Dual-Stage Quaternion Estimator: An advanced method for orientation and angular kinematics estimation based on IMU sensor fusion

Truong Tat Thuan*, Tran Ngoc Binh

Institute of Automation, Academy of Military Science and Technology, 89B Ly Nam De, Hoan Kiem, Hanoi, Vietnam.

*Corresponding author: thuan.truongtat@gmail.com

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ABSTRACT

In modern orientation and control applications such as robots, unmanned vehicles, or pan-tilt stabilization systems, accurately estimating orientation angles, angular velocities, and angular accelerations from inertial measurement unit (IMU) data is a significant challenge due to noise, gyroscope bias, and system nonlinearity. Solutions involving Extended Kalman Filter (EKF) have shown many advantages in significantly improving "filtering" quality; however, most only focus on orientation angles and angular velocities, neglecting angular acceleration—a crucial quantity in complex motion conditions. This paper proposes an improved filter, the "Dual-Stage Quaternion Estimator" (DSQE), which uses a two-stage structure with stage 1 applying an Unscented Kalman Filter (UKF) based on quaternions and stage 2 using a linear Kalman filter to estimate angular dynamics, including angular jerk. This method improves the accuracy of orientation estimation thanks to the second-order Taylor expansion and the UKF's ability to handle nonlinearity, while also providing accurate orientation angles in complex motion conditions. Simulation results show that DSQE with UKF outperforms traditional methods, especially in scenarios with strong vibrations or high accelerations.

Keywords: Unscented Kalman filter; Quaternion; IMU; Platform stabilization; Angular velocity; Angular acceleration; Angular jerk; Sensor fusion.

1. INTRODUCTION

Currently, the use of Two-Stage Kalman Filters (2SKF) to improve orientation estimation in systems using low-cost Inertial Measurement Units (IMUs) is becoming increasingly popular and is applied in many fields: UAVs and aviation [4, 5, 9], biomedical devices [1, 2, 7, 8], as well as various types of robots and in industry [3, 6]. The core of the 2SKF structure is to separate the processing into two phases:

The first stage typically performs a rough estimation of orientation/quaternion using an Extended Kalman Filter (EKF) [1-4], or a Multiplicative Kalman Filter (MKF) [7], or a combined Kalman compensation filter [8]. The second stage focuses on correcting errors arising from noise, bias, or specific physical conditions by leveraging additional information from accelerometers, magnetometers [1], or physical models such as aerodynamics [9], mass [3], or biological body structure [7].

These filters mostly use variants of EKF, with additional specific mechanisms such as: Smooth Variable Structure Filter (SVSF) to increase stability in noisy environments [4]; mass observer combined with a quaternion filter in a vehicle with trailer systems [3]; personalization for each user through biomechanical correction [2].

Despite achieving many improvements, these methods also have some notable limitations. Some of them depend on auxiliary data beyond IMU: Methods using aerodynamic constraints [9], mass models [3], or biological information [2, 7], while helping to improve accuracy, limit their widespread applicability in dynamically changing environments. Systems using multiple processing layers, such as EKF + SVSF [4] or EKF combined with an observer [3] require high processing power, making them difficult to integrate into embedded devices or real-time applications. Although two stages help reduce short-term errors, these methods still struggle to completely eliminate long-term drift, especially when the magnetometer is noisy [1, 6]. Especially, most filters only focus on orientation angles and angular velocities, neglecting angular acceleration and angular jerk. Furthermore, the EKF filter does not guarantee accurate prediction of strongly nonlinear models. This approach simplifies the prediction model but, conversely, makes it difficult for the system to predict motions with large accelerations and jerks.

