

## Direction estimation and tracking of rapidly moving RF emitters

Le Van Bao, Nguyen Quang Vinh\*, Pham Cong Tu

Institute of Missile, Academy of Military Science and Technology, 17 Hoang Sam, Nghia Do, Hanoi, Vietnam.

\*Corresponding author: vinhquang2808@gmail.com

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### ABSTRACT

*Accurately localizing dynamic targets in complex and rapidly changing environments, such as unmanned aerial vehicles (UAVs), mobile platforms, or unstable moving objects-poses significant challenges in both civilian and military applications. This paper proposes an approach that integrates software-defined radio (SDR) technology with circularly polarized microstrip antennas to enhance localization performance under harsh conditions, including weak signals, multipath propagation, and disturbances caused by rapid motion. A simulation model is developed, and experimental analyses are conducted using an improved MUSIC algorithm for direction-of-arrival (DOA) estimation, combined with a second-order filtering technique. The results demonstrate a strong correlation between the simulated data and practical measurements, confirming the effectiveness of the proposed method.*

**Keywords:** Circularly polarized microstrip antenna; MUSIC algorithm; ESPRIT algorithm; DOA; SDR; Kalman filter.

### 1. INTRODUCTION

Radio frequency (RF) source localization is a critical problem with numerous applications in fields such as defense, aviation, telecommunications, security, and search and rescue operations. The objective of this problem is to accurately determine the direction of arrival (DOA) of signals emitted by the source, thereby enabling the precise localization of the emitter. However, in practical scenarios where the source may move randomly or operate in environments characterized by interference and multipath propagation, traditional localization methods employing directional antennas or simple linear antenna arrays often exhibit limited accuracy and flexibility.

To overcome these limitations, integrating software-defined radio (SDR) technology with circularly polarized microstrip antennas has emerged as an effective solution. SDR enables flexible acquisition, processing, and analysis of RF signals through software, reducing reliance on traditional hardware implementations. Meanwhile, circularly polarized microstrip antennas are capable of maintaining stable signal reception even when the source is moving or its orientation varies, owing to their inherent resilience to polarization mismatches.

Notably, R. Zekavat and R. M. Buehrer provided a comprehensive survey of wireless localization techniques (for both indoor and outdoor scenarios), emphasizing sensor fusion and integration capabilities in rapidly varying channels [1]. Hanan Awad Hassan Ali and Shinnazar Seytnazarov proposed a system for recognizing human movement directions using Wi-Fi signals combined with machine learning and deep learning algorithms [2]. Cahl C.B. et al. compared the performance of three DOA estimation algorithms (Beamscan, conventional Beamforming, and MUSIC) in HF radar systems, demonstrating that MUSIC offers superior resolution, particularly under weak signal conditions and large angular deviations [3]. Khumane, D. simulated a DOA estimation system employing the MUSIC algorithm alongside the LMS algorithm for beamforming, thereby enhancing detection capability and interference suppression in smart antenna systems [4]. Another study has integrated MUSIC with adaptive beamforming to improve DOA estimation performance in multipath environments, where MUSIC is responsible for generating the pseudo-spectrum while beamforming reinforces the desired signal and mitigates interference [5].

Huang et al. conducted a study on DOA estimation using the MUSIC algorithm alone and evaluated its performance through software simulations, clarifying the influence of the number of antenna elements, element spacing, and SNR levels on the resolution of the algorithm. However, this study was limited to instantaneous DOA estimation and did not address trajectory tracking over time or implementation on practical hardware platforms [6].

However, these existing studies still lack an integrated approach that combines high-resolution, high-speed DOA estimation with continuous tracking capability in dynamic environments. This paper proposes an improved method that integrates the MUSIC algorithm with classical beamforming and a second-order Kalman filter to smooth and stabilize the DOA tracking trajectory over time. Furthermore, the system is implemented on a practical hardware platform using SDR and circularly polarized microstrip antennas, enabling the performance of the proposed algorithm to be evaluated under realistic conditions where signals may exhibit strong fluctuations or be affected by interference. The results obtained from both simulations and experiments demonstrate the validity and effectiveness of the proposed model.

## 2. DEVELOPMENT OF THE ALGORITHMIC MODEL

### 2.1. DOA estimation algorithm model

Consider an antenna array consisting of  $M$  identical antenna elements receiving  $N$  narrowband signals arriving from different directions  $\theta_1, \theta_2, \theta_3, \dots, \theta_N$ .

