Research on methods for measuring and controlling the modulation index of a frequency-modulated interferometer for high-precision displacement measurement

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ABSTRACT

This paper presents the theoretical foundation combined with experimental implementation to develop an algorithm for controlling and measuring the modulation index (m) of a Frequency-Modulated Interferometer (FMI) used for high-precision displacement measurements. In FMI-based displacement measurement systems, the modulation index m and synchronous demodulators (Lock-in Amplifiers – LIAs) are two critical factors that distinguish this method from traditional interferometric techniques. While the modulation index m determines the measurement range limit, the LIAs ensure high accuracy within that range. In FMI systems, the modulation index is embedded in all harmonic components. When measuring over a large range, the harmonic amplitudes vary, causing m to change with displacement, which reduces the signal-to-noise ratio (SNR) of the interferometric signal pairs and consequently degrades measurement accuracy. A method for controlling the value of m is proposed based on the relationship between the initial measurement range, m, and Bessel functions. Accordingly, the quantitative value of m can also be accurately measured based on the intensity ratios of consecutive even and odd interferometric signal pairs. An experimental verification of the method was conducted with an initial measurement range of 25 cm. The modulation index m was successfully controlled and measured as m = 2.575, then adjusted to reach the special value m = 2.631.

Keywords: FMI; Modulation index; LIAs; Measurement range.

1. INTRODUCTION

Single-frequency and dual-frequency interferometers can only measure displacement, not absolute distance, which limits their measurement range [1-5]. In contrast, the FMI offers a superior displacement measurement range by combining absolute distance measurement with high-resolution displacement detection - even in complex measurement environments - while still maintaining accuracy comparable to that of traditional interferometers [6-10].

In FMI systems, the modulation index (*m*) is embedded in all harmonic components. When the measurement range changes, the intensity of the harmonic signals also changes, leading to a reduction in the signal-to-noise ratio (SNR) and, consequently, a decrease in measurement accuracy. To maintain high accuracy, modulation index *m* must be precisely determined and controlled throughout the measurement process.

Studies [9, 10] have proposed solutions for measuring the dynamic modulation index *m* using the intensity of the *i*th-order harmonic component in the noise signal spectrum. Sudarshanam et al. suggested estimating the value of *m* through the power spectrum of the interferometric signal.

$$m^2 = \frac{4i(i+1)V_i V_{i+1}}{(V_i + V_{i+2})(V_{i-1} + V_{i+1})} \tag{1}$$

where $i > 1$ is an integer representing the order of the harmonic component, and the intensity refers to that of the ith harmonic.

This method is simple and easy to implement using commonly available spectrum analyzers, and it does not require synchronous or phase-sensitive detection circuits.

The limitation of this method is that $V_i > 0$ (always positive), as spectrum analyzers only display the absolute magnitude of spectral components. Therefore, the sign of the Bessel function cannot be determined (since *m* is embedded in all harmonic components of the Bessel function), which restricts the measurable range of *m*. Additionally, the accuracy is limited by the 1/f noise voltage of the spectrum analyzer [10] - a common type of noise at low frequencies - which can obscure low-amplitude spectral components, reducing measurement accuracy under low SNR conditions. As a result, this method is not well-suited for measurement systems requiring high resolution or precise tracking of small changes in *m* or PZT (Piezoelectric Transducer) displacement.

The above limitations are improved in studies [8, 9], which propose determining the value of modulation index *m* based on the Bessel function. However, *m* is only qualitatively estimated by observing the shape of the Lissajous figure and adjusting the input parameters to obtain a clear Lissajous pattern (circular or elliptical). It should be noted that *m* typically varies only slightly (especially in FMI systems using a Piezoelectric Transducer – PZT oscillator), so the *m* range that produces a clear Lissajous figure is quite broad. Achieving high accuracy at specific critical *m* values (m =2.63; 3.768...) derived from the Bessel function remains a significant challenge.

To thoroughly address this issue, the value of *m* must be precisely controlled and quantitatively determined in real time. In that case, the accuracy of displacement measurements using FMI can be significantly improved if *m* is well-controlled to reach the special values of the Bessel function.