Thus, although 2SKF is a promising research direction for improving the performance of orientation systems using low-cost IMUs, to be widely applicable, it is necessary to focus on overcoming weaknesses in computational complexity, dependence on auxiliary data, drift handling, and especially the ability to apply in conditions with large accelerations and jerks. To overcome these limitations, this paper proposes an improved filter, the "Dual-Stage Quaternion Estimator" (DSQE), which uses an Unscented Kalman Filter (UKF) in stage 1 to estimate orientation and gyroscope bias, combined with stage 2 using a linear Kalman filter to estimate angular velocity, angular acceleration, and angular jerk. This method leverages the second-order Taylor expansion and the UKF's ability to handle nonlinearity to achieve higher accuracy in complex motion conditions.

2. PROBLEM

2.1. Theoretical foundations of the DSQE filter

2.1.1. Overall structure

The Dual-Stage Quaternion Estimator (DSQE) filter is designed with two main stages:

- **Stage 1 (Quaternion UKF)**: Estimates orientation in quaternion form $\mathbf{q} = [q_0, q_1, q_2, q_3]^T$ and gyroscope bias $\mathbf{b}_g = [b_x, b_y, b_z]^T$, using data from the gyroscope, accelerometer, and magnetometer, while also receiving true angular velocity $\boldsymbol{\omega}$, angular acceleration $\boldsymbol{\alpha}$ and angular jerk $\mathbf{j}$ from stage 2.
- **Stage 2 (Angular KF)**: Estimates true angular velocity $\boldsymbol{\omega}$, angular acceleration $\boldsymbol{\alpha}$, and angular jerk $\mathbf{j}$ from bias-corrected gyroscope data $\mathbf{b}_g$.

The two stages interact bi-directionally: Stage 1 provides bias $\mathbf{b}_g$ to stage 2 for data correction, while stage 2 returns $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, and $\mathbf{j}$ to improve kinematic prediction in stage 1.

2.2. Stage 1: Quaternion UKF

2.2.1. System model

Objective: Estimate quaternion orientation $\mathbf{q}$ and gyroscope bias $\mathbf{b}_g$ from the true angular velocity ω_{true}, angular acceleration α, and angular jerk $\mathbf{j}$ (provided by stage 2).

State vector:

$$\mathbf{x}_k = [q_0, q_1, q_2, q_3, b_x, b_y, b_z]^T \in \mathbb{R}^7$$

Process model:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \omega_{\text{true},k}, \alpha_k, \mathbf{j}_k) + \mathbf{w}_{p,k}$$

where $\omega_{\text{true},k} = \omega_k - \mathbf{b}_k$ is the bias-corrected angular velocity, $\mathbf{w}_{p,k}$ is the process noise with covariance $\mathbf{Q} = \text{diag}(\mathbf{Q}_q, \mathbf{Q}_b)$.

Specifically, quaternion prediction is calculated using a second-order Taylor expansion:

$$\begin{aligned} \mathbf{q}_{k+1} &= \mathbf{q}_k + \frac{T}{2}\mathbf{q}_k \otimes \omega_{\text{true},k} + \frac{T^2}{4}\mathbf{q}_k \otimes \alpha_k + \frac{T^2}{8}\mathbf{q}_k \otimes \omega_{\text{true},k}^2 + \mathbf{w}_q \\ \mathbf{b}_{k+1} &= \mathbf{b}_k + \mathbf{w}_b \end{aligned} \tag{1}$$

where $\mathbf{w}_q$ and $\mathbf{w}_b$ are the noise components for the quaternion and bias.

Matrix form of the process model:

$$\mathbf{q}_{k+1} = \left[\mathbf{I}_4 + \frac{1}{2}\mathbf{\Omega}(\omega_{\text{true},k})T + \frac{1}{8}\mathbf{\Omega}(\omega_{\text{true},k})^2T^2 + \frac{1}{4}\mathbf{\Omega}(\alpha_k)T^2\right]\mathbf{q}_k + \mathbf{w}_q \qquad (2)$$
$$\mathbf{b}_{k+1} = \mathbf{b}_k + \mathbf{w}_b$$

where:

$$\mathbf{\Omega}(\omega) = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}, \quad \mathbf{\Omega}(\alpha) = \begin{bmatrix} 0 & -\alpha_x & -\alpha_y & -\alpha_z \\ \alpha_x & 0 & \alpha_z & -\alpha_y \\ \alpha_y & -\alpha_z & 0 & \alpha_x \\ \alpha_z & \alpha_y & -\alpha_x & 0 \end{bmatrix}$$

