A study on control command synthesis algorithm for single-channel flying equipment with linear steering vane

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ABSTRACT

This paper presents the algorithm and mathematical foundation for control command synthesis applied to single-channel flying equipment equipped with a linear steering vane. The proposed algorithm is developed based on optimal control principles, with a cost function aiming to minimize control energy. From the resulting control law, a relationship is established between the control signal and the normal force acting on the flying equipment. Simulation results are conducted to demonstrate the superiority of the proposed control law over relay-based control, particularly in applications involving single-channel flying equipment.

Keywords: Single-channel flying equipment; Command control; Linear steering machine; Algorithm.

1. INTRODUCTION

The controlled flight of flying equipment (FE) in space is inherently complex, particularly for single-channel flying equipment (SCFE) such as the Igla and Igla-S. Subjected to continuous control inputs, the FE remains in a dynamic state, simultaneously experiencing overloads along all axes, while the target of control is a randomly maneuvering object. Controlling an FE through a single channel (using a pair of rudders) represents a special case of polar-coordinate-based control. In this method, the FE is guided using a pair of rudders while being forcibly spun around its longitudinal axis at a predetermined angular velocity. When this angular velocity becomes excessively high, there is a risk of target lock loss. To prevent this, it is necessary to rapidly reduce the angular rate of the LOS vector by applying a control strategy based on the principle of rapid response. This control principle has been implemented in SCFE of earlier generations that use relay-steering machines, such as the A72, A87, and Igla systems. Several foreign studies [1] have briefly mentioned this principle in the context of designing and developing flying vehicles. However, they do not provide a detailed analysis or a rigorous mathematical justification of the control principle as applied to the aforementioned type of flying equipment. Study [8] develops an autopilot structure built in the body-fixed coordinate system, implementing a serial compensation method to eliminate the cross-coupling effects between the pitch and yaw channels. Study [9, 10], designed a digital controller for a high-spinning rolling flying equipment around its longitudinal axis. In contrast, domestic studies [2-4] have conducted in-depth analyses, clarifying the mathematical foundation of the rapid-response control principle for SCFE equipped with a relay-steering machine. These studies are based on Pontryagin's optimal control theory, employing a cost function that minimizes control time. However, for SCFE, existing research has yet to clearly address the differences in control laws when using a linear-steering machine compared to a relay-steering machine, especially in the context of adaptive command control system synthesis. In addition, study [3] proposed a sinusoidal harmonic control signal form for SCFE equipped with a linear steering machine and demonstrated that the average control force is proportional to the control signal. However, the mathematical foundation and detailed proof of this proposition have not yet been fully clarified. Study [5] introduced a harmonic-function-based control law in which the range and directional channels of a two-channel flying equipment are controlled independently based on the principle of optimal normal force. The available published materials, including monographs, international scientific journals, and technical documentation, have yet to clearly and comprehensively present the control principles and mathematical foundations of systems employing linear steering machines in SCFE.

In this paper, the authors present the mathematical foundation for synthesizing control commands for SCFE equipped with a linear steering machine. An optimal control approach is employed to derive a control law tailored to the characteristics of the system, while also determination of normal force required for flight control. Additionally, the paper conducts simulation experiments within a closed-loop control structure utilizing a linear steering machine to validate the effectiveness of the proposed control algorithm.

2. A CONTROL COMMAND SYNTHESIS ALGORITHM BASED ON MATHEMATICAL FOUNDATIONS FOR SINGLE-CHANNEL FLYING EQUIPMENT USING LINEAR STEERING MACHINES

Single-channel flying equipment (SCFE) represents a special case within the broader class of conventionally controlled flying equipment. Accordingly, their state equations are formulated entirely based on flight dynamics theory, taking into account the specific characteristics of this flying equipment type and incorporating gyroscopic effects into the control equations around the center of mass.

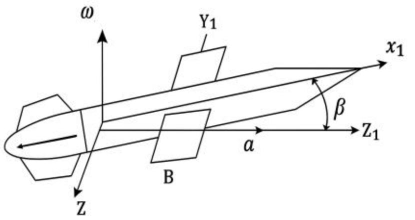

Figure 1. *Single-channel control flying equipment type.*

Consider the equations of motion around the center of mass of the flying equipment:

$$\dot{\omega}_{x1} + a_{x1}^{\omega_{x1}}.\omega_{x1} = -a_{xe}.\delta_e \tag{1}$$

$$\dot{\omega}_{y1} + a_{y1}^{\omega_{y1}}.\omega_{y1} + a_{x1}^{z1}.\omega_{x1}.\omega_{z1} + a_{y1}^{\beta}.\beta + a_{y1}^{\dot{\beta}}.\dot{\beta} = a_{yH}.\delta_H \tag{2}$$

$$\dot{\omega}_{z1} + a_{z1}^{\omega_{z1}}.\omega_{z1} - a_{x1}^{y1}.\omega_{x1}.\omega_{y1} + a_{z1}^{\alpha}.\alpha + a_{z1}.\dot{\alpha} = a_{zB}.\delta_B \tag{3}$$

In there: Equations (1), (2), and (3) correspond to the control equations for the rotation of the flying equipment about the axes 0x1, 0y1, 0z1.

