the conic constant K = 0, and all coefficients A[i] = 0. The aspheric surface in this paper is constructed based on the aspheric coefficients as follows: $k = 0$; $R = 19\ mm$; $A_2 = -7.55\times10^{-6}$; $A_4 = 1.803\times10^{-7}$; $A_6 = -6.818\times10^{-9}$; $A_8 = 5.121\times10^{-11}$; $A_{10} = -1.355\times10^{-13}$.

2.2. Machining process experiment

The aluminum workpieces used in this paper have a height of h = 19 mm, an outer diameter of Ø = 30 mm, and a spherical radius of R = 19.5 mm, as specifically described in figure 1. The material composition of the aluminum used in the machining process was determined using the Brucker QB spectrometer, and the specific chemical composition of the workpiece is presented in table 1.

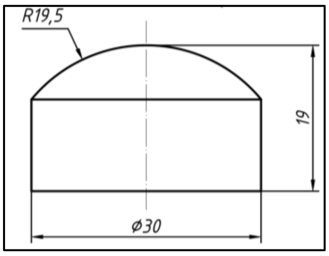

Figure 1. *Workpiece.*

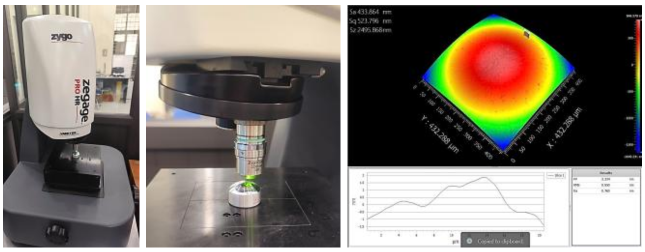

Figure 2. *Measuring SR on the ZEGAGE PRO HR.*

Table 1. *The chemical composition of the aluminum.*

Run	Mg	Si	Mn	Ti	Z	Cu	Fe	Cr	Ni	Pb	Sn	Al%
Avg	1.379	0.189	0.488	0.15	0.026	4.038	0.149	0.042	0.005	0.002	0.008	93.624

Surface roughness is measured by the 3D optical profiler ZeGage Pro HR, which uses light wave measurement technology. This non-contact device is capable of measuring micro and nano-scale roughness, as described in figure 2. The experiments were conducted on the ultra-precision diamond lathe Nanoform® X, and the technological system is specifically described in figure 3.

Figure 3. *Machining system, workpiece and tool.*

2.3. Box-Behnken experimental model (BBD)

BBD is a widely used response surface methodology in optimization problems. BBD is a design that includes a central point with points at the midpoints of the edges [12]. Analysis of Variance (ANOVA) is subsequently conducted to assess the significance of the model. The independent parameters considered for optimization are detailed in table 2. The number of experiments can be calculated using the following formula:

$$N = k^2 + k + cp \tag{2}$$

The encoded variable values are obtained and used in the equation:

$$x_i = \frac{X_i - X_{io}}{\Delta X_i};\ i\ = 1,2,3,....k \tag{3}$$

Where, x_i are the encoded values, X_i is the actual value of the independent variables, X_{io} is the actual value in the central plane and ΔX_i is the step change. The regression equation is represented as follows:

$$y = \beta_0 + \sum_{i=1}^{k}\beta_i x_i + \sum_{i=1}^{k}\beta_{ii} x_i^2 + \sum_{i<j}\beta_{ij} x_i x_j + \varepsilon \tag{4}$$

Where, Y is the response variable, β represents a set of regression coefficients (also known as constants), k is the number of independent variables, and ε is a constant, unobserved error vector.