Measurement model:

$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k \qquad (3)$$

where $\mathbf{z}_k = [a_x, a_y, a_z, m_x, m_y, m_z]^T$ is the measurement vector from the accelerometer and magnetometer, $\mathbf{v}_k$ is the measurement noise with covariance $\mathbf{R} = \text{diag}(R_a, R_a, R_a, R_m, R_m, R_m)$. The measurement function $h(\mathbf{x}_k)$ maps the state to the predicted acceleration and magnetic field:

$$\mathbf{a} = \mathbf{R}(\mathbf{q}) \cdot \mathbf{g}, \quad \mathbf{m} = \mathbf{R}(\mathbf{q}) \cdot \mathbf{m}_{\text{ref}} \qquad (4)$$

where $\mathbf{R}(\mathbf{q})$ is the rotation matrix from the quaternion, $\mathbf{g}$ is the gravity vector, and $\mathbf{m}_{\text{ref}}$ is the reference magnetic field vector.

2.2.2. UKF algorithm

The UKF operates in two steps: Prediction and update.

Prediction step:

1. Generate a set of *2n+1* sigma points (*n=7*) based on the current state $\hat{\mathbf{x}}_{k|k-1}$ and covariance $\mathbf{P}_{k|k-1}$:

$$\mathbf{x}_k^{(i)} = \hat{\mathbf{x}}_{k|k-1} + \left(\sqrt{(n+\lambda)\mathbf{P}_{k|k-1}}\right)_i, \quad \mathbf{x}_k^{(n+i)} = \hat{\mathbf{x}}_{k|k-1} - \left(\sqrt{(n+\lambda)\mathbf{P}_{k|k-1}}\right)_i \qquad (5)$$

where $\lambda = \alpha^2(n+\kappa) - n$, and weights determined by α, β, κ.

2. Propagate the sigma points through the nonlinear process function $f(\cdot)$:

$$\mathbf{x}_{k+1|k}^{(i)} = f(\mathbf{x}_k^{(i)}, \omega_{\text{true},k}, \alpha_k, \mathbf{j}_k) \qquad (6)$$

3. Calculate the predicted state and covariance:

$$\hat{\mathbf{x}}_{k+1|k} = \sum_{i=0}^{2n} W_m^{(i)} \mathbf{x}_{k+1|k}^{(i)}, \qquad (7)$$

$$\mathbf{P}_{k+1|k} = \sum_{i=0}^{2n} W_c^{(i)} (\mathbf{x}_{k+1|k}^{(i)} - \hat{\mathbf{x}}_{k+1|k})(\mathbf{x}_{k+1|k}^{(i)} - \hat{\mathbf{x}}_{k+1|k})^T + \mathbf{Q} \qquad (8)$$

Update step:

1. Propagate the sigma points through the nonlinear measurement function $h(\cdot)$:

$$\mathbf{z}_k^{(i)} = h(\mathbf{x}_{k|k-1}^{(i)}) \qquad (9)$$

2. Calculate the predicted measurement and covariance:

$$\hat{\mathbf{z}}_k = \sum_{i=0}^{2n} W_m^{(i)} \mathbf{z}_k^{(i)}, \quad \mathbf{P}_{\mathbf{zz},k} = \sum_{i=0}^{2n} W_c^{(i)} (\mathbf{z}_k^{(i)} - \hat{\mathbf{z}}_k)(\mathbf{z}_k^{(i)} - \hat{\mathbf{z}}_k)^T + \mathbf{R} \qquad (10)$$

3. Calculate the cross-covariance:

$$\mathbf{P}_{\mathbf{xz},k} = \sum_{i=0}^{2n} W_c^{(i)} (\mathbf{x}_{k|k-1}^{(i)} - \hat{\mathbf{x}}_{k|k-1})(\mathbf{z}_k^{(i)} - \hat{\mathbf{z}}_k)^T \qquad (11)$$