2. THEORETICAL FOUNDATION FOR THE CONTROL AND MEASUREMENT METHOD OF THE MODULATION INDEX IN FMI

2.1. Principle of high-precision displacement measurement using FMI

The schematic principle is based on the Michelson interferometer diagram:

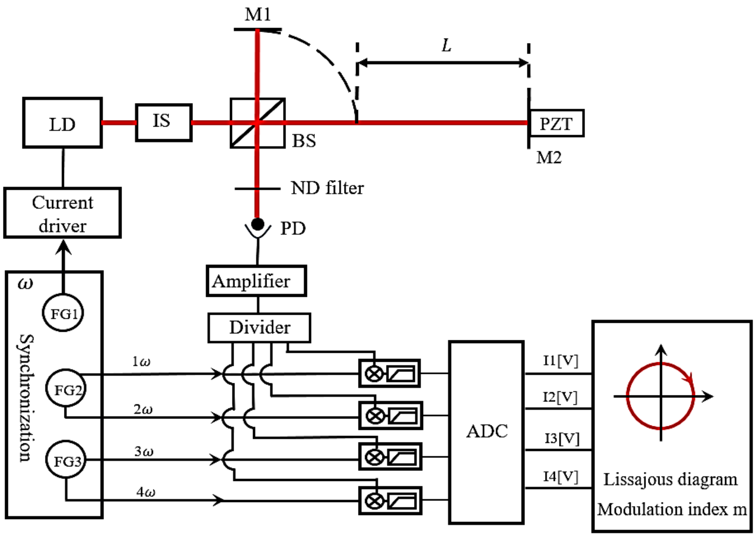


Figure 1. *Schematic diagram of the displacement measurement principle of the FMI. PD: Detector; ND filter: Neutral density filter; BS: Beam splitter; LD: Laser diode; IS: Isolator; M1: Reference mirror; M2: Measurement mirror.*

- The obtained interferometric signal intensity of FMI in previous studies [7-10]:

$$I(\Delta L,t)=I_0\left\{1+V\left\{\begin{array}{l}\cos\left(\varphi_0+\dfrac{4\pi nf_0\Delta L}{c}\right)\times\left[J_0(m)+2\sum_{k=1}^{\infty}J_{2k}(m)\times\cos(2k\times 2\pi f_m t)\right]\\-\sin\left(\varphi_0+\dfrac{4\pi nf_0\Delta L}{c}\right)\times 2\sum_{k=1}^{\infty}J_{2k-1}(m)\times\sin\left[(2k-1)\times 2\pi f_m t\right]\end{array}\right\}\right\} \quad (2)$$

- The modulation index (m) is determined:

$$m=\frac{2\pi\Delta fL}{c} \quad (3)$$

In which: V, Δf, f_m, L, and J_n are the interferometric signal contrast, modulation depth, modulation frequency, measurement range, and the Bessel function solution, respectively.

The concept of measurement range in FMI is understood as the total initial optical path length between the two interferometer arms plus the displacement during the measurement process, expressed as:

$$L=L_0+\Delta L \quad (4)$$

In which: L_0, ΔL represent the measurement range, the initial distance between the two interferometer arms (which does not change over time), and the displacement of the measured object.

The interferometric signal obtained from equation 2 is processed using synchronous demodulation techniques (LIAs) to extract pairs of even and odd harmonic components with quadrature phase. Assume the harmonic signal pair contains the components $J_{2k-1}(m)$ and $J_{2k}(m)$ with reference signals at frequencies $(2k-1)\omega_m$ and $2k\omega_m$, respectively. The intensity signal pair contains the desired even-odd harmonic components:

$$I_{2k-1}=-VI_0J_{2k-1}(m)\sin\left(\frac{4\pi nf_0\Delta L}{c}\right) \quad (5)$$

$$I_{2k}=VI_0J_{2k}(m)\cos\left(\frac{4\pi nf_0\Delta L}{c}\right) \quad (6)$$

The Lissajous figure is constructed from Equations (5) and (6) to determine the phase shift and direction of the measured object. The displacement ΔL is calculated as:

$$\Delta L=\frac{c}{4\pi nf_0}\left\{arctan\left\{-\frac{J_{2k}(m)I_{2k-1}}{J_{2k-1}(m)I_{2k}}\right\}\right\} \quad (7)$$