Table 2. *Values the experimental variables coding levels.*

Parameters	**Unit**	**Symbol**	**Level (-1)**	**Level (0)**	**Level (+1)**	ΔX_i
Spindle speed (S)	*rev/min*	x_1	1000	1500	2000	500
Feed rate (F)	*mm/min*	x_2	5	15	25	5
Depth of cut (ap)	*µm*	x_3	2	5	8	3

The parameter ranges in table 2 were selected by the research team based on the advice of the cutting tool distributor and the authors' trial cutting process in determining the effective machining range. The experimental matrix, based on the BBD method and experimental data (table 3), was used to develop a second-order multivariable regression equation 5.

$$Ra = 0,9863 - 0.1374x_1 + 0.1505x_2 - 0.1044x_2x_3 + 0,128x_2^2 + 0,135x_3^2\ (nm) \tag{5}$$

This equation can be used to make predictions and assess the individual or paired effects of factors on surface roughness. The model run data is specifically described in table 4 with a reliability of 95.36%.

Table 3. *Experiment matrix and experimental results.*

No.	Encryption value			Result SR
	n (rev/min)	F (mm/min)	ap (µm)	
1	-1	-1	0	1.0305
2	1	-1	0	0.826
3	-1	1	0	1.3515
4	1	1	0	1.0645
5	-1	0	-1	1.0725
6	1	0	-1	0.9305
7	-1	0	1	1.3815
8	1	0	1	0.9155
9	0	-1	-1	0.9335
10	0	1	-1	1.4645
11	0	-1	1	1.243
12	0	1	1	1.3565
13	0	0	0	0.9425
14	0	0	0	0.9055
15	0	0	0	1.111

Table 4. *ANOVA analysis for the surface roughness quadratic model.*

Source	Sum of Squares	Degree of freedom	Mean Square	F-value	p-value	
Model	0.5690	9	0.0632	11.43	**0.0077**	**significant**
X_1 - cutting speed	0.1511	1	0.1511	27.31	**0.0034**	**significant**
X_2 - Feed rate	0.1812	1	0.1812	32.75	**0.0023**	**significant**
X_3 - Depth of cut	0.0307	1	0.0307	5.55	0.0651	insignificant
X_1X_2	0.0017	1	0.0017	0.3075	0.6031	insignificant
X_1X_3	0.0262	1	0.0262	4.74	0.0813	insignificant
X_2X_3	0.0436	1	0.0436	7.88	**0.0377**	**significant**
X_1^2	0.0079	1	0.0079	1.43	0.2854	insignificant
X_2^2	0.0606	1	0.0606	10.95	**0.0213**	**significant**
X_3^2	0.0673	1	0.0673	12.15	**0.0175**	**significant**
Residual	0.0277	5	0.0055			
Lack of Fit	0.0037	3	0.0012	0.1019	0.9517	
Pure Error	0.0240	2	0.0120			
Cor Total	0.5967	14				
R^2 = 0.9536, Adj. R^2 = 0.8702, Adj. R^2 = 0.8111, Adeq. precision = 10.56, C.V. %= 6,75 %						

The model's suitability of fit to the data is 0.9536, a value close to 1, indicating that the influence of the technological parameters on the response variable is well represented by the multiple regression equation. The linear component of the parameter X_1 is statistically significant (p-value = 0.0034), making it straightforward to evaluate its effect on the response variable. X_2 not only influences the response through its linear and quadratic components but also through its interaction with other parameters. The linear effect of X_2 is statistically significant (p-value = 0.0023), indicating its critical role. The nonlinear (quadratic) effect of X_2 (X_2^2) is also significant (p-value = 0.0213), suggesting that the depth of cut affects surface roughness in a nonlinear (parabolic) manner. This implies that both low and high values of X_2 may increase surface roughness, with an optimal value located somewhere in between. Furthermore, the interaction effect between X_2 and X_3 (X_2X_3) is significant (p-value = 0.0377), showing that the influence of feed rate (X_2) depends on the level of depth of cut (X_3); in other words, the effect of feed rate on surface roughness varies depending on the depth of cut. Although the linear component of X_3 is not statistically significant, X_3 still plays an important role due to its significant quadratic effect (X_3^2) (p-value = 0.0175), indicating that depth of cut affects surface roughness in a nonlinear manner. This again suggests a parabolic trend, where both low and high values of X_3 can increase roughness, with a minimum roughness at an intermediate value. Therefore, removing X_3 from the model would result in the loss of this important curvature effect. Additionally, the interaction effect between X_2 and X_3 (p-value = 0.0377) confirms that the influence of X_3 depends on the level of X_2. In other words, the effect of depth of cut on surface roughness varies depending on the feed rate - reinforcing the important role of X_3, even if its linear contribution is not statistically strong.