4. Calculate the Kalman gain:

$$\mathbf{K}_k = \mathbf{P}_{\mathbf{xz},k}\mathbf{P}_{\mathbf{zz},k}^{-1} \tag{12}$$

5. Update the state and covariance:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{z}_k - \hat{\mathbf{z}}_k), \quad \mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{P}_{\mathbf{zz},k}\mathbf{K}_k^T \tag{13}$$

6. Normalize the quaternion:

$$\mathbf{q}_{k|k} \leftarrow \frac{\mathbf{q}_{k|k}}{|\mathbf{q}_{k|k}|} \tag{14}$$

Interaction with stage 2: Stage 1 receives $\omega_{\text{true},k}$, α_k, and $\mathbf{j}_k$ from stage 2 to improve prediction, and provides $\mathbf{b}_k$ to stage 2 to correct the measurement.

2.3. Stage 2: Angular KF

2.3.1. System model

Objective: Estimate true angular velocity ω_{true}, angular acceleration α, and angular jerk $\mathbf{j}$ from gyroscope data that has been bias-corrected from stage 1.

State vector:

$$\mathbf{x}_k = [\omega_{\text{true},x}, \omega_{\text{true},y}, \omega_{\text{true},z}, \alpha_x, \alpha_y, \alpha_z, j_x, j_y, j_z]^T \in \mathbb{R}^9$$

Process model: The angular dynamics model is expressed in linear state matrix form:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \tag{15}$$

where $\mathbf{w}_{k-1} = [w_{1,k-1}, w_{2,k-1}, w_{3,k-1}]^T$ is a Gaussian process noise with expectation $E[\mathbf{w}_{k-1}] = \mathbf{0}$ and covariance $E[\mathbf{w}_{k-1}\mathbf{w}_{k-1}^T] = \mathbf{Q}_2 = \text{diag}(0.0001\mathbf{I}_{3\times3}, 0.01\mathbf{I}_{3\times3}, 0.1\mathbf{I}_{3\times3})$.

Specifically, the model is built based on the following equations:

$$\begin{gathered} \omega_{\text{true},k} = \omega_{\text{true},k-1} + \alpha_{k-1}T + \frac{1}{2}\mathbf{j}_{k-1}T^2 + \mathbf{w}_{1,k-1} \\ \alpha_k = \alpha_{k-1} + \mathbf{j}_{k-1}T + \mathbf{w}_{2,k-1} \\ \mathbf{j}_k = \mathbf{j}_{k-1} + \beta\dot{\omega}_{k-1} + \mathbf{w}_{3,k-1} \end{gathered} \tag{16}$$

where:

- T: Sampling period;
- β: Adjustment coefficient for the effect of angular acceleration on angular jerk, chosen based on the system's dynamic characteristics (usually in the range 0.1 to 1.0);
- $\dot{\omega}_{k-1} = \alpha_{k-1}$: Angular acceleration at step $k-1$, used to model the change in angular jerk;
- $\mathbf{w}_{1,k-1}, \mathbf{w}_{2,k-1}, \mathbf{w}_{3,k-1}$: Gaussian noise components corresponding to angular velocity, angular acceleration, and angular jerk.

Matrix form of the process model:

$$\mathbf{A} = \begin{bmatrix} \mathbf{I}_{3\times3} & T\mathbf{I}_{3\times3} & \frac{T^2}{2}\mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & T\mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \beta\mathbf{I}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}$$

Measurement model: The measurement is taken from the gyroscope, which has been bias-corrected from stage 1:

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \tag{17}$$

where:

- $\mathbf{z}_k = \omega_{\text{IMU},k} - \mathbf{b}_k = [\omega_{\text{IMU},x}, \omega_{\text{IMU},y}, \omega_{\text{IMU},z}]^T$: Angular velocity measured from IMU after bias correction;
- $\mathbf{H} = [\mathbf{I}_{3\times3}, \mathbf{0}_{3\times3}, \mathbf{0}_{3\times3}]$: Measurement matrix, mapping the state to angular velocity;
- $\mathbf{v}_k$: Measurement noise with covariance $\mathbf{R}_2 = \text{diag}(0.0001\mathbf{I}_{3\times3})$.

