Optimization of overhead crane main girder considering deflection constraint using differential evolution

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ABSTRACT

Overhead cranes are one of the commonly used lifting and transporting equipment for hoisting, lowering, and moving goods within factories and warehouses. The main girder plays the most important role, directly bearing the loads during lifting operations and resisting various forces during crane operation. In this paper, the authors present an optimal design method for the main girder using the Differential Evolution method, taking into account the deflection constraint of the girder in two cases: hollow rectangular box section and I-shaped section. The research results of this paper provide an important scientific and practical basis for selecting reasonable structural parameters of the crane main girder, while also contributing to improving the efficiency of design, fabrication, and operation of the equipment. In addition, the results of this paper serve as a reference for comparing and evaluating the optimization of the two types of cross-sections, providing a basis for selecting the girder's cross-sectional shape according to bending strength and deflection criteria.

Keywords: Overhead crane; Main girder; Optimal design; Deflection; Differential evolution.

1. INTRODUCTION

Overhead cranes are specialized lifting and transporting equipment widely used in factories, warehouses, manufacturing plants, mechanical workshops, construction sites, etc., with the functions of lifting, lowering, and moving goods horizontally and vertically within a certain working range. The use of overhead cranes helps reduce manual labor, increase productivity, ensure operational safety, and improve production efficiency. Overhead cranes are complex structural equipment, in which the main girder is the primary load-bearing component, responsible for supporting the hoist and carrying the entire load of goods, as well as dynamic loads arising during operation. The main girder of an overhead crane is commonly designed with typical cross-sectional types such as I-beam, rectangular box beam, or C-shaped beam. These cross-sectional forms are defined by specific geometric parameters such as length, width, height, and thickness of the section, among others. Therefore, the optimal design of the main girder plays a particularly important role throughout the entire design process of the overhead crane. A well-designed girder not only ensures the structural strength, stiffness, and stability but also helps to reduce weight, save materials, lower production costs, and improve operational efficiency.

There have been many studies related to the optimal design of overhead cranes. Ref. [1] investigated the design of an optimal controller for the hoisting mechanism, introducing a method based on the Riccati equation in optimal control. This method contributes to improving control performance while ensuring safe and stable operation of the crane's hoisting mechanism. Ref. [2] presented a study on time-optimal control for overhead cranes using input shaping (IS) techniques, where oscillations of the main girder during operation were effectively minimized and suppressed. Several investigations [3-5] focused on nonlinear control methods for gantry and overhead cranes, aiming to suppress vibrations and reduce oscillations. In these works, controller parameters were further optimized using the Particle Swarm Optimization (PSO) algorithm. In addition, Ref. [6] examined the design of a Fuzzy PD controller to achieve accurate control, reduce operational errors, and mitigate vibrations and oscillations during crane operation. Ref. [7] proposed an optimal design for a welded I-shaped main girder, where strength and stability criteria were incorporated. The optimization of the cross-sectional dimensions was achieved through a combined approach of mathematical analysis and the Genetic Algorithm (GA). The validity of the method was confirmed by a 3D finite element model in ANSYS Workbench, and both approaches produced consistent results. Ref. [8] addressed the box girder of a 550-ton gantry crane, employing the Finite Element Analysis (FEA) method via ANSYS rather than evolutionary algorithms. Based on the optimization results, the maximum stress and displacement obtained from classical strength of materials theory were compared with those predicted by ANSYS simulations. The study presented in Ref. [9] focused on the supporting girder of the traveling mechanism, in which several alternative cross-sectional designs were introduced and evaluated under strength and stability requirements. Ref. [10] employed the Differential Evolution (DE) method for the optimal design of the main girder. Nevertheless, the study did not fully account for the governing constraints, as the deflection limit was excluded, the self-weight of the girder was ignored, and only static loading conditions were considered without dynamic load effects.

Thus, it can be seen that no previous study has investigated the optimal design of the main girder of an overhead crane using the DE method while considering the deflection constraint and accounting for dynamic load factors according to relevant standards. In this paper, the authors conduct an optimal design study of the crane's main girder, taking into account the deflection constraint using the DE method, with two cross-sectional types: hollow rectangular box section and I-shaped section. Based on this, the authors present the deflection analysis results of the girder and propose the optimal design space. The research results of this study provide a scientific basis for selecting the design parameters of the crane's main girder, thereby assessing the advantages and disadvantages of each cross-sectional type as a reference for selecting the girder's cross-sectional shape that satisfies both strength and technical requirements.