2.4. Particle Swarm Optimization algorithm (PSO)

PSO is a swarm optimization algorithm that initializes a population, monitors the initialization process, and updates positions. The solutions follow the best positions in the problem space and represent potential positions in the search space. Each individual updates its position based on its personal best position P_{best} and the global best position G_{best} of the population. The updates of the velocity and position of the individual are adjusted by random coefficients, helping the particles move closer to these best positions [11]. Figure 4 shows the PSO algorithm diagram. The steps for implementing the PSO algorithm are described as follows.

Step 1: Start (initialize the algorithm).

Step 2: Initialize the population

Build the population size I (number of individuals), algorithm iterations maximum number (T), search space dimensions' number.

Step 3: Calculate the objective function.

For each individual, calculate the objective function value at the current position.

Step 4: Create a search loop.

Step 5: Update velocity and position.

Each iteration, the individual updates its velocity based on the trajectory towards the best position it has ever achieved and the trajectory towards the best position the entire swarm has ever achieved. Each individual velocity is updated based on its own best position ($pBest$) and the swarm best position ($gBest$). The formula for the individual velocity i at iteration $t+1$ is presented:

$$v_i(t+1) = \omega . v_i(t) + c_1.r_1.(pBest_i - x_i(t)) + c_2.r_2.(gBest - x_i(t)) \tag{6}$$

Where, v_i^t is the velocity of individual i at time t; ω is the inertia weight; c_1 is the cognitive coefficient, c_2 is the social coefficient; r_1, r_2 are random numbers used during the velocity update process; P_{best}^t is the best personal position of individual i up to the current time; G_{best}^t is the global best position among all individuals, X_i^t is the current position of individual i at time t; v_i^{t+1} is the velocity of individual i after being updated at time $t+1$. The position of each individual is updated by adding its new velocity to the current position:

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{7}$$

Where, the index t represents the number of iterations.

Step 6: Update $pBest$ và $gBest$ best value

Compare the current position objective function value with $pBest$. If better, update $pBest$ to the current position. Compare the $pBest$ to update, if any $pBest$ is better than $gBest$, then update $gBest$.

Step 7: Check stop conditions

Step 8: Stop searching and output the results

The PSO algorithm will stop when the final result is achieved (maximum number of iterations).

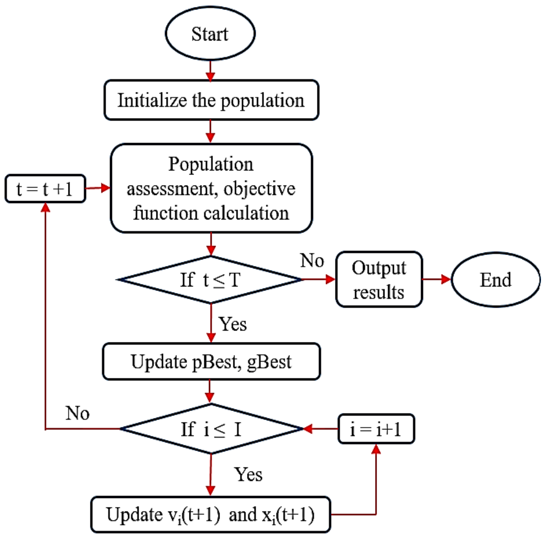

Figure 4. *PSO algorithm diagram.*

2.5. Results of surface roughness optimization

Figure 5 depicts the "Best Cost in PSO" curve over the number of iterations. It shows how the optimal cost gradually decreases and stabilizes over iterations, reflecting the process of the algorithm searching for the optimal solution in the range of 0 to 100 iterations. The objective function is a mathematical equation constructed from the Box-Behnken model and presented as

$$Ra = 0{,}557 - 6.10^{-4} n + 2{,}083.10^{-7} F - 3{,}48.10^{-9}.F.a_p + 1{,}28.10^{-9} F^2 - 1{,}5.10^{-8} a_p{}^2 \ (nm) \tag{8}$$