2.3.2. Kalman filter algorithm

The linear Kalman filter is applied in stage 2, with the following steps:

Prediction step:

1. Predict state:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}\hat{\mathbf{x}}_{k-1|k-1} \tag{18}$$

2. Predict covariance:

$$\mathbf{P}_{k|k-1} = \mathbf{A}\mathbf{P}_{k-1|k-1}\mathbf{A}^T + \mathbf{Q}_2 \tag{19}$$

Update step:

1. Calculate measurement error (innovation):

$$\mathbf{y}_k = \mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1} \tag{20}$$

2. Calculate measurement error covariance:

$$\mathbf{S}_k = \mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^T + \mathbf{R}_2 \tag{21}$$

3. Calculate Kalman gain:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}^T\mathbf{S}_k^{-1} \tag{22}$$

4. Update state:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{y}_k \tag{23}$$

5. Update covariance:

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H})\mathbf{P}_{k|k-1} \tag{24}$$

Interaction with stage 1: Stage 2 provides $\omega_{\text{true},k}$, α_k, and $\mathbf{j}_k$ o stage 1 to improve quaternion prediction, and receives bias $\mathbf{b}_k$ from stage 1 to correct the measurement $\mathbf{z}_k$.

***Process:**

Predict state and covariance, update with Kalman gain, and provide ω and α to stage 1.

Interaction with first-stage filter

- **Output:** The second-stage filter provides ω_k (true angular velocity) and α_k (angular acceleration) to the first-stage filter (Quaternion UKF) for more accurate quaternion prediction.
- **Input:** Receives gyroscope bias b_k from the first-stage filter to correct the measurement Z_k.

3. SIMULATION AND RESULTS

3.1. Simulation setup

- **Simulation scenarios:** 2 scenarios corresponding to medium motion and high acceleration motion with a sampling period: $T = 0.005$ s.

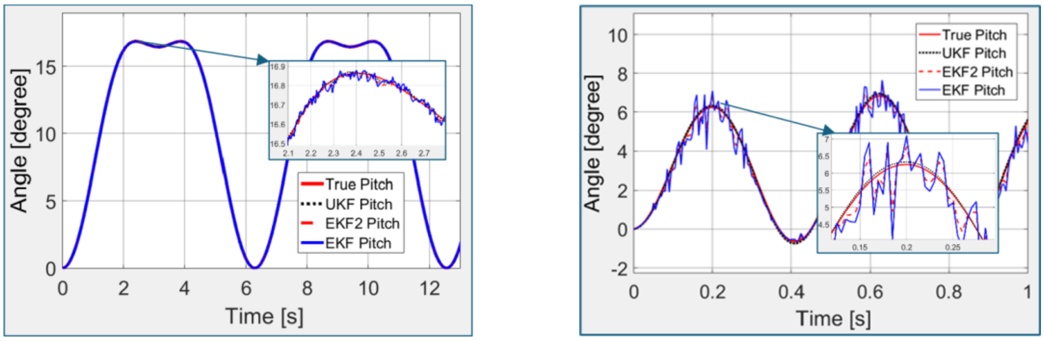
  Medium oscillation:
  - Angular velocity: $\boldsymbol{\omega}_{\mathbf{true}}(t) = [0.5\sin(t), 0.3\sin(t), 0.2\sin(t)]^T$.
  - Angular acceleration: $\boldsymbol{\alpha}_{\mathbf{true}}(t) = [0.5\cos(t), 0.3\cos(t), 0.2\cos(t)]^T$.
  - Angular jerk: $\mathbf{j}_{\text{true}}(t) = [-0.5\sin(t), -0.3\sin(t), -0.2\sin(t)]^T$.