In equation (7), ΔL depends on the intensities of the harmonic components ($2k$-1), $2k$, and the two Bessel function parameters $J_{2k}(m)$, $J_{2k-1}(m)$. When m is well controlled to reach the special points $\left[J_{n-1}(m)=J_n(m)\right]$ shown in figure 3, ΔL depends only on the interferometric signal intensity, allowing the measurement to achieve high accuracy.

$$\Delta L=\frac{c}{4\pi nf_0}\left\{arctan\left\{-\frac{I_{2k-1}}{I_{2k}}\right\}\right\} \quad (8)$$

2.2. Developing a control method and measurement algorithm for the modulation index (*m*)

2.2.1. Modulation index control method

To ensure high accuracy of the FMI across different measurement ranges, the value of m needs to be properly controlled to achieve a high SNR of the harmonic pairs and to accurately calculate displacement. The steps are as follows:

Step one: The static value of m is determined from the Bessel function and through its relationship with the optical path difference L_0 between the two interferometer arms. At this step, the *m* range with high SNR is achieved, corresponding to the modulation depth range obtained by adjusting the modulation amplitude. A fixed value is calculated to achieve the desired *m* according to equation (3). This amplitude is then fixed to maintain a stable *m*.

Step two: The dynamic value of *m* changes with the displacement (ΔL), with three cases:

(i) In the case where $\Delta L \ll L_0$, commonly in FMI systems using high-precision PZT oscillators, this change is negligible, and the LIAs still achieve a good SNR ratio.

(ii) For displacements (ΔL) significantly large compared to L_0 (commonly applied in measurements on actual sliding grooves), the dynamic value of *m* at step two changes considerably with ΔL, which can reduce the SNR of the harmonic components. It is necessary to adjust the modulation amplitude (Δf) to achieve an *m* value with a high SNR ratio.

(iii) In some applications where $L_0 = 0$ (figure 2), equation (3) becomes:

$$m = \frac{2\pi \Delta f \Delta L}{c} \tag{9}$$

Accordingly, the amplitude is controlled to obtain the range of m values from the Bessel function. To ensure high accuracy, m must reach specific $\left(J_n(m) = J_{n-1}(m)\right)$. At the first special point $J_1(m) = J_2(m)$ when $m \cong 2.63$, as the displacement increases, m increases according to equation (9) until it reaches the next special point (m = 3.769), and so on for the special points within the chosen m range. In the case of large displacements, outside the m range, the interference signal may be lost; therefore, identifying the special points of m plays an important role, acting as markers on the scale to ensure a continuous signal and extend the displacement range.

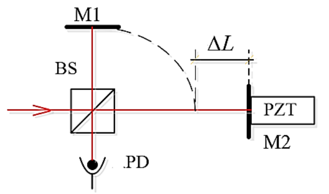


Figure 2. *Displacement measurement range with* $L_0 = 0$.

2.2.2. Developing a measurement program for the modulation index (m)

The value of *m* is determined by the signal processor based on the Bessel function values as follows:

$$m = \frac{2nJ_k(m)}{J_{k-1}(m) + J_{k+1}(m)} \tag{10}$$

Simultaneously using 4 LIAs to acquire and process the intensity signals of 4 harmonic components, from I_1 to I_4, which are expanded from equations (5) and (6) as follows:

$$I_1 = I_0 V J_1(m) \sin\left(\frac{4\pi n}{\lambda_0}\Delta L\right);\ I_2 = I_0 V J_2(m) \cos\left(\frac{4\pi n}{\lambda_0}\Delta L\right);$$

$$I_3 = I_0 V J_3(m) \sin\left(\frac{4\pi n}{\lambda_0}\Delta L\right); I_4 = I_0 V J_4(m) \cos\left(\frac{4\pi n}{\lambda_0}\Delta L\right).$$

It can be inferred that:

$$\frac{I_1(m)}{I_3(m)} = \frac{J_1(m)}{J_3(m)}; \frac{I_2(m)}{I_4(m)} = \frac{J_2(m)}{J_4(m)} \tag{11}$$

From this ratio pair, it is entirely possible to accurately determine *m* according to the Bessel function graph, figure 3. Accordingly, the steps to measure the value of *m* are as follows:

Four LIAs simultaneously acquire and process the signals of four consecutive harmonic components with high SNR, as shown in figure 1.