Due to the specific nature of the control method, the aerodynamic configuration of this type of flying equipment exhibits the following distinctive characteristics: It has a length much greater than its body diameter and exhibits a high degree of symmetry with respect to all three axes 0x, 0y and 0z. The rotating fins are rigidly fixed to the body and positioned at the tail section of the flying equipment (FE) to maintain a constant $\omega_{x1} = \omega_0$ rotational speed around the longitudinal axis. Only a single pair of control fins is used for guiding the flying equipment, arranged in a "duck" aerodynamic configuration, in which one of the two rudders can be considered to have zero deflection angle ($\delta_H = 0$). The controlled flight duration is short; both the spin rate around the longitudinal axis and the flight speed during the target-tracking phase are maintained constant. The linear steering machine ensures that the output is proportional to the input signal.

The objective is to formulate and solve the problem of determining the control law for a Single-channel flying equipment (SCFE) equipped with a linear steering machine.

In there:

$$a_{x1}^{\omega_{x1}}\omega_{x1} + a_{xe}\delta_e = 0 \Rightarrow \delta_e = -\frac{a_{x1}^{\omega_{x1}}}{a_{xe}}\omega_{x1} \Rightarrow \omega_{x1} = \omega_0 = k_{\delta_e}.\delta_e \quad (4)$$

$$a_{y_1}^{\omega_{y1}} = a_{z_1}^{\omega_{z1}} = a_1 \; ; \; a_{x_1}^{z_1} = a_{x_1}^{y_1} = a_2 \; ; \; a_{y1}^{\beta} = a_{z1}^{\alpha} = a_3 \; ; \; a_{y_1}^{\dot{\beta}} = a_{z_1}^{\dot{\alpha}} = a_4 \; ; \; a_{yH} = a_{zB} = a_{BH} \quad (5)$$

Since $\omega_{x1} = \omega_0 = const$, the coefficients in expressions (2) and (3) are therefore:

$$a_{x_1}^{z_1}\omega_{x1} = a_{x_1}^{y_1}\omega_{x1} = a_2\omega_{x1} = const \quad (6)$$

Expressions (1), (2), and (3) represent the control equations for the SCFE type. Under the given assumption, the focus is placed on equations (2) and (3), which can be rewritten in the following form:

$$\dot{\omega}_{y1} + a_1.\omega_{y1} + a_2.\omega_{x1}.\omega_{z1} + a_3.\beta + a_4.\dot{\beta} = 0 \quad (7)$$

$$\dot{\omega}_{z1} + a_1.\omega_{z1} - a_2.\omega_{x1}.\omega_{y1} + a_3.\alpha + a_4.\dot{\alpha} = a_{BH}.\delta_B \quad (8)$$

When the angles of attack and sideslip α , β are small, as in the case of SCFE, like the Igla-S, the relationship between the angular velocity of the velocity vector Ω and the angular velocity of the flying equipment around its center of mass ω is given by:

$$\omega_{y1} = \Omega_y + \dot{\beta} \quad (9)$$

$$\omega_{z1} = \Omega_z + \dot{\alpha} \quad (10)$$

Since the TBB maintains a sufficiently high and constant flight speed ($V = const$) and possesses a highly symmetrical aerodynamic shape, the following is derived based on flight theory:

$$\Omega_z = A\alpha \quad (11)$$

$$\Omega_y = A\beta \quad (12)$$

Assume that: $A_\alpha = A_\beta = A; \frac{g_y}{V} = \frac{g_z}{V} = 0$

By combining equations (9) through (12), we obtain:

$$\omega_{y1} = A\beta + \dot{\beta} \quad (13)$$

$$\omega_{z1} = A\alpha + \dot{\alpha} \quad (14)$$

Taking the derivatives of equations (13) and (14):

$$\dot{\omega}_{y1} = A\dot{\beta} + \ddot{\beta} \quad (15)$$

$$\dot{\omega}_{z1} = A\dot{\alpha} + \ddot{\alpha} \quad (16)$$

Substituting equations (13), (14), (15), and (16) into equations (7) and (8), we obtain:

$$\ddot{\beta} + A\dot{\beta} + a_1(A\beta + \dot{\beta}) + a_2\omega_{x1}(A\alpha + \dot{\alpha}) + a_3\beta + a_4\dot{\beta} = 0 \quad (17)$$

$$\ddot{\alpha} + A\dot{\alpha} + a_1(A\alpha + \dot{\alpha}) - a_2\omega_{x1}(A\beta + \dot{\beta}) + a_3\alpha + a_4\dot{\alpha} = a_{BH}\delta_B \quad (18)$$