2. FORMULATION OF THE OPTIMAL DESIGN PROBLEM FOR THE CRANE MAIN GIRDER

The structural optimization model for the main girder (figure 1) can be considered as a simply supported beam resting on two supports, *A* and *B*.

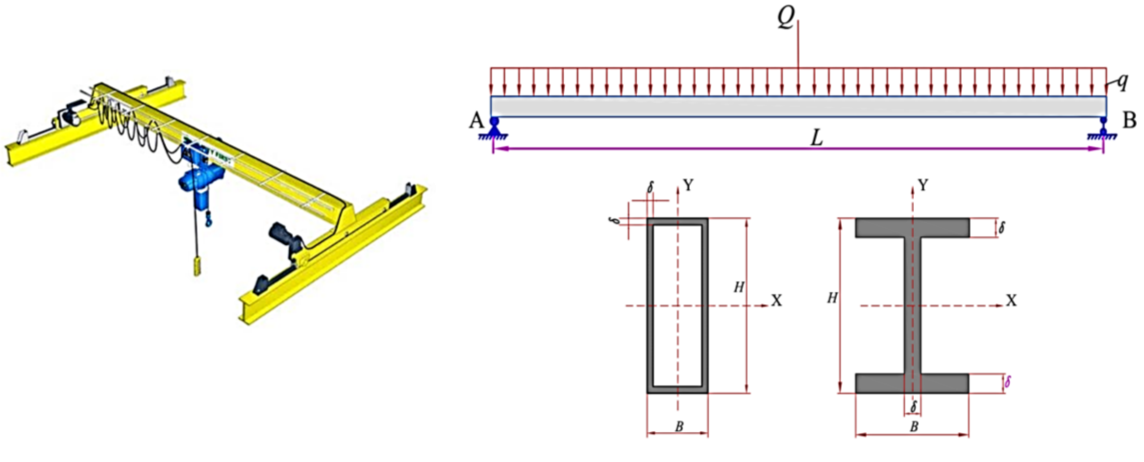


Figure 1. *Calculation model of the overhead crane main girder.*

The nominal lifting load *Q* (N) is applied at the most critical position, located at the mid-span of the beam. The self-weight of the beam is converted into a uniformly distributed load q (N/m) along the entire length of the beam L (m). The crane's operating condition is assumed to be a medium-duty service condition. Consider a rectangular hollow box section and an I-section with width B (m), height H (m), and thickness δ (m). The mass of the trolley is neglected.

According to [11], under medium-duty service conditions, the maximum deflection of the main girder (y_{max}) of the overhead crane must satisfy the following condition:

$$y_{\max} = y_{q(\max)} + y_{Q(\max)} = \frac{5qL^4}{384EI_x} + \frac{kQL^3}{48EI_x} \le \left[y_b\right] = \frac{L}{500} \tag{1}$$

Where: k is the safety factor, $k = 1.2$ [12]; E is the elastic modulus of the material, (GPa); q is the uniformly distributed load along the entire length of the girder caused by the self-weight of the girder (N/m); I_x is the moment of inertia of the cross-section about the x-axis, determined based on the girder's cross-sectional geometry (m^4).

The quantity q is determined by the following equation: $q = \gamma g A$. Where: γ is the density of the girder material (kg/m^3), g is the gravitational acceleration (m/s^2), A is the cross-sectional area of the girder, determined based on the girder's cross-section (m^2):

The quantity A is determined by the equation:

i) For the hollow rectangular box section: $A = BH - (B-2\delta)(H-2\delta)$

ii) For the I-shaped section: $A = 2B\delta + (H-2\delta)\delta$

The quantity I_x is determined by the equation:

i) For the hollow rectangular box section: $I_x = \frac{1}{12}\left[BH^3 - (B-2\delta)(H-2\delta)^3\right]$

ii) For the I-shaped section: $I_x = \frac{1}{6}B\delta^3 + \frac{1}{2}B\delta(H-\delta)^2 + \frac{1}{12}\delta(H-2\delta)^3$

According to [11], the strength condition of the girder must satisfy:

$$\sigma_{\max} = \frac{M_{\max}}{W_x} = \frac{kQL}{4} + \frac{qL^2}{8} \le \left[\sigma_b\right] \tag{2}$$

Where: k is the safety factor, $k = 1.2$ [12]; W_x is the section modulus of the girder's cross-section about the x-axis, determined based on the girder's cross-sectional geometry (m^3):

i) For the hollow rectangular box section: $W_x = \frac{I_x}{H/2} = \frac{1}{6H}\left[BH^3 - (B-2\delta)(H-2\delta)^3\right]$

ii) For the I-shaped section: $W_x = \frac{I_x}{H/2} = \frac{2}{H}\left[\frac{1}{6}B\delta^3 + \frac{1}{2}B\delta(H-\delta)^2 + \frac{1}{12}\delta(H-2\delta)^3\right]$

Where: $[\sigma_b]$ is the allowable bending stress of the girder material, (N/m^2).