After acquiring the intensity ratios of consecutive even-odd harmonic signal pairs according to equation (11), the data is processed using a MATLAB program (see appendix) to reconstruct the Bessel function corresponding to the accurately quantified value of *m*.

From this value, input parameters can be adjusted to obtain the intensity ratio corresponding to the desired *m*, thereby improving the accuracy of displacement measurements.

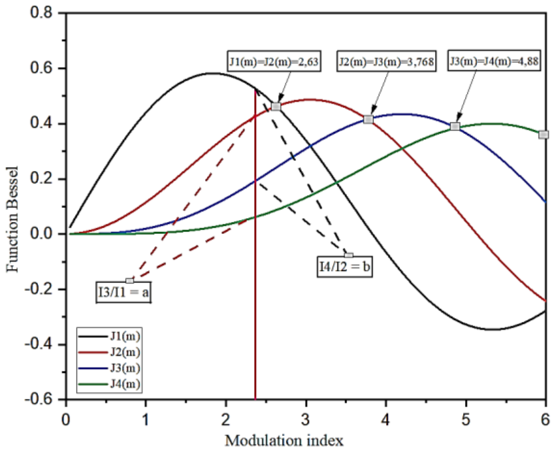


Figure 3. *Determining the value of m from the ratio of interferometric signal intensity pairs.*

3. EXPERIMENT AND RESULTS

3.1. Developing the experimental setup

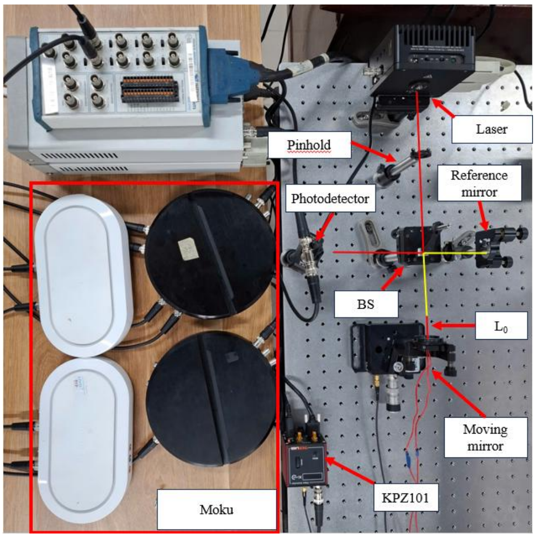


Figure 4. *Experimental model for displacement measurement using FMI.*
BS: Beam splitter; KPZ: Voltage controller.

The experimental setup (figure 4) was assembled with an initial measurement range $L_0 = 25$ cm. A laser diode (LD) (model LDM56/M, Thorlabs Inc.) was stabilized using the TED200C and LDC200C controllers to regulate temperature and current. The LD frequency was directly modulated by adjusting the pump current. To generate displacement for the measurement mirror, a PZT actuator (model NFL5D20P/M, Thorlabs) with a resolution of 0.6 nm was used, driven by a sinusoidal input signal to produce harmonic oscillations. The PZT was controlled by a compatible driver (KPZ101, Thorlabs Inc.) that generates sinusoidal pulse signals to create oscillations.

The interferometric signal was detected by a photodetector (PDA36A-EC, Thorlabs Inc.) and sent to a data acquisition system (Moku:Lab and Moku:Go, Liquid Instruments Inc.). This system integrates over 12 devices into one platform, including function generators, LIAs, digital filters, and a spectrum analyzer. The output signals from the LIAs, containing the amplitudes of harmonic signals, were digitized using an ADC (PCI 6259, NI Inc.).