By transforming expressions (17) and (18), we obtain:

$$\ddot{\beta} + (A + a_1 + a_4)\dot{\beta} + (a_1A + a_3)\beta + a_2\omega_{x1}\dot{\alpha} + a_2A\omega_{x1}\alpha = 0 \quad (19)$$

$$\ddot{\alpha} + (A + a_1 + a_4)\dot{\alpha} + (a_1A + a_3)\alpha - a_2\omega_{x1}\dot{\beta} - a_2A\omega_{x1}\beta = a_{BH}\delta_B \quad (20)$$

By transforming equations (19) and (20), assigning the corresponding coefficients, and combining with equation (4), we obtain the state control equations of the single-channel flying equipment (SCFE):

$$\begin{cases} \omega_{x1} = \omega_0 = k_{\delta_e}.\delta_e \\ \ddot{\beta} + C_1\dot{\beta} + C_2\beta + C_3\omega_{x1}\dot{\alpha} + C_4\omega_{x1}\alpha = 0 \\ \ddot{\alpha} + C_1\dot{\alpha} + C_2\alpha - C_3\omega_{x1}\dot{\beta} - C_4\omega_{x1}\beta = a_{BH}\delta_B \end{cases} \quad (21)$$

Remark: The key difference compared to the dynamic equations (21) control conventional flying equipment lies in the fact that, in the dynamic model of an SCFE using a linear steering machine, only one pair of control rudders is present, and the model explicitly accounts for the angular rotation rate of the flying equipment around its longitudinal axis.

Controlling flying equipment in space is a complex process, involving numerous parameters. However, it can ultimately be characterized by the input variable, namely, the control fin deflection angle and the output variables, which are the angles between the velocity vector and the vehicle's longitudinal axis, i.e., the angle of attack α and the sideslip angle β. These parameters play a crucial role, as they determine the control limits and the resulting control overloads. Other parameters, such as pitch, elevation, or heading angles of the flying equipment, serve merely as consequential outcomes of the control process. According to the system of equations (21), since the flying equipment is equipped with only a single pair of control rudders and is forcibly rotated around its longitudinal axis at an angular velocity ω_{x1}, the control process is governed by a single moment vector generated by this fin pair. This moment vector rotates around the TBB's longitudinal axis at the same angular velocity ω_{x1} and its direction continuously changes depending on the deflection orientation of the control rudders.

It is assumed that the flying equipment remains stabilized during the control process, and oscillations induced by the component vectors of the control moment are neglected. Under the above assumptions, the system of equations (21) can be reduced to a single unified control equation for the single-channel flying equipment using only one pair of control fins, which takes the following form:

$$\ddot{\alpha} + C_1\dot{\alpha} + C_2\alpha = a_{BH}\delta_B \quad (22)$$

In there: α angle of attack of the single-channel flying equipment.

Problem statement: Based on the system of equations (21) describing the controlled motion of the flying equipment and considering the constraints encountered during the guided homing phase of flying equipment toward the target, the task is to determine an appropriate control law for the rudder δ_B deflection suitable for single-channel flying equipment equipped with a linear steering machine. The goal is to ensure that the desired flight trajectory is accurately achieved.

Solve the control problem for a single-channel flying equipment using a linear steering machine based on the Maximum Principle, with the objective of optimizing control energy. The problem is solved under the criterion of minimizing the integral of the squared magnitude of the control signal over a given control time interval $T = t_k - t_0$. This is an optimal control problem in which the Pontryagin Maximum Principle is applied to determine the optimal control law, ensuring that the control energy consumption is minimized while still satisfying the guidance requirements to reach the target.

According to the solution approach for this type of problem, it is necessary to transform the state equation of the control object (22) into a system of differential equations by defining the state variables.

$x_1 = \alpha$: Angle of attack of the single-channel flying equipment;

$x_2 = \dot{\alpha}$: The angle of attack rate of the single-channel flying equipment;

Transform expression (22) into the following system of equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -C_1 x_2 - C_2 x_1 + a_{BH}.\delta_B \end{cases} \tag{23}$$

Expression (23) written in matrix form:

$$\dot{x} = A.x + B.u \tag{24}$$

In there: The matrix $A = \begin{bmatrix} 0 & 1 \\ -C_2 & C_1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ a_{BH} \end{bmatrix}$, $u = a_{BH}$

Initial and terminal conditions of the problem: $x_1(t_0) = x_{t0} = \alpha(t_0)$, $x_2(t_0) = 0$, $x_1(t_k) = x_{1k} = \alpha^*(t_k)$, $x_2(t_k) = x_{2k}$ satisfying the boundary conditions of the problem. The control process must simultaneously satisfy the constraint conditions, including the maximum deflection limit of the control rudder vane: $|\delta_B| \le \delta_{gh} = \delta_{\max}$. Optimal control problem with control constraints.