The objective of the optimal design problem for the overhead crane main girder is to minimize the girder's mass while ensuring that the bending stress and deflection constraints are satisfied.

The mass of the girder is determined by the following formula:

$$m = \gamma V \tag{3}$$

Where: γ is the density of the girder material (kg/m^3); V is the volume of the girder material (m^3), determined based on the girder's cross-sectional shape (m^3):

i) For the hollow rectangular box section: $V = \left[BH - (B-2\delta)(H-2\delta)\right]L$

ii) For the I-shaped section: $V = \left[2B\delta + (H-2\delta)\delta\right]L$

Thus, for a given type of main girder, under a specified nominal lifting load, bending strength condition, and deflection constraint, the weight of the main girder depends on its geometric parameters, material properties, and the crane's operating condition. Accordingly, to meet the above requirements, the objective function of the optimal design problem for the main girder is selected as the minimization of its mass. The optimization problem is stated as follows: Determine the geometric parameters of the main girder (such as width, height, and thickness of the cross-section) to minimize the girder's weight, while satisfying the bending strength and deflection constraints under given conditions, including nominal lifting load, cross-sectional type, girder length, and material properties.

The mass-minimization optimization problem is analyzed as follows:

i) Objective function:

$$m = \gamma V \rightarrow min \tag{4}$$

ii) Independent variables of the objective function: B, H và δ.

iii) Constraints:

$$\begin{cases} \sigma_{\max} = \dfrac{M_{\max}}{W_x} \le [\sigma_b]; y_{\max} \le [y_b] = \dfrac{L}{500} \\ B_U \le B \le B_L; H_U \le H \le H_L; \delta_U \le \delta \le \delta_L \end{cases} \tag{5}$$

Where: J_U and J_L (J represents B, H, δ) are the upper and lower bounds of the continuous design variables.

iv) Given parameters: Q, L, γ, E, σ_b, the number of girders and the type of main girder cross-section.

3. RESULTS AND DISCUSSION

Input parameters: Q = 10×10^4 N; L = 12 m; γ = 7.85×10^3 kg/m^3; E = 210 Gpa; [σ_b] = 188×10^6 N/m^2; B_L = 0.5 m; B_U = 0.1 m; H_L = 0.6 m; H_U = 0.2 m; δ_L = 0.025 m; δ_U = 0.015 m; Number of generations: M = 30; Population size per generation: N = 3000; Number of girders 1. Run 1 - Run 5 denote the first to fifth random runs, respectively.

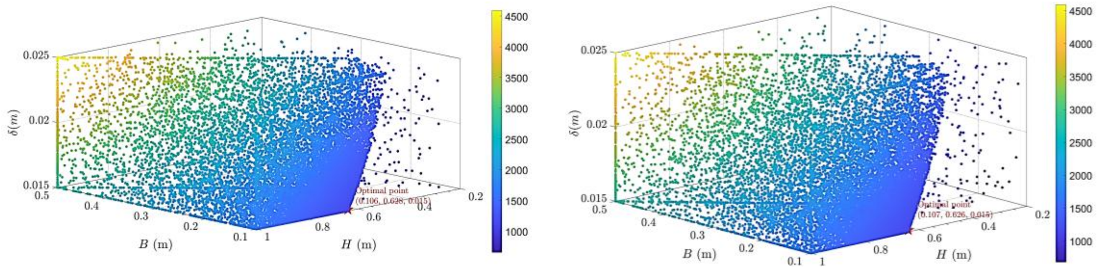
Table 1 presents the optimal results of the girder cross-section. The survey results show that, to achieve the optimal mass while still satisfying the strength conditions, the thickness of the steel plate (δ) tends to be selected near its lower bound.