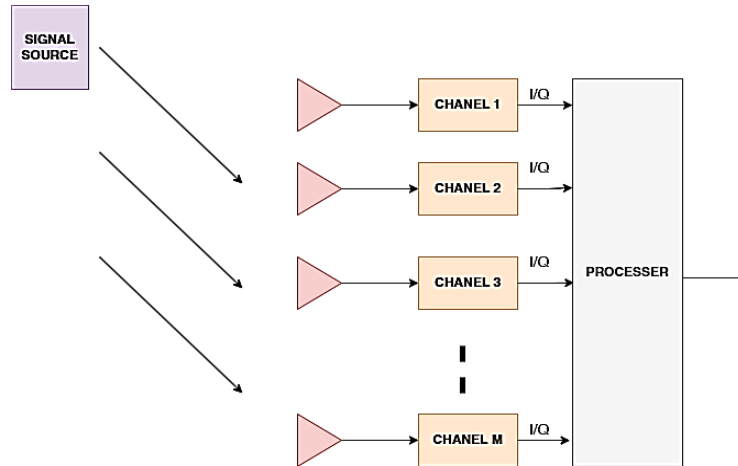


Figure 1. Illustration of the DOA estimation model.

The  $i$ -th signal impinging on the antenna array at time  $t$  is represented in figure 1 as:

$$S_i(t) = S_i e^{j(\omega t + \varphi_i)}, i = 1, 2, 3, \dots, N \quad (1)$$

where  $S_i$ ,  $\omega$ , and  $\varphi_i$  denote the amplitude, angular frequency, and phase, respectively, which are assumed to be uniformly distributed across the elements of the array. The signal column vector can then be expressed as:

$$\mathbf{S}(t) = [S_1(t), S_2(t), S_3(t), \dots, S_N(t)]^T \quad (2)$$

The array steering vector corresponding to the  $i$ -th source is defined as follows:

$$a(\theta_i) = [1, e^{j\omega_i \tau_1(\theta_i)}, e^{j\omega_i \tau_2(\theta_i)}, e^{j\omega_i \tau_3(\theta_i)}, \dots, e^{j\omega_i \tau_{M-1}(\theta_i)}]^T \quad i = 1, 2, 3, \dots, N \quad (3)$$

where  $\tau_m = (m - 1) \cdot \frac{d}{c} \sin(\theta_i)$ ,  $m = 1, 2, 3, \dots, M$  with  $d$  being the spacing between antenna elements and  $c$  the propagation speed. The element spacing is typically chosen such that  $d \leq \lambda/2$ , where  $\lambda$  denotes the wavelength of the incoming signal.

The array steering vector can thus be rewritten as:

$$\mathbf{a}(\theta_i) = \left[ 1, e^{j\frac{2\pi}{\lambda_\omega}d \sin(\theta_i)}, e^{j\frac{2\pi}{\lambda_\omega}2d \sin(\theta_i)}, \dots, e^{j\frac{2\pi}{\lambda_\omega}(M-1)d \sin(\theta_i)} \right]^T, i = 1, 2, \dots, N \quad (4)$$

where  $f_c$  is the carrier frequency and  $\lambda_\omega$  is the wavelength of the signal impinging on the array.

The signal received at the  $m$ -th antenna element is given by:

$$x_m(t) = x_1(t) \exp\left(-j\frac{2\pi}{\lambda_\omega}(m-1)\frac{d}{c} \sin(\theta_i)\right); i = 1, 2, 3 \dots M \quad (5)$$

where  $x_1$  denotes the signal received by the first antenna element. The array steering vector is represented as:  $\mathbf{A} = [a(\theta_1), a(\theta_2), a(\theta_3), \dots, a(\theta_N)]$ .

By applying additive white Gaussian noise  $n_i(t)$  across all elements, the received signal can then be expressed as:

$$\mathbf{x}(t) = [x_1(t), x_2(t), x_3(t), \dots, x_M(t)]^T = \mathbf{A}S(t) + \mathbf{n}(t) \quad (6)$$

By discretizing over time (sampling) with  $K$  snapshots, we obtain:

$$\mathbf{X}(k) = \mathbf{A}\mathbf{S}(k) + \mathbf{n}(k) \quad k = 1, 2, 3, \dots, K \quad (7)$$

Since the DOA parameter inherently possesses a spatial (angle-of-arrival) nature, it is necessary to employ the spatial correlation matrix to achieve accurate DOA estimation.