Cost function:

$$J = \int_0^{t_k} u^2(t).dt = \int_0^{t_k} \delta_B^2(t).dt \tag{25}$$

Constraint on control: $|\delta_B| \le \delta_{\max}$, $\forall t \in [0, t_k]$

Boundary condition: $x_1(t_0) = x_{t0} = \alpha(t_0)$, $x_2(t_0) = 0$, $x_1(t_k) = x_{1k} = \alpha^*(t_k)$, $x_2(t_k) = x_{2k}$

From the system of equations (23) and the given assumptions, the cost functional Π to be minimized is determined as follows:

$$\Pi = \delta_B(t_k) + \lambda_1 x_1(t_k) + \lambda_2 x_2(t_k) \tag{26}$$

λ - The constants are determined based on the terminal conditions of the problem.

The condition for the functional Π to reach its minimum is that the Hamiltonian function (H) must attain a maximum during the control process. The Hamilton function: $H = \sum_{i=1}^{n+1} \psi_i f_i + L$ is defined as follows:

$$H = \delta_B^2 + \psi_1.x_2 + \psi_2.(-C_1 x_2 - C_2 x_1 + a_{BH}\delta_B) \tag{27}$$

In there: ψ_i - Auxiliary functions; f_i - Coefficient of the dynamical system; $L = \delta_B^2$ Instantaneous objective function.

Conjugate equation:

$$\begin{cases} \dot{\psi}_1 = -\dfrac{\partial H}{\partial x_1} = \psi_2.C_2 \\ \dot{\psi}_2 = -\dfrac{\partial H}{\partial x_2} = -\psi_1 + \psi_2.C_1 \end{cases} \tag{28}$$

Find the optimal control law subject to the constraint of minimizing the Hamiltonian over the admissible control set $|\delta_B| \le \delta_{\max}$. At each time t, the optimal control $u(t)$ must minimize the Hamiltonian H over the set of admissible controls, subject to the given control constraints. Consider the unknown function $u(t)$ to be determined.

$$u^*(t) = \delta_B^*(t) = \arg\min H(x(t), \psi(t), u) = \arg \underbrace{\min}_{|\delta_B| \le \delta_{\max}} [\delta_B^2 + \psi_2.a_{BH}\delta_{BH}] \tag{29}$$

So that the Hamiltonian (H) reaches an extremum:

$$\frac{\partial H}{\partial u} = \frac{\partial H}{\partial \delta_B} = 2\delta_B + \psi_2.a_{BH} = 0 \tag{30}$$

From expression (30), we derive the minimizing solution in the absence of constraints:

$$\delta_B^0 = -\frac{a_{BH}}{2}.\psi_2(t) \tag{31}$$

By applying the amplitude constraint:

$$\delta_B^*(t) = \begin{cases} -\frac{a_{BH}}{2}.\psi_2(t), \left| -\frac{a_{BH}}{2}.\psi_2(t) \right| \le \delta_{\max} \\ \delta_{\max}.sign(-\psi_2(t)), \left| -\frac{a_{BH}}{2}.\psi_2(t) \right| > \delta_{\max} \end{cases} \tag{32}$$

Thus:

$$\delta_B^*(t) = sat(-\frac{a_{BH}}{2}.\psi_2(t), \delta_{\max}) \tag{33}$$

Transforming the system of equations (28):

$$\ddot{\psi}_2 = -\psi_2.C_2 + \dot{\psi}_2.C_1 \tag{34}$$

The characteristic equation of (34) is given by the form: $k^2 - C_1 k + C_2 = 0$

Inference: $\Delta = C_1^2 - 4C_2$

According to [8], the value of $C_1 = (A + a_1 + a_4) << C_2 = (a_1 A + a_3) \Rightarrow \Delta < 0$ Therefore, the solution of (34) is a complex solution of the following form:

$$\psi_2(t) = A_1 e^{C_1 t}.\sin(\sqrt{(4C_2 - C_1^2)}t - B_0) \tag{35}$$

A_1, B_0: The constants are determined based on the conditions of the problem.

Substituting expression (35) into (31), the optimal control expression is determined as follows:

$$\delta_B^0 = -\frac{a_{BH}}{2}.A_1 e^{C_1 t} \sin(\sqrt{(4C_2 - C_1^2)}t - B_0) \tag{36}$$

With expression (36), it is assumed that: At the initial control time $t_0 = 0$, since the function does not change sign, expression (36) can be simplified as follows:

$$\delta_B^0 = A_0.\sin(\sqrt{(4C_2 - C_1^2)}t - B_0) \tag{37}$$

Expression (37) defines the control law for a single-channel flying equipment equipped with a linear steering machine, based on the criterion of optimal control energy. Figure 2 illustrates the control law in the form of a sinusoidal harmonic function.