***Table 1.** Optimal results of the girder cross-section.*

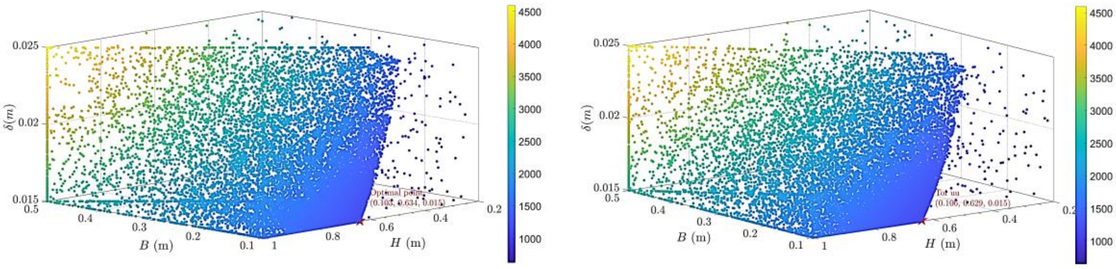
Run	I-shaped cross-section				Hollow rectangular box cross-section				Δm(kg)
	B_I (m)	H_I (m)	δ_I (m)	m_I (kg)	B_{bx} (m)	H_{bx} (m)	δ_{bx} (m)	m_{bx} (kg)	
Run 1	0.1062	0.6282	0.0150	1145.47	0.1525	0.4533	0.0150	1627.04	481.57
Run 2	0.1074	0.6258	0.0150	1145.53	0.1490	0.4568	0.0151	1627.02	481.49
Run 3	0.1033	0.6340	0.0151	1145.49	0.1544	0.4514	0.0150	1626.07	480.58
Run 4	0.1059	0.6288	0.0150	1145.48	0.1497	0.4561	0.0150	1627.03	481.55
Run 5	0.1057	0.6293	0.0151	1145.47	0.1475	0.4583	0.0151	1627.12	481.65

This finding is entirely consistent with the convergence regions shown in figures 2 and 3. Specifically, for the I-shaped cross-section, the optimal point is concentrated around (B, H, δ) = (0.11, 0.63, 0.015), while for the hollow rectangular box cross-section, the optimal point is concentrated around (B, H, δ) = (0.15, 0.46, 0.015. This confirms the validity of the optimal weight design of the main girder using the DE method, providing a scientific basis for selecting appropriate geometric parameters according to the girder's cross-sectional type during the design and manufacturing process. In addition, at the optimal points, it can be observed that while satisfying the strength conditions, the mass of the I-shaped girder is significantly lower than that of the hollow rectangular box girder, with a difference of approximately Δm = 481 (kg). This indicates that the I-shaped cross-section offers a more optimal mass. The obtained results are in reasonable agreement with the findings reported in Ref. [10].

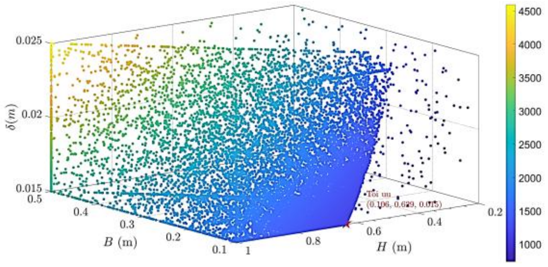
For the I-shaped section (figure 2): By observing the optimal region for the I-shaped cross-section, it can be seen that the optimal design space of the main girder lies within the dark blue area. The optimal region is concentrated within the ranges $0.4 \leq H \leq 1$, $0.1 \leq B \leq 0.3$, $0.015 \leq \delta \leq 0.0175$. When the thickness of the steel plate (δ) increases, the mass of the girder also increases rapidly. Therefore, during the optimization process, the thickness of the steel plate (δ) is usually selected near its lower bound. This observation is consistent with the results shown in table 1.