  Strong oscillation:
  - Angular velocity: $\boldsymbol{\omega}_{\mathbf{true}}(t) = [1.5\sin(20t), 0.9\sin(15t), 0.6\sin(10t)]^T$.
  - Angular acceleration: $\boldsymbol{\alpha}_{\mathbf{true}}(t) = [30\cos(20t), 13.5\cos(15t), 6\cos(10t)]^T$.
  - Angular jerk: $\mathbf{j}_{\text{true}}(t) = [-600\sin(20t), -202.5\sin(t), -60\sin(10t)]^T$.
- **Compared Filters**:
  - Filter 1: EKF with state variable $\mathbf{x} = [\mathbf{q}, \mathbf{b}]^T$.
  - Filter 2: DSQE-EKF, with stage 1 using EKF with first-order Taylor expansion..
  - Filter 3: DSQE-UKF, with stage 1 using UKF with second-order Taylor expansion.

- **Simulated Sensor**: MPU-9250 IMU on a pan-tilt platform mounted on a mobile vehicle..
- **Noise Signals and Covariance Matrices**:
    - Process noise (**Q**) for stage 1: $Q_q = 0.0001$ (rad/s)^2, $Q_b = 0.00001$ (rad/s)^2.
    - Measurement noise (**R**) for stage 1: $R_a = 0.05$ (m/s^2)^2, $R_m = 0.1$ (μT)^2.
    - Process noise (**Q₂**) for stage 2: $\mathbf{Q}_2 = \mathrm{diag}(0.0001\mathbf{I}_{3\times3}, 0.01\mathbf{I}_{3\times3}, 0.1\mathbf{I}_{3\times3})$.
    - Measurement noise (**R₂**) for stage 2: $\mathbf{R}_2 = \mathrm{diag}(0.0001\mathbf{I}_{3\times3})$.
    - Matrix **Q** for stage 1: $\mathbf{Q} = \mathrm{diag}(Q_q\mathbf{I}_{4\times4}, Q_b\mathbf{I}_{3\times3})$.
    - Matrix **R** for stage 1: $\mathbf{R} = \mathrm{diag}(R_a, R_a, R_a, R_m, R_m, R_m)$.
- **Coefficient** β: Set $\beta = 0.5$ to simulate the effect of angular acceleration on angular jerk.

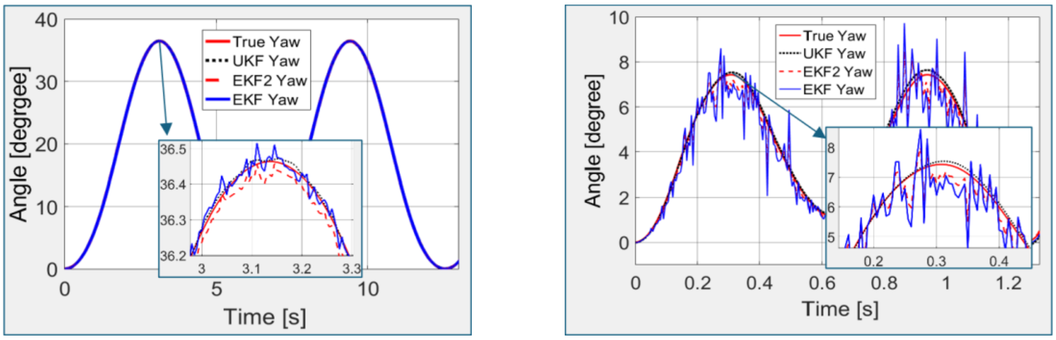
3.2. Results

Figures 1 and 2 show the estimated Pitch and Yaw angles of the 3 filters under two conditions: medium oscillation and strong oscillation. Meanwhile, figures 3 and 4 show the RMSE when estimating these angles compared to their true values.