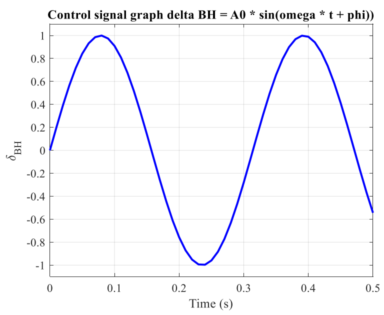

***Figure 2.** Control law for the control surface of an SCFE with a linear steering machine.*

In the case of $\omega = \sqrt{(4C_2 - C_1^2)} = 20$ rad/s; $\varphi = B_0 = 0$ rad. According to the problem conditions and the theorem on *n* intervals [7], from the differential equation (22), within the time interval from $0 \div T$, the rudder vane deflection angle will change sign once (shown in figure 2). The value of *T* is exactly the period of the sine function (34). During the control process, the flying equipment and the plane of the control rudder vane are subjected to forced rotation around the vertical axis with an angular velocity ω_{x1}. Therefore, the control process of the single-channel flying equipment should be regarded as a control process based on successive rotation cycles. Therefore, it is necessary to coordinate the sign change of the rudder vane deflection angle according to the harmonic control law (34) with the forced rotation cycle of the flying equipment around the vertical axis $0X_1$.

According to [2], the expression is given as follows:

$$\sqrt{(4C_2 - C_1^2)}t = \omega_{x1}t \tag{38}$$

By combining expressions (38) and (37), we obtain:

$$\delta_B^0 = A_0.\sin(\omega_{x1}t - B_0) \tag{39}$$

When considering the constraint $|\delta_B| \le \delta_{\max}$, thus:

$$\delta_B^*(t) = \begin{cases} A_0.\sin(\omega_{x1}t - B_0), |A_0.\sin(\omega_{x1}t - B_0)| \le \delta_{\max} \\ \delta_{\max}.sign(-A_0.\sin(\omega_{x1}t - B_0)), |A_0.\sin(\omega_{x1}t - B_0)| > \delta_{\max} \end{cases} \tag{40}$$

Remark: In all practical control systems-including the control system of an aircraft actuator, the control signal is always constrained by the physical limits of the actuating element. Specifically, to ensure feasibility and mechanical durability, the value of the control signal must not exceed the boundary limits in terms of torque, force, or angular displacement of the steering mechanism. These constraints can be applied in two ways: Input signal amplitude limitation and Hard mechanical limits on the rotational angle of the control surface.

According to [2, 3], and the proof above, when the control signal is in the form of: $\delta(t) = A_0.\sin(\omega_{x1}t - \varphi)$ then the normal force acting on the single-channel guided flying equipment, which is also the control force, varies harmonically following a sine law. Indeed:

The control force in the elevation channel during the rotation cycle of the single-channel flying equipment:

$$F_{ytb} = mV.\frac{d\theta}{dt}\bigg|_{TB} = \int_0^T K_\theta.\cos(\omega_x t).\delta(t).dt = -\frac{K_{p\alpha}.A_0}{2}\sin\varphi \tag{41}$$

The control force in the azimuth channel during the rotation cycle of the single-channel flying equipment:

$$F_{ztb} = mV.\frac{d\psi}{dt}\bigg|_{TB} = \int_0^T K_\psi.\sin(\omega_x t).\delta(t).dt = \frac{K_{p\alpha}.A_0}{2}\cos\varphi \tag{42}$$

The average normal force controlling the trajectory of the center of the single-channel flying equipment has the form:

$$F_{tb} = \sqrt{F_{ytb}^2 + F_{ztb}^2} = \frac{K_{p\alpha}.A_0}{2} \tag{43}$$

Remark: If the control signal varies sinusoidally with a frequency equal to the rotation frequency of the single-channel flying equipment, the resulting aerodynamic force will also vary sinusoidally and generate a nonzero average force to control the trajectory.

According to the proportional navigation guidance method, the generation of control commands for the flying equipment's center in the elevation and azimuth channels is determined by the following functions:

$$u_1 = f_h(t), u_2 = f_z(t) \tag{44}$$

The functions $f_h(t)$ and $f_z(t)$ are formed based on information about the target and the internal state of the single-channel flying equipment. In that case, the amplitude of the control signal $A_0 = \sqrt{u_1^2 + u_2^2}$ and control phase $\gamma_0 = \arccos(\frac{u_1}{\sqrt{u_1^2 + u_2^2}})$:

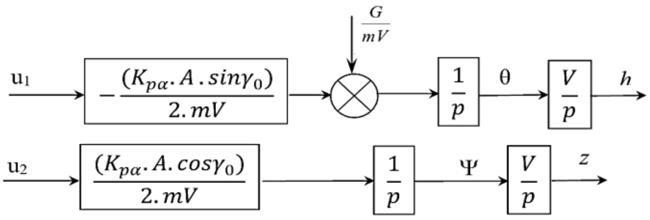

Figure 3. *Open-loop control model of the trajectory inclination and azimuth angles of the SCFE.*

The synthesis of control commands for the single-channel flying equipment using continuously moving control rudders vane is carried out by simultaneously adjusting the phase and amplitude of the rudder vane rotation pulse, which is a distinctive feature of this type of linear steering machine.