a) Run 1. *b) Run 2.*



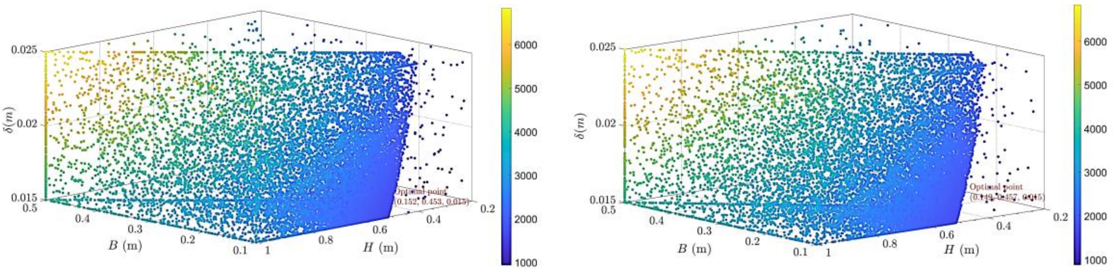
c) Run 3. *d) Run 4.*



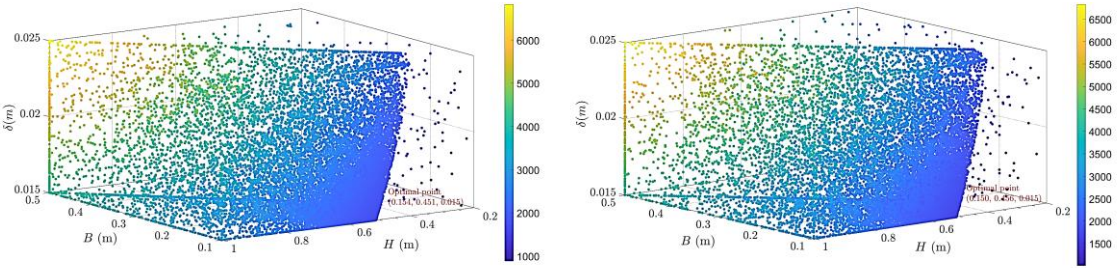
e) Run 5.

Figure 2. *Optimal design space and optimal point for the I-shaped cross-section.*

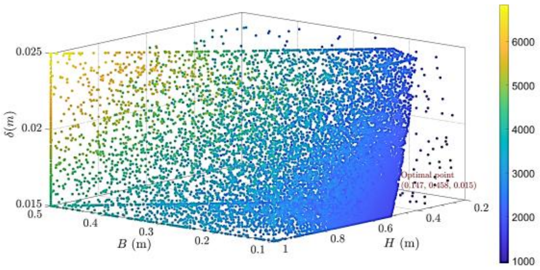
For the hollow rectangular box section (figure 3): By observing the optimal region for the hollow rectangular box cross-section, it can be seen that the distribution rule is relatively similar to that of the I-shaped cross-section. The optimal design space of the main girder lies within the dark blue area. The optimal region is concentrated within the ranges $0.4 \leq H \leq 1$, $0.1 \leq B \leq 0.3$, $0.015 \leq \delta \leq 0.02$. When the thickness of the steel plate (δ) increases, the mass of the girder also increases rapidly. Therefore, during the optimization process, the thickness of the steel plate (δ) is usually selected near its lower bound. This observation is consistent with the results shown in table 1.



a) Run 1. b) Run 2.



c) Run 3. d) Run 4.



e) Run 5.

Figure 3. *Optimal design space and optimal point for the hollow box cross-section.*

Deflection graphs for different cases:

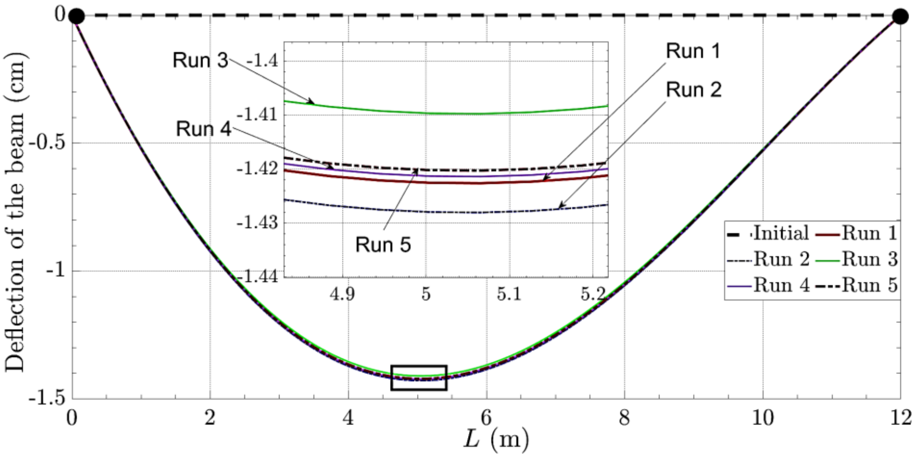


Figure 4. *Deflection of the main girder in the investigated cases (I-shaped cross-section).*

The deflection results of the main girder for the two cases using the I-shaped (figure 4) and hollow rectangular box cross-sections (figure 5). The survey results indicate that the deflection behavior of the main girder in the investigated cases with the I-shaped and hollow rectangular box cross-sections is quite similar. All the deflection values obtained from the analysis are smaller than the allowable deflection limit. The maximum deflection when using the I-shaped cross-section is approximately 1.45 cm, which is smaller than that of the hollow rectangular box cross-section, approximately 1.98 cm. Therefore, it can be concluded that using the I-shaped cross-section is more optimal than the hollow rectangular box cross-section in terms of both mass and strength conditions.