We define the correlation matrix as follows [7]:

$$\mathbf{R} = E\{\mathbf{X}(k)\mathbf{X}^H(k)\} = \mathbf{A}E\{\mathbf{S}(k)\mathbf{S}^H(k)\}\mathbf{A}^H + \sigma^2\mathbf{I} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2\mathbf{I} \quad (8)$$

where  $R_s$  denotes the source signal correlation matrix,  $\sigma^2$  is the noise variance (white Gaussian noise), and  $\mathbf{I}$  is the identity matrix. If the signals are narrowband and mutually uncorrelated,  $R_s$  is typically assumed to be a diagonal matrix. Estimation is performed using time averaging, under the assumption that the signals are ergodic. Signals across multiple snapshots are “stacked” to form:  $\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}$ , where  $\mathbf{X}$  is the received data matrix of dimensions  $M \times N$  (each column corresponding to a snapshot),  $\mathbf{S}$  is the source signal matrix of dimensions  $(N \times K)$ , and  $\mathbf{N}$  is the noise matrix of dimensions  $(M \times K)$ .

The expression for estimating the correlation matrix is given by:

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k)\mathbf{x}^H(k) = \frac{1}{K} \mathbf{X} \mathbf{X}^H \quad (9)$$

The estimated matrix  $\hat{\mathbf{R}}$  encapsulates the spatial relationships among the antenna elements and serves as the primary input for algorithms such as MUSIC, ESPRIT, Root-MUSIC, and others.

MUSIC is one of the most widely used and effective algorithms for DOA estimation, owing to its high resolution and ability to separate closely spaced signal sources. However, in practical environments with multiple emitters, strong interference, or when real-time processing of rapidly varying signals is required, the traditional MUSIC algorithm may encounter limitations in computational speed and its ability to suppress out-of-sector noise.

The spatial correlation matrix  $\hat{\mathbf{R}}$  is estimated from the received data  $\mathbf{X}$  according to the following expression:

$$\hat{\mathbf{R}} = \frac{1}{K} \mathbf{X} \mathbf{X}^H \quad (10)$$

This correlation matrix is then decomposed into its eigencomponents through eigenvalue decomposition:

$$\hat{\mathbf{R}} = \hat{\mathbf{V}}\hat{\Lambda}\hat{\mathbf{V}}^H \quad (11)$$

The MUSIC algorithm exploits the orthogonality between the noise subspace and the steering

vectors  $\mathbf{a}(\theta)$  corresponding to the actual directions of arrival. When  $\mathbf{a}(\theta)$  aligns closely with the true signal direction, it becomes nearly orthogonal to the noise subspace, resulting in a small denominator in the MUSIC spectrum function and thereby producing a prominent peak in the power spectrum [7].

$$\mathbf{P}_{MUSIC}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{V}_n\mathbf{V}_n^H\mathbf{a}(\theta)} \quad (12)$$

Here,  $\mathbf{V}_n$  denotes the matrix containing the eigenvectors of the noise subspace. The estimated DOAs correspond to the angles associated with the most prominent peaks of the  $\mathbf{P}_{MUSIC}(\theta)$  spectrum. The number of these peaks equals the number of signal sources impinging on the antenna array.

Although the conventional MUSIC algorithm offers high accuracy in DOA estimation, it requires scanning the power spectrum over the entire angular domain with sufficiently fine resolution. This leads to substantial computational complexity, especially in real-time applications or when the number of antenna elements becomes large.

The computational complexity of MUSIC is given by the following expression [8]:

$$KM^2 + O(M^3) + MK^3(M - N) \quad (13)$$

In this context,  $K$  denotes the number of snapshots,  $M$  the number of antenna elements in the array, and  $N$  the number of signal sources. The term  $KM^2$  represents the number of operations required for estimating the covariance matrix. The term  $O(M^3)$  reflects the computational complexity of the eigen-decomposition step, while  $(MK)^3(M-N)$  constitutes the most computationally intensive part, which is associated with scanning the entire angular domain and evaluating the MUSIC spectrum.

To further reduce complexity, particularly in real-time tracking systems, this paper proposes integrating a preliminary angle estimation step using digital beamforming (DB). This stage functions as a primary spatial filter, allowing rapid identification of the angular region with the strongest signal energy in the azimuth domain. Consequently, the MUSIC algorithm is constrained to scan only within a narrow angular window rather than the entire domain, thereby significantly reducing the number of evaluation points for the MUSIC spectrum. The proposed estimation framework is illustrated in figure 2.