According to the relationship in equation (3), the m range (approximately 1.5 to 7 radians) is identified to have a high SNR for the harmonic signals, corresponding to a range of modulation depths (Δf). A desired value of $m \approx 2.63$ corresponds to a modulation depth of (Δf = 500 MHz) (see figure 5). By adjusting and fixing the pump current amplitude to this value, m can be kept stable during the measurement process.

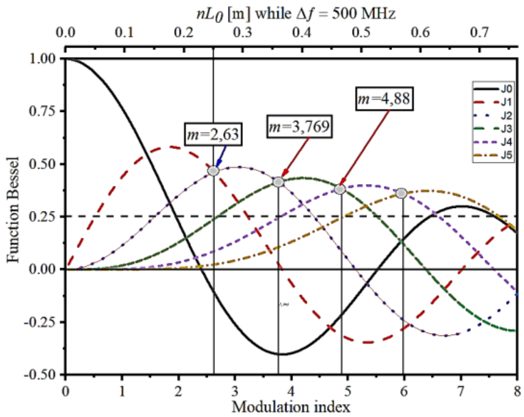
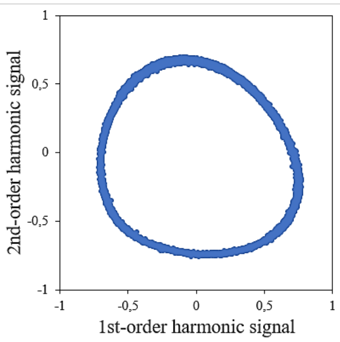


Figure 5. *The value m = 2.63 is stabilized with an initial measurement range* (nL_0).

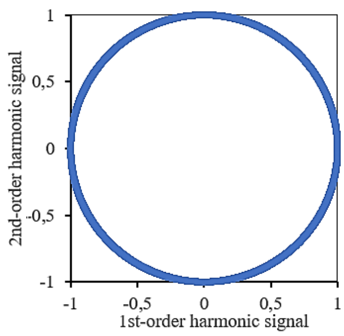
Table 1. *Experimental conditions for FMI displacement measurement with* L_0 *= 25 cm.*

Modulation frequency of the laser diode	20 kHz
Modulation depth (Δf) of the laser	500 MHz
Unbalanced length, L_0	25 cm
Laser wavelength	633 nm
Cut-off frequency of the low-pass filter	50 Hz
PZT control frequency	400 mHz
PZT control amplitude	1,5 µm

The LIAs acquired four consecutive intensity values with ratios of $I_3/I_1 = 0{,}5215$ and $I_4/I_2 = 0{,}0356$. These values were processed in MATLAB to obtain the modulation index m (figure 7a) and the corresponding Lissajous figure (figure 6a).



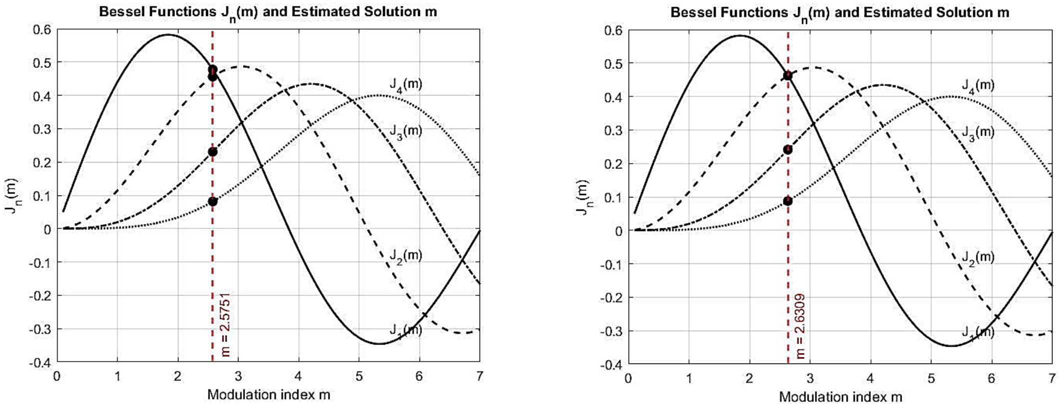
a. Lissajous figure with m = 2.575.



b. Lissajous figure with m = 2.631.