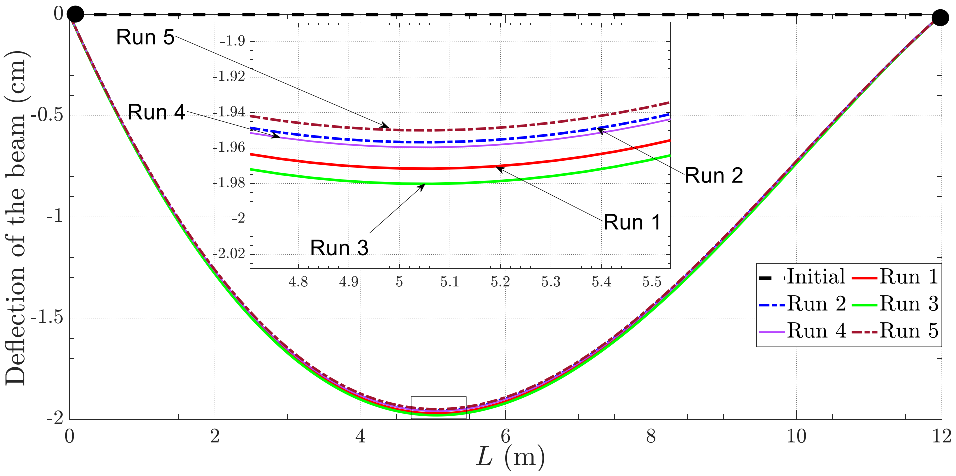


Figure 5. *Deflection of the main girder in the investigated cases (hollow rectangular box cross-section).*

4. CONCLUSIONS

In this study, the optimal design problem of the overhead crane main girder, considering the deflection constraint, was formulated and solved using the DE method. Based on this, the optimal design space was explored, and the optimal points were identified. The survey results show that, for the I-shaped cross-section, the optimal point is concentrated around (B, H, δ) = (0.11, 0.63, 0.015), while for the hollow rectangular box cross-section, the optimal point is concentrated around (B, H, δ) = (0.15, 0.46, 0.015). The mass of the girder using the I-shaped cross-section is significantly smaller than that using the hollow rectangular box cross-section, and the deflection of the girder with the I-shaped cross-section is also smaller. Therefore, it can be concluded that the I-shaped cross-section is more optimal for the overhead crane main girder compared to the hollow rectangular box cross-section. The obtained results provide a foundation for developing optimal design methods for mechanical components in general and beam-like components in particular, according to the specified design requirements.

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TÓM TẮT

Nghiên cứu tối ưu hóa dầm chính cầu trục có xét đến điều kiện độ võng sử dụng phương pháp tiến hóa vi phân

Cầu trục là một trong những thiết bị nâng chuyển dùng để nâng, hạ, di chuyển hàng hóa trong phạm vi nhà xưởng, kho bãi, được sử dụng tương đối phổ biến hiện nay. Dầm chính đóng vai trò quan trọng nhất, trực tiếp chịu tải trọng trong quá trình nâng hạ và các lực tác động khi cầu trục hoạt động. Trong bài báo này, nhóm tác giả trình bày phương pháp thiết kế tối ưu dầm chính bằng phương pháp tiến hóa vi phân có xét thêm điều kiện độ võng của dầm trong hai trường hợp mặt cắt hình hộp chữ nhật rỗng và chữ I. Kết quả nghiên cứu của bài báo là cơ sở khoa học và thực tiễn quan trọng để lựa chọn bộ thông số kết cấu hợp lý cho dầm chính cầu trục, đồng thời góp phần nâng cao hiệu quả thiết kế, gia công và khai thác thiết bị. Ngoài ra, kết quả của bài báo được dùng để đánh giá so sánh tối ưu hai dạng mặt cắt, làm cơ sở tham khảo trong việc lựa chọn hình dạng tiết diện dầm theo điều kiện độ bền uốn và độ võng.

Từ khoá: Cầu trục; Dầm chính; Thiết kế tối ưu; Độ võng; Tiến hóa vi phân.