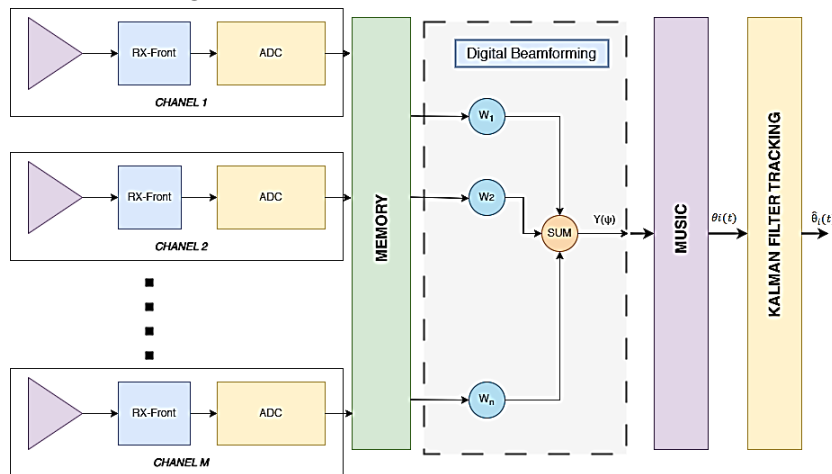


Figure 2. Improved angle estimation algorithm model.

To further reduce computational complexity, particularly in real-time tracking systems, an initial coarse angle estimation can be performed using basic beamforming. This beamforming step provides a rough estimate of the angle of arrival, which is then used to constrain the angular search

range in the MUSIC algorithm - a technique known as “narrowed scanning.” The estimation model is illustrated in figure 2.

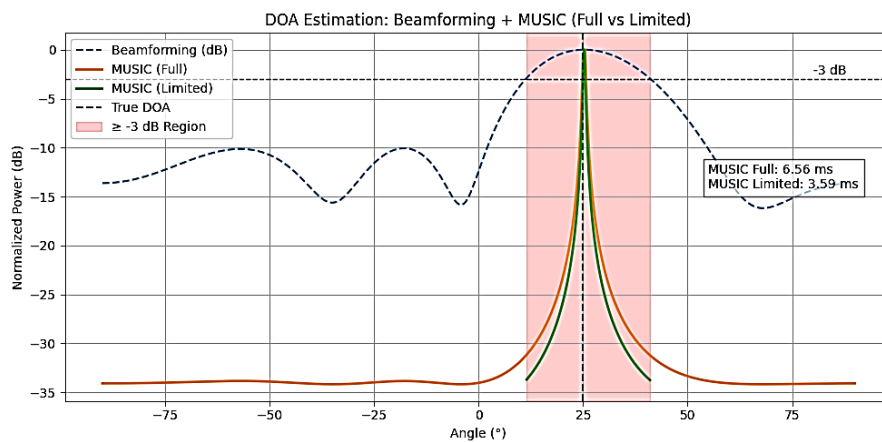
In a digital beamforming architecture, the signals from each antenna element, after ADC, are multiplied by weighting coefficients  $w_i$  and summed, as expressed by the following equation:

$$Y(\psi) = \sum_{i=1}^M w_i x_i(t) \quad (14)$$

This process enables the system to rapidly obtain an initial estimate of the angle of arrival, also referred to as coarse DOA estimation. This estimated information is then used to restrict the search space of the MUSIC algorithm to a narrow angular region around the estimated value, rather than across the entire domain. The narrowed scanning region is determined based on a 3 dB spectral power threshold, corresponding to directions where the output beamforming energy falls no more than 3 dB below its peak. This enhancement provides the algorithm with greater practical applicability compared with conventional MUSIC, which requires dense angular spectrum scanning.

A simulation is conducted using a four-element array with inter-element spacing  $d=\lambda/2$ , a source operating at a center frequency of 1.6 GHz, and an angular scanning range from  $-90^\circ$  to  $90^\circ$ . The results obtained from the full-range conventional MUSIC algorithm and the proposed improved MUSIC algorithm are presented in figure 3.

As observed in figure 3, the conventional MUSIC algorithm, when scanning over the entire angular range from  $-90^\circ$  to  $90^\circ$ , produces a very sharp peak in the power spectrum at the true DOA. This clearly demonstrates the high resolution capability of MUSIC under conditions of strong signals and moderate noise levels.