3. DISCUSSION AND ANALYSIS

3.1. Simulation scenarios for single-channel flying equipment with relay steering machine and linear steering machine

In the case of a maneuvering diving target with acceleration, the maneuvering trajectory is described by the following system of equations:

$$\begin{cases} x_m(t) = 3000 + 200t(m) \\ h_m(t) = 2000 - 30t^2(m) \\ z_m(t) = 0(m) \end{cases} \tag{45}$$

$$h_0 = 0m, x_0 = 500m, z_0 = 10m, \theta_0 = \frac{\pi}{6} rad, \psi_0 = \frac{0,1}{30} rad \tag{46}$$

3.2. Methodology and simulation tools

For the single-channel homing system with a relay actuator: According to [6], the proportional navigation guidance method has: $u_1 = \dot{\varepsilon}_c, u_2 = \dot{\beta}_c$. Then:

$$\begin{cases} mV\dfrac{d\theta}{dt} = k_1.\dot{\varepsilon}_c - G.\cos\theta \\ -mV\dfrac{d\psi}{dt} = k_2.\dot{\beta}_c \end{cases} \tag{47}$$

For the single-channel flying equipment with a linear steering machine: $u_1 = k_{\dot{\varepsilon}}.\dot{\varepsilon}_c, u_2 = k_{\dot{\beta}}.\dot{\beta}_c$. Then:

$$\begin{cases} mV\dfrac{d\theta}{dt} = -\dfrac{k_{p\alpha}.A_0}{2}.\sin\gamma_0 - G.\cos\theta \\ -mV\dfrac{d\psi}{dt} = \dfrac{k_{p\alpha}.A_0}{2}.\cos\gamma_0 \end{cases} \tag{48}$$

A comparative evaluation of the two types of steering machines (relay and linear) with respect to the following aspects: Flying equipment target trajectory in space, Lateral and longitudinal overloads of the flying equipment during target tracking, Control force over time, Accumulated energy consumption over time:

Relay steering machine:

$$E(t)=\int_0^t[(k_1.\dot{\varepsilon}_c-G.\cos\theta)^2+(k_2.\dot{\beta}_c)^2]dt \tag{49}$$

Linear steering machine:

$$E(t)=\int_0^t[(-\frac{k_{p\alpha}.A_0}{2}.\sin\gamma_0-G.\cos\theta)^2+(\frac{k_{p\alpha}.A_0}{2}.\cos\gamma_0)^2]dt \tag{50}$$

- Control surface deflection angle over time:

Relay steering machine:

$$\begin{cases}\delta_\theta(t)=\dfrac{(k_1.\dot{\varepsilon}_c-G.\cos\theta)}{k_\theta}\\ \delta_\psi(t)=\dfrac{(k_2.\dot{\beta}_c)}{k_\psi}\end{cases} \tag{51}$$

Linear steering machine:

$$\begin{cases}\delta_\theta(t)=\dfrac{(-\frac{k_{p\alpha}.A_0}{2}.\sin\gamma_0-G.\cos\theta)}{k_\theta}\\ \delta_\psi(t)=\dfrac{(\frac{k_{p\alpha}.A_0}{2}.\cos\gamma_0)}{k_\psi}\end{cases} \tag{52}$$

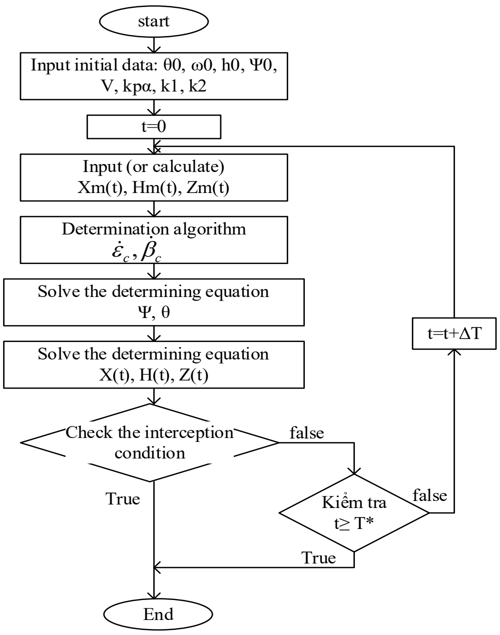

Figure 4. *An algorithm for simulating the target tracking process of the flying equipment.*