a) Medium oscillation. *b) Strong oscillation.*

Figure 1. *Pitch angle estimation results for 3 filters.*

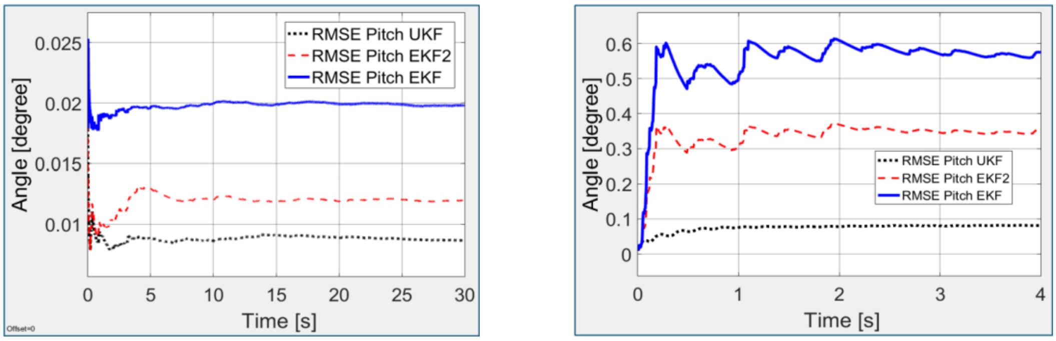


a) Medium oscillation. *b) Strong oscillation.*

Figure 2. *Yaw angle estimation results for 3 filters.*

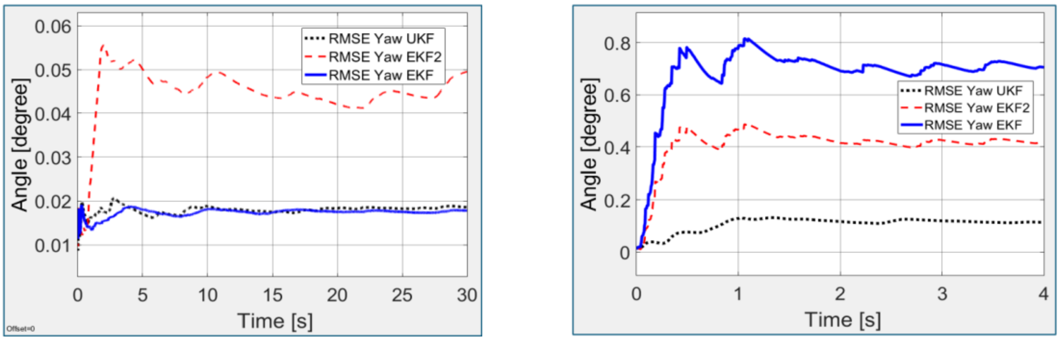
From the estimation results in figures 1 and 2, it can be seen that under normal conditions, the performance of all three filters is very good, with negligible differences in both Pitch and Yaw angles, and the effectiveness of using DSQE UKF is slightly better than the other two filters, but the difference is not significant. However, under strong oscillation conditions, it can be seen that only DSQE UKF demonstrates good estimation capability, while the noise from the two DSQE EKF and conventional EKF filters is very large.

Figures 3 and 4 clearly show the difference with specific data, and the summarized results are presented in table 1. While under normal conditions, the RMSE of Yaw and Pitch angles for DSQE UKF is not superior to the other two filters (especially the Yaw angle RMSE is even larger than that of conventional EKF). In the case of strong oscillation, the application of DSQE shows a significant difference, with RMSE significantly reduced compared to the other two filters.



a) *Medium oscillation.* b) *Strong oscillation.*

Figure 3. *Pitch angle RMSE for 3 filters.*



a) *Medium oscillation.* b) *Strong oscillation.*

Figure 4. *Yaw angle RMSE for 3 filters.*

Table 1. *RMSE of 3 filters under normal and strong oscillation conditions.*

	Normal condition		**Strong oscillation**	
	RMSE Pitch [degree]	**RMSE Yaw [degree]**	**RMSE Pitch [degree]**	**RMSE Yaw [degree]**
DSQE UKF	**0.008**	**0.018**	**0.08**	**0.12**
DSQE EKF	0.012	0.05	0.35	0.42
EKF	0.02	0.017	0.57	0.7
Oei [3]	0.132*	N/A	0.132*	N/A
Sabatelli [6]	0.19*	0.19*	0.19*	0.19*
Eliahu [7]	0.395*	N/A	0.395*	N/A