**Figure 3.** Simulation of the conventional MUSIC algorithm versus the improved MUSIC algorithm.

To reduce computational complexity and shorten processing time, an improved version of the algorithm is employed, in which the scanning region is restricted to within -3 dB around the spectral peak. This range is illustrated by the pink-shaded area in the graph, indicating the angular sector with relatively high power ( $\geq -3$  dB) surrounding the maximum. When the MUSIC scan is performed solely within this region, the resulting DOA estimates are nearly identical to those obtained from a full-range scan, while the processing time is reduced by almost 50%, from 6.56 ms to 3.59 ms.

Thus, constraining the scanning region based on the -3 dB threshold allows the MUSIC algorithm to maintain its estimation accuracy while optimizing performance, making it well-suited for real-time DOA tracking applications or embedded systems with limited resources.

## 2.2. Development of a real-time DOA tracking model using the improved MUSIC algorithm combined with a second-order Kalman filter

To smooth the instantaneous estimates obtained from the improved MUSIC algorithm and enable real-time tracking of the direction of arrival, this paper proposes the application of a second-order Kalman filter. This is a linear filter that optimizes estimation errors according to the minimum mean square error (MMSE) criterion, making it well-suited for systems with linear dynamics and Gaussian measurement noise.

The system state at time step  $k$  is represented by the vector:

$$\mathbf{x}(k) = \begin{bmatrix} \theta(k) \\ \dot{\theta}(k) \end{bmatrix} \quad (15)$$

where  $\theta(k)$  denotes the DOA value at time  $k$  and  $\dot{\theta}(k)$  represents the rate of change of the DOA. It is assumed that the angular velocity  $\dot{\theta}(k)$  remains approximately constant over short time intervals, which aligns with the practical observation that the signal variation is smooth.

The discrete-time linear state transition model is given by:

$$\mathbf{x}(k+1) = \mathbf{F} \cdot \mathbf{x}(k) + \mathbf{w}(k) \quad (16)$$

where  $\mathbf{F} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$ , with  $T=1$  second representing the sampling interval, and  $\mathbf{w}(k) \sim \mathcal{N}(0, Q)$  is the process noise, modeling uncertainties in the motion.

At each time step  $k$ , MUSIC provides a measurement of the estimated DOA, denoted as  $z(k)$ . The measurement model is expressed by the equation:

$$\mathbf{z}(k) = \mathbf{H} \cdot \mathbf{x}(k) + \mathbf{v}(k) \quad (17)$$

where  $\mathbf{H}=[1 \ 0]$  indicates that only the angle  $\theta$  is measured, while the rate of change of the angle is not directly observed.  $\mathbf{v}(k) \sim \mathcal{N}(0, Q)$  represents the Gaussian measurement noise.

The process noise (discrete white noise) is given by:

$$\mathbf{Q} = \mathbf{Q}_{white} = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix} \cdot \sigma^2, \text{ với } \sigma^2 = 0.5 \quad (18)$$

At each time frame gian  $k$ , the Kalman filter is executed through the following steps:

### Prediction:

$$\hat{\mathbf{x}}^-(k) = \mathbf{F} \cdot \hat{\mathbf{x}}(k-1) \quad (19)$$

$$\mathbf{P}^-(k) = \mathbf{F} \cdot \mathbf{P}(k-1) \cdot \mathbf{F}^T + \mathbf{Q} \quad (20)$$

Update with measurement  $z(k)$ :

$$\mathbf{K}(k) = \mathbf{P}^-(k) \cdot \mathbf{H}^T \cdot (\mathbf{H} \cdot \mathbf{P}^-(k) \cdot \mathbf{H}^T + \mathbf{R})^{-1} \quad (21)$$

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}^-(k) + \mathbf{K}(k) \cdot (\mathbf{z}(k) - \mathbf{H} \cdot \hat{\mathbf{x}}^-(k)) \quad (22)$$

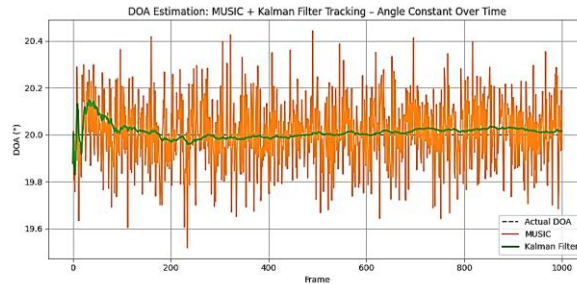
$$\mathbf{P}(k) = (\mathbf{I} - \mathbf{K}(k) \cdot \mathbf{H}) \cdot \mathbf{P}^-(k) \quad (23)$$