The limits of the technological parameters are as follows: spindle speed $1000 \le n \le 2000$ (rev/min); feed rate $5 \le F \le 25$ (mm/min); depth of cut $2 \le ap \le 8$ (µm). The Particle Swarm Optimization (PSO) algorithm is implemented in MATLAB software to optimize surface roughness. The search for optimal roughness is conducted through key parameters such as: Maximum iterations MaxIt = 100; Population size npop = 100; Inertia weight $\omega = 1$; Acceleration coefficient for individual particles $c_1 = 2$; Acceleration coefficient for the entire swarm $c_2 = 2$. When running the program, the optimal roughness achieved is 0.8 nm, obtained under the machining conditions of $n = 2000$ rpm, $F = 8$ mm/min and $ap = 4.2$ µm. Figure 6 and figure 7 illustrate that (n) significantly influences surface roughness; as (n) increases, surface roughness decreases with a substantial slope. Spindle speed is one of the crucial factors determining surface quality, but within the range, its influence is only linear. Figure 8 depicts the influence of (F) and (ap) on surface roughness as a non-linear quadratic curve exhibiting a minimum point. However, their impact is not as pronounced as that of (n), as indicated by the less steep slopes shown in the figures.

Qualitatively comparing the slopes of (F) and (a_p), the slope of (a_p) is observed to be larger, thereby suggesting a greater influence of (a_p) on surface roughness compared to (F). Some causes explaining the influence of (a_p) and (F) on surface quality are as follows: Decreasing the feed rate (F) reduces the distance between the peaks formed by the cutting tool, increases the overlapping area between two consecutive cutting paths, and thereby reduces surface waviness, leading to better surface roughness.