Furthermore, table 1 also compares with data from studies [3, 6, 7] (where the asterisk indicates that the application conditions for this data are unclear, especially for study [6] where the authors did not clearly distinguish orientation angles). The results demonstrate the superiority of DSQE UKF compared to previous studies; even under strong oscillation conditions, the best value from Oei [3], with an RMSE of 0.132 degrees, is still larger than the Yaw angle RMSE of the proposed method (0.12 degrees).

Thus, the application of the DSQE UKF filter has demonstrated superior performance in estimating Yaw and Pitch angles under strong oscillation conditions. This is a crucial feature, especially for two-axis pan-tilt systems, which is highly necessary when they are mounted on mobile vehicles where real-world movements can involve high accelerations and jerks.

4. CONCLUSIONS

The Dual-Stage Quaternion Estimator filter with UKF in stage 1 and a linear Kalman filter in stage 2, integrating a new angular jerk model, is an advanced solution for IMU sensor fusion. This method provides an accurate estimation of orientation angles, especially in applications with harsh operating conditions. The use of UKF effectively handles nonlinearities, combined with a second-order Taylor expansion and a bi-directional interaction mechanism, delivering superior performance. The new angular jerk model ($\mathbf{j}_k = \mathbf{j}_{k-1} + \beta\dot{\omega}_{k-1} + \mathbf{w}_{3,k-1}$) enhances the ability to describe complex dynamics, improving accuracy in strong oscillation scenarios. Future research can focus on optimizing computational resources for deploying DSQE-UKF on real-time embedded systems and adjusting the coefficient β to suit specific applications.

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TÓM TẮT

Bộ lọc Dual-Stage Quaternion Estimator: Một phương pháp ước lượng định hướng và động học góc tiên tiến dựa trên tổng hợp dữ liệu IMU

Trong các ứng dụng định hướng và điều khiển hiện đại như robot, phương tiện không người lái, hoặc hệ thống ổn định pan-tilt, việc ước lượng chính xác các góc định hướng, vận tốc góc và gia tốc góc từ dữ liệu cảm biến quán tính (IMU) là một thách thức lớn do nhiễu, sai lệch con quay hồi chuyển và tính phi tuyến của hệ thống. Các giải pháp xây dựng các bộ lọc EKF đã thể hiện được nhiều ưu điểm khi cải thiện đáng kể chất lượng "lọc", tuy nhiên, hầu hết chỉ tập trung vào các góc định hướng và vận tốc góc, bỏ qua gia tốc góc – một đại lượng quan trọng trong các điều kiện chuyển động phức tạp. Bài báo này đề xuất một bộ lọc cải tiến "Dual-Stage Quaternion Estimator" (DSQE), sử dụng cấu trúc hai giai đoạn với giai đoạn 1 áp dụng Unscented Kalman Filter (UKF) dựa trên quaternion và giai đoạn 2 sử dụng bộ lọc Kalman tuyến tính để ước lượng động học góc, bao gồm cả độ giật góc. Phương pháp này cải thiện độ chính xác trong ước lượng định hướng nhờ khai triển Taylor bậc hai và khả năng xử lý phi tuyến của UKF, đồng thời cung cấp các góc định hướng chính xác trong các điều kiện chuyển động phức tạp. Kết quả mô phỏng cho thấy DSQE với UKF vượt trội so với các phương pháp truyền thống, đặc biệt trong các kịch bản rung lắc mạnh hoặc gia tốc lớn.

Từ khoá: Bộ lọc Kalman; Quaternion; IMU; Ổn định bệ; Vận tốc góc; Gia tốc góc; Sensor fusion.