## 3. RESULTS AND DISCUSSION

### 3.1. Simulation

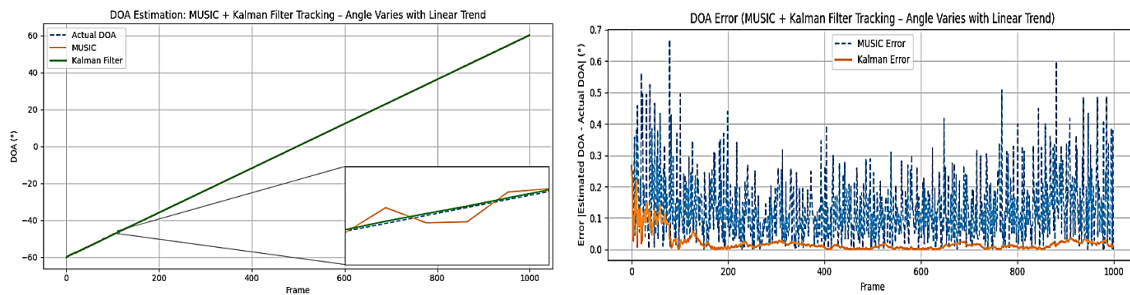
A simulation is conducted with a two-element antenna array over 1000 iterations, each consisting of 200 snapshots. The incident signal is simulated with an SNR of 10 dB at a center frequency of  $f = 1.6$  GHz.

a. Simulation of the signal source at a fixed angle of  $20^\circ$



**Figure 4.** DOA tracking performance under constant angle of arrival.

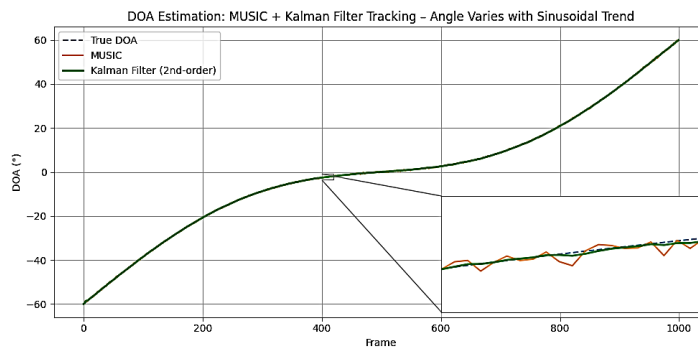
b. Simulation of a signal source with a linearly time-varying angle from  $-60^\circ$  to  $60^\circ$ . The second-order Kalman filter provides the results shown in figure 5



**Figure 5.** a) Simulation of linear DOA variation tracking using MUSIC and the Kalman filter. b) Estimation error between the tracked DOA and the true value during the MUSIC and Kalman filter tracking process.

c. Simulation of a signal source with a sinusoidally time-varying angle from  $-60^\circ$  to  $60^\circ$ . The second-order Kalman filter provides the results shown in figure 6

An examination of the simulation results for the stationary source, linearly varying DOA, and harmonically oscillating DOA presented in figures 4, 5, and 6, respectively, reveals that the MUSIC algorithm provides accurate instantaneous estimates but is highly sensitive to noise. In contrast, the second-order Kalman filter demonstrates excellent smoothing capability, significantly reducing errors and fluctuations in the DOA estimates. The second-order Kalman filter maintains stable results, closely follows the true trend, and is well-suited for real-time implementation. Therefore, this approach is appropriate for applications in localization systems, phased array radar, and target tracking.



**Figure 6.** Simulation of DOA tracking when the DOA varies following a linear trend with superimposed harmonic oscillations.

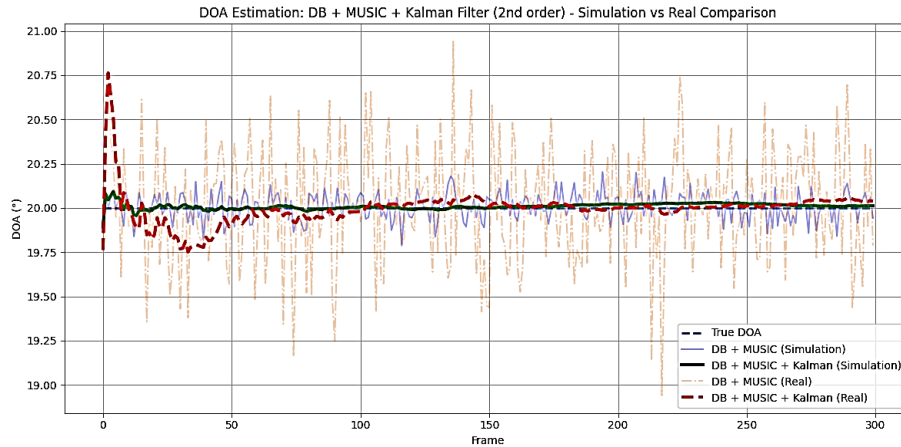
### 3.2. Implementation of the improved MUSIC algorithm combined with a Kalman filter on a microstrip antenna array and SDR