Figure 6. *Lissajous figure generated from two pairs of 1st- and 2nd-order harmonic signals.*

To achieve the desired special value of the modulation index m, the pump current amplitude of the laser is adjusted to obtain the corresponding modulation depth, or alternatively, the optical path difference L_0 can be varied. In this case, the author keeps L_0 fixed and adjusted the pump amplitude. With a laser pump current tuning sensitivity of approximately 300 MHz/mA, a modulation depth of $(\Delta f = 502{,}3MHz)$ is achieved, resulting in intensity ratios of $I_3/I_1 = 0{,}5218$ and $I_4/I_2 = 0{,}1887$. The corresponding modulation index is measured as $m \approx 2.631$ (figure 7b), with the associated Lissajous figure shown in figure 6b.



a) Before adjustment, the measured modulation index was m = 2.575.

b) After adjustment, the measured modulation index was m ≈ 2.631.

Figure 7. *The value of m measured from the intensity ratio of the interference signals was calculated using a program developed in MATLAB.*

The measurement results show that the value of m can be accurately determined from the measured intensity ratios, which play an important role in improving the displacement measurement accuracy using the FMI.

4. CONCLUSIONS

The paper successfully presents a method to control and develop an algorithm for measuring the modulation index m, combining both theoretical foundations and high-precision experimental verification. The measured m values are compared with Lissajous figures to evaluate both qualitative and quantitative aspects. However, the research has so far only succeeded in an experimental model to assess the feasibility of the theory with very small displacements ΔL generated by the PZT oscillator. In real-world environments, larger ΔL, vibrations, and significant sliding deviations reduce the SNR considerably, lowering accuracy, possibly causing signal loss, and posing many challenges that need to be addressed. This also lays the groundwork for deeper scientific research aimed at mastering high-precision displacement measurement using FMI in the future.

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TÓM TẮT

Nghiên cứu phương pháp kiểm soát và đo chỉ số điều biến của giao thoa kế điều biến tần số để đo dịch chuyển với độ chính xác cao

Bài báo trình bày cơ sở lý thuyết kết hợp thực nghiệm, xây dựng thuật toán để kiểm soát và đo chỉ số điều biến (m) của giao thoa kế điều biến tần số (FMI) đo dịch chuyển với độ chính xác cao. Trong hệ FMI đo dịch chuyển, giá trị m và bộ trích xuất đồng bộ (LIAs) là hai yếu tố quan trọng tạo điểm khác biệt so với các phương pháp giao thoa truyền thống. Trong khi giá trị m quyết định giới hạn phạm vi đo thì bộ LIAs đảm bảo độ chính xác cao cho phạm vi đó. Trong FMI, chỉ số điều biến nhúng trong tất cả các hàm điều hòa, khi phạm vi đo lớn, cường độ hàm điều hòa thay đổi, m thay đổi theo độ dịch chuyển làm giảm tỷ lệ tín/tạp (SNR) của các cặp tín hiệu giao thoa, làm giảm độ chính xác. Một phương pháp kiểm soát giá trị m thông qua mối quan hệ giữa khoảng đo ban đầu (L_0)*, m và hàm Bessel được đề xuất. Theo đó, giá trị định lượng của m cũng được đo chính xác dựa trên tỷ lệ cường độ các cặp tín hiệu giao thoa chẵn, lẻ liên tiếp. Thực nghiệm kiểm chứng phương pháp với khoảng đo ban đầu 25 cm, giá trị m được kiểm soát và đo được m=2,575, hiệu chỉnh đạt điểm đặc biệt* $m \approx 2{,}631$ *của hàm Bessel.*

Từ khoá: FMI; Chỉ số điều biến m; LIAs; Phạm vi đo.