3.3. Simulation results and discussion

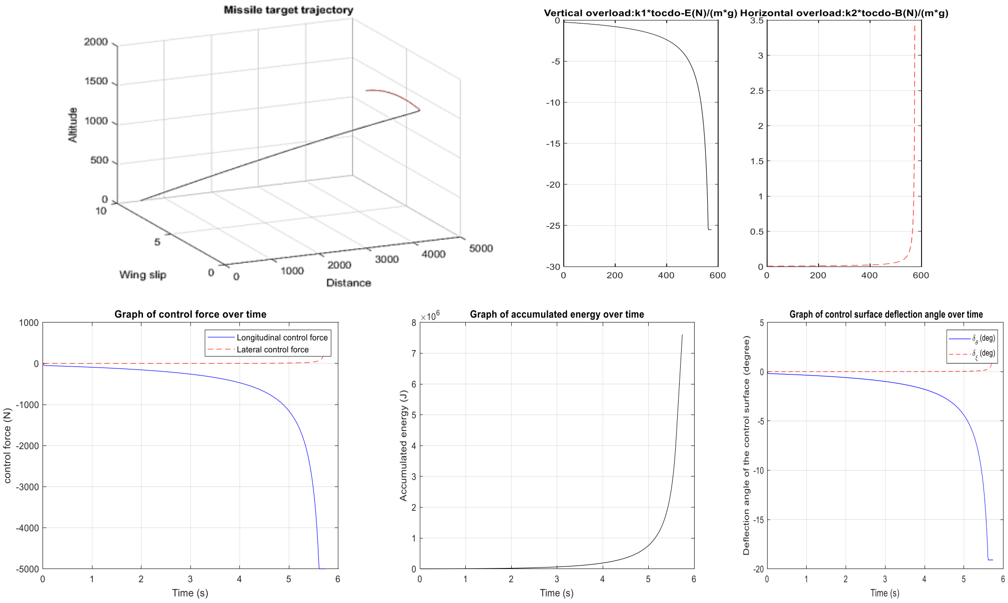

***Figure 5.** Simulation results of the SCFE with relay steering machine in tracking a maneuvering target.*

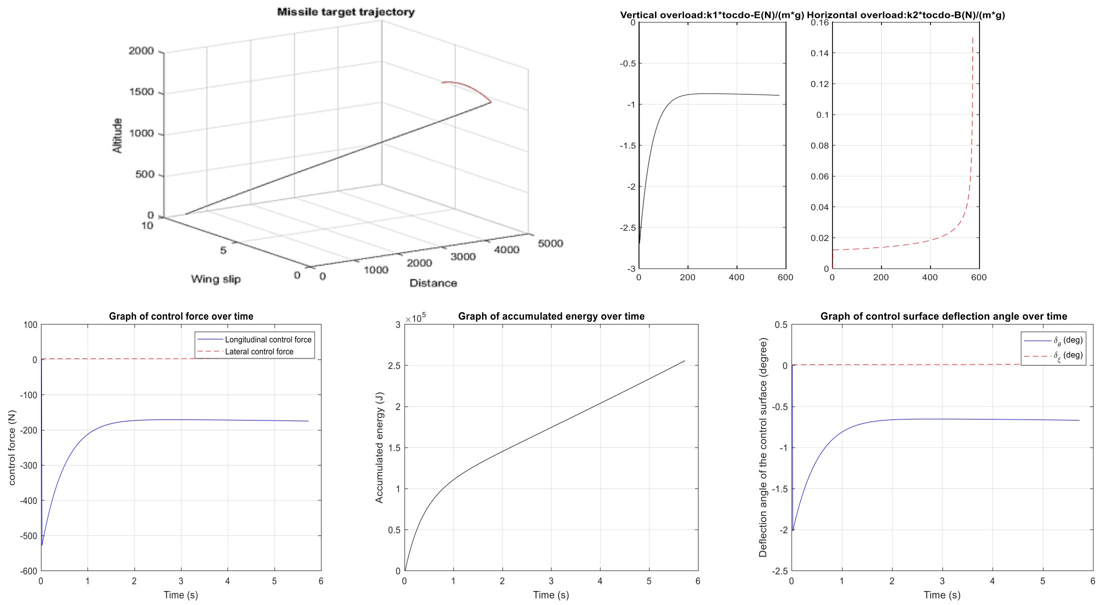

***Figure 6.** Simulation results of the SCFE with a linear steering machine in tracking a maneuvering target.*

The simulation results show that:

***Missile–target trajectory in space:** The linear actuator provides smoother and more accurate guidance results. Relay steering machine: The flight path exhibits a significantly curved trajectory. There is a clear difference between the missile and target trajectories, especially in the final phase. This indicates suboptimal target tracking, possibly caused by strong control oscillations. Linear steering machine: The trajectory is smoother and closer to the target. This demonstrates a softer control response and better tracking performance.

***Lateral and longitudinal overload:** The linear actuator manages overload more effectively, offering better safety and stability. Relay steering machine: Longitudinal overload: Drops to nearly - 27 g, far beyond normal limits ⇒ extremely dangerous. Lateral overload: Spikes up to around 3.2 g, with strong oscillations near the end of the trajectory. This indicates unstable control and high energy consumption. Linear steering machine: Longitudinal overload: Oscillates around - 2.5 g and stabilizes after 2 seconds. Lateral overload: Very small (below 0.15 g), with almost no oscillation.