To experimentally capture signals and perform real-time DOA estimation, the system was implemented on an SDR hardware platform. The SDR enables acquisition of raw RF signals from the antenna array and facilitates complete processing in software, thereby allowing seamless integration with the DOA estimation algorithms and the Kalman filter.

**Hardware setup:**

- **SDR receiver:** bladeRF, supporting RF spectrum acquisition over a frequency range from 325 MHz to 3.8 GHz, with a sampling rate up to 40 MSPS, featuring two independent receive channels and capable of internal to ensure phase consistency between channels.
- **Antenna array:** two microstrip patch antenna elements arranged in a ULA configuration with spacing  $d = \lambda/2$  at the frequency  $f_c = 1.6$  GHz.
- **Signal processing computer:** an MSI laptop equipped with an 11th-generation Intel® Core™ i7-11800H processor at 2.30 GHz, 16 GB RAM, running Windows 11.
- **Continuous wave transmitter operating at 1.6 GHz.**

The experimental results with the transmitter positioned at a distance of 600 m are shown in figure 10 and table 1.



**Figure 7.** Results of implementing the algorithmic model using software-defined radio (SDR) and microstrip patch antennas.

**Table 1.** Results of the algorithmic model for DOA measurement at 20 degrees using both simulation and implementation with software-defined radio and microstrip patch antennas.

Frame	Actual DOA (°)	DB + MUSIC (°) (simulated)	DB + MUSIC + Kalman (°) (simulated)	Error simulated (°)	DB + MUSIC (°) (real)	DB + MUSIC + Kalman (°) (real)	Error real (°)
0	20.00	19.89	19.89	0.11	19.76	19.76	0.24
1	20.00	20.09	20.08	0.08	20.48	20.43	0.43
2	20.00	20.00	20.05	0.05	20.70	20.76	0.76
3	20.00	20.06	20.07	0.07	20.38	20.63	0.63
4	20.00	20.08	20.09	0.09	20.28	20.51	0.51
5	20.00	19.99	20.05	0.05	19.94	20.26	0.26
6	20.00	20.05	20.06	0.06	20.25	20.26	0.26
7	20.00	19.89	19.89	0.11	19.76	19.76	0.24

As shown in figure 7 and table 1, the MUSIC algorithm, when applied in a simulated environment (SNR = 10 dB), is able to estimate the angle of arrival at 20 degrees with reasonable accuracy, although some fluctuations remain due to noise. In contrast, when implemented on actual hardware, the estimation error of the MUSIC algorithm increases significantly, reflecting the impact of real-world noise on the algorithm's accuracy.

When incorporating the second-order Kalman filter, both in simulation and in real-world experiments, a significant reduction in estimation error is observed. Particularly in practical environments, the combination of the MUSIC algorithm with the second-order Kalman filter demonstrates effective smoothing and temporal prediction, resulting in more stable outcomes compared to using MUSIC alone.

Although a slight discrepancy between the mean errors in simulation and in practice remains, it is relatively small, indicating that the simulation model closely reflects actual deployment conditions. This confirms that the combined MUSIC and second-order Kalman filtering approach is effective, stable, and well-suited for practical implementation using SDR and microstrip patch antennas.

Moreover, the MUSIC algorithm integrated with beamforming offers a balanced solution between computational speed and accuracy, making it highly suitable for embedded systems or applications requiring rapid processing. Experimental results further reveal that when combined with the second-order Kalman filter, the system's capability for real-time directional tracking is markedly enhanced, owing to the Kalman filter's ability to predict and smooth the estimated values. The Kalman filter effectively suppresses minor noise or sudden fluctuations in the DOA outputs of MUSIC, particularly under high-noise conditions or when the DOA changes gradually over time.

#### 4. CONCLUSIONS

This study has presented an efficient enhanced model for direction-of-arrival (DOA) estimation, which integrates an initial coarse angular scan via beamforming with the high-resolution MUSIC algorithm, followed by a second-order Kalman filter for real-time DOA tracking. Experimental results demonstrate that the proposed algorithmic model achieves an optimal balance between estimation accuracy and computational efficiency.