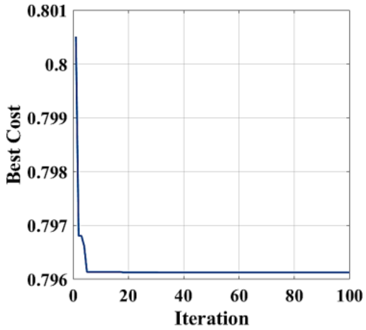

Figure 5. *Optimal surface roughness value in PSO algorithm.*

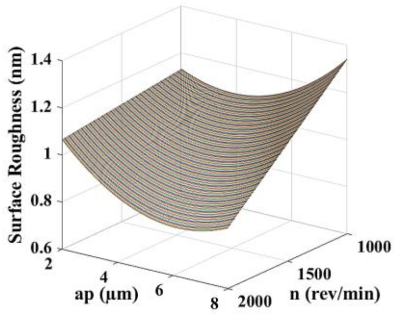

Figure 6. *The influence of* ***ap*** *and* ***n*** *on surface roughness.*

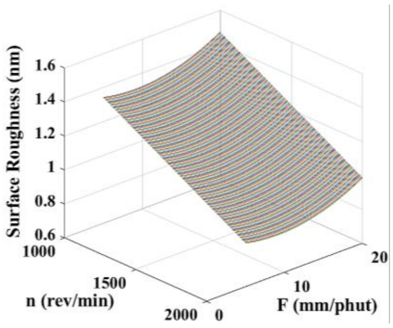

Figure 7. *The influence of* ***n*** *and* ***F*** *on surface roughness.*

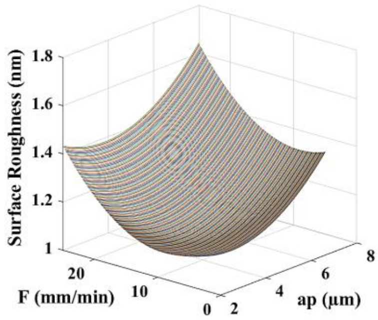

Figure 8. *The influence of* ***ap*** *and* ***F*** *on surface roughness.*

Conversely, if the feed rate is excessively low, it can cause tool wear, increase the temperature in the cutting zone, leading to thermal deformation of the surface and altered material elastic properties, potentially increasing surface roughness. When a_p is reduced, cutting force and cutting heat also decrease, leading to less tool vibration and resulting in a smoother and more uniform machined surface. However, if the depth of cut is too small, the tool may not cut into the material but only compress the surface layer or cut completely and deform the surface material layer, negatively affecting surface smoothness.

4. CONCLUSIONS

The research paper investigates UPT technology with an experimental research approach, utilizing the Box-Behnken method to design the experimental model while simultaneously employing the PSO algorithm to optimize SR during UPT of spherical aluminum surfaces. The analysis results and optimization indicate the following: The experimental model demonstrates high reliability, with the $R^2 = 0.9536$ value indicating a strong fit of the regression model to the experimental values. The achieved SR value is optimal at 0.8 nm, corresponding to the parameter set of $n = 2000$ (rpm), $F = 8$ (mm/min), and $a_p = 4.2$ (µm), indicating high effectiveness in enhancing the quality of UPT of spherical surfaces. The influence of cutting parameters was analyzed in detail: a_p and F exhibit a non-linear quadratic (parabolic) relationship with SR, while spindle speed (n) shows a linear relationship with surface roughness. The influence n has the largest impact on surface roughness, followed by a_p, and finally F. The results demonstrate that the BBD and PSO algorithms have effectively addressed the optimization problem and provided a good option for building the regression model. This paves the way for further research aimed at improving and controlling the quality of machining lens materials and lens molds, in conjunction with advanced technologies such as machine learning and artificial intelligence, to further enhance productivity and machining accuracy.

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TÓM TẮT

Nghiên cứu tối ưu hóa độ nhám bề mặt phi cầu Al khi tiện kim cương đơn điểm sử dụng mô hình thiết kế thực nghiệm Box-Behnken và thuật toán PSO

Bài báo trình bày kết quả tối ưu hóa chất lượng bề mặt phi cầu Al khi tiện kim cương một điểm (SPDT) trên cơ sở phương pháp thực nghiệm Box-Berken Design (BBD). Bộ dữ liệu thực nghiệm gồm 15 thí nghiệm được thiết lập từ mô hình BBK cùng với đó là sự hỗ trợ của module ANOVA trong phần mềm chuyên dụng DESIGN EXPERT. Hàm mục tiêu độ nhám được thiết lập, một phương trình hồi quy đa biến bậc hai xác định độ nhám của bề mặt phi cầu được hình thành trên dữ liệu thu được sau khi tiến hành đầy đủ các thí nghiệm. Hàm mục tiêu hình thành trên cơ sở mối quan hệ giữa độ nhám bề mặt phi cầu với các thông số cắt: tốc độ trục chính (n-vòng/phút); tốc độ chạy dao (F-mm/phút); chiều sâu cắt (ap-mm). Kết quả mô hình hóa với độ tin cậy cho thấy sự phù hợp cao các dữ liệu của mô hình và giá trị thực nghiệm. Thông qua việc sử dụng thuật toán tối ưu bầy đàn (PSO), giá trị tối ưu độ nhám bề mặt đạt được là 0,8 nm, đạt được với điều kiện gia công n = 2000 vòng/phút, $F = 8$ *mm/phút và* $ap = 4.2$ *μm. Nghiên cứu này có ý nghĩa quan trọng trong việc nâng cao chất lượng bề mặt quang học trong gia công siêu chính xác và tạo nền tảng tin cậy trong xây dựng mô hình thực nghiệm để tối ưu, đánh giá chính xác quá trình gia công.*

Từ khoá: SPDT; Mặt phi cầu; Nhám bề mặt; Box-Berken; PSO.