***Control force over time:** The linear actuator produces smooth control forces, making it suitable for precise control. Relay steering machine: The control force increases negatively at a very high rate, reaching nearly **-** 5000 N at 6 seconds (vertical direction). Strong oscillations occur, with large variations and loss of control at the end of the process. Linear steering machine: The force gradually stabilizes around - 200 N, with small oscillations and a well-regulated trend.

***Cumulative energy consumption:** The linear actuator is significantly more energy-efficient. Relay steering machine: Initially increases slowly, but after 4 seconds it spikes exponentially, reaching nearly 8×10^6 J. This reflects extremely high energy consumption due to strong control oscillations at the end of the process. Linear steering machine: Increases steadily in a more linear fashion, reaching approximately 2.8×10^5 J at 6 seconds. More suitable for systems with energy constraints.

***Fin deflection angle over time:** The small and stable deflection angle in the linear system helps maintain aerodynamic stability and ensures better control. Relay steering machine: The vertical control deflection angle increases sharply in the negative direction, reaching nearly - 20 degrees, with strong oscillations. This reflects the on/off switching behavior of the relay, resulting in unsmooth control. Linear steering machine: The control angle remains small (around - 2 degrees), with smooth changes and quick stabilization.

Thus, the system using a linear actuator demonstrates significantly superior performance in terms of efficiency, stability, and energy consumption compared to the relay actuator. The relay actuator induces strong oscillations and poses a high risk of instability during the tracking of maneuvering targets.

4. CONCLUSIONS

This paper develops an algorithm for solving the dynamic equations of the SCFE using a linear steering machine, providing the mathematical foundation for synthesizing the corresponding control law. An optimal control method is applied based on the criterion of minimal control energy. Based on the synthesized control law, the relationship between the control signal and the normal force is established.

The experimental results were validated through simulations using MATLAB. In the simulation setup with a target engagement scenario, the comparison between the SCFE using a relay steering machine and the one using a linear steering machine shows that:

For the same target, the SCFE using a linear steering machine demonstrates significantly lower control energy consumption, lateral overload, and longitudinal overload. This confirms the advantages of the linear steering machine in terms of efficiency and its ability to ensure that the control signal is proportional to the fin deflection angle.

REFERENCES

[1]. В.М. Кашим, А.Л. Лифиц: *"Основы проектирования переносных зенитных ракетных комплексов"*. Гродно ГрГУ им. Я. Купалы, Москва, (2014).

[2]. Nguyen Van Son. *"Regarding the control problem of single-channel guided flying equipment",* Technical PhD thesis, Academy of Military Science and Technology, (2007).

[3]. Phan Van Tu. *"Construct method and control algorithms for the class Single-channel flying equipment",* Technical PhD thesis, Academy of Military Science and Technology, (2010).

[4]. Nguyen Duc Cuong, Tran Duc Thuan. *"Guided flying equipment and anti-ship cruise missiles",* People's Army Publishing House, (2015).
[5]. To Ba Thanh. *"Researching on synthesizing of guiding and steering control algorithm for a class of two-channel aerial vehicle rotating around the longitudinal axis",* Technical PhD thesis, Academy of Military Science and Technology, (2024).
[6]. Казаков И .Е, Мишаков А Ф: *"Авиационые управляемые ракеты",* ВВИА им проф. Н.Е. Жуковского, (1988).
[7]. Пугачёв В. С: *"Основы автоматического управления",* Наука Мосва, (1974).
[8]. Q. F. Gao et al., *"Autopilot design of single channel rolling missile".* Beijing Ligong Daxue Xuebao/Transaction of Beijing Institute of Technology, (2011).
[9]. E.C. Suiçmez, A.T. Kutay. *"Single channel digital controller design for a high spinningrate rolling airframe missile".* Aeronautical Journal -New Series- 126(1305), (2022).
[10]. T.-V. Chelaru, Valentin Pana, Cristian Emil Constantinescu, Costin Ene, Chelaru Adrian: *"Stability of Single-Channel Homing Rolling Aerospace Vehicle".* Aerospace ISSN: 2226-4310, (2024).

TÓM TẮT

Nghiên cứu giải thuật tổng hợp lệnh điều khiển cho thiết bị bay một kênh với máy lái tuyến tính

Bài báo trình bày giải thuật và cơ sở toán học cho việc tổng hợp lệnh điều khiển áp dụng cho thiết bị bay một kênh sử dụng máy lái tuyến tính. Giải thuật được xây dựng dựa trên nguyên lý điều khiển tối ưu với tiêu chuẩn tối thiểu hóa năng lượng điều khiển. Từ luật điều khiển thu được, bài báo thiết lập mối quan hệ giữa tín hiệu điều khiển và lực pháp tuyến tác động lên thiết bị bay. Kết quả mô phỏng được thực hiện nhằm minh chứng tính ưu việt của luật điều khiển đề xuất so với luật điều khiển rơ le, đặc biệt trong ứng dụng với thiết bị bay một kênh.

Keywords: Thiết bị bay một kênh; Lệnh điều khiển; Máy lái tuyến tính; Giải thuật.