Specifically, employing beamforming to roughly determine the angular sector effectively constrains the search space of the MUSIC algorithm, thereby significantly reducing processing time without compromising estimation accuracy. Additionally, the second-order Kalman filter smooths and stabilizes the DOA tracking trajectory over time, proving particularly effective under conditions of fluctuating signals or noise. Comparative analyses indicate that the improved MUSIC processing chain combined with the Kalman filter yields results nearly identical to those of the full-range MUSIC approach, while reducing computation time by more than 50%, thus meeting the demands of real-time, high-speed tracking applications and mobile embedded systems.

This model is particularly well-suited for hardware platforms with limited resources, such as systems employing SDR, embedded microprocessors, compact radar devices, or radio-based localization equipment.

#### REFERENCES

- [1]. Zekavat, R., and R. M. Buehrer, "*Handbook of Position Location: Theory, Practice and Advances*". Wiley, (2005). ISBN 9780471656403.
- [2]. Ali, H. A. H., and S. Seytnazarov, "*Human walking direction detection using wireless signals, machine and deep learning algorithms*," *Sensors*, Vol. 23, No. 24, (2023). <https://doi.org/10.3390/s23249647>
- [3]. Cahl, C. B., S. A. Hausman, and R. M. Headrick, "*Comparison of Beamscan, Beamforming, and MUSIC Algorithms for HF Radar Direction Finding*". NOAA Technical Memorandum OAR-PMEL-164, (2023). <https://repository.library.noaa.gov/view/noaa/60117>

- [4]. Khumane, D., and S. Jagade, "Performance Evaluation of DOA Estimation Using MUSIC Algorithm and LMS Beamforming Algorithm," International Journal of Engineering Research & Technology (IJERT), Vol. 8, No. 9, pp. 1–5, (2019). <https://www.ijert.org/performance-evaluation-of-doa-estimation-using-music-algorithm-and-lms-beamforming-algorithm>
- [5]. Vocal Technologies Ltd., "Adaptive Beamforming Using MUSIC Pseudo-Spectrum," Vocal.com, (2018). <https://vocal.com/beamforming-2/adaptive-beamforming-using-music-pseudo-spectrum>
- [6]. Huang, H., S. Salous, and J. McGeehan, "Hardware analysis for Capon and MUSIC DOA estimation algorithms," in Proc. IEEE Antennas Propag. Soc. Int. Symp., (2009). <https://doi.org/10.1109/APS.2009.5171760>
- [7]. Salama, A. A., "Direction of Arrival Estimation: A Tutorial Survey of Classical and Modern Methods," arXiv preprint arXiv:2508.11675. (2025). <https://arxiv.org/abs/2508.11675>
- [8]. Madhava, V. M., S. N. Jagadeesha, and T. Yerriswamy, "A Comparative Study of DOA Estimation Algorithms with Application to Tracking Using Kalman Filter," Signal & Image Processing: An International Journal (SIPIJ), pp. 16–17, (2015).

### TÓM TẮT

#### **Nghiên cứu xác định và theo dõi hướng đến của nguồn tín hiệu vô tuyến biến đổi nhanh**

*Việc định vị chính xác các mục tiêu động trong môi trường phức tạp và thay đổi nhanh, chẳng hạn như máy bay không người lái (UAV), nền tảng di động hoặc các vật thể chuyển động không ổn định, đặt ra những thách thức đáng kể trong cả ứng dụng dân sự và quân sự. Bài báo này đề xuất một phương pháp tích hợp vô tuyến được xác định bằng phần mềm (SDR) với ăng-ten vi dải phân cực tròn để nâng cao hiệu suất định vị trong các điều kiện khắc nghiệt, bao gồm tín hiệu yếu, lan truyền đa đường và nhiễu động do chuyển động nhanh. Một mô hình mô phỏng được phát triển và phân tích thực nghiệm được thực hiện bằng cách sử dụng thuật toán MUSIC cải tiến để ước tính hướng đến (DOA), kết hợp với kỹ thuật lọc bậc hai. Kết quả cho thấy mối tương quan chặt chẽ giữa dữ liệu mô phỏng và các phép đo thực tế, xác nhận tính hiệu quả của phương pháp được đề xuất.*

**Từ khóa:** Anten vi dải phân cực tròn; Thuật toán MUSIC; Thuật toán ESPRIT; DOA; SDR; Bộ lọc Kalman.