Design and optimization of axial length in a -1× magnification image-inverting optical system

Lai Thanh Tuan, Dang Thanh Dat, Nguyen Thai Dung,
Nguyen Quang Hiep, Le Van Nhu*

Le Quy Don Technical University, 236 Hoang Quoc Viet, Nghia Do, Hanoi, Vietnam.
*Corresponding author: levannhuktq@gmail.com

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ABSTRACT

This paper focuses on optimizing the axial size of a -1x magnification image-inverting optical system. The objective is to shorten the distance between the object plane and the image plane. This is achieved by shifting the rear principal plane forward and moving the front principal plane backward. An inverting lens system with a focal length of 40.48 mm has been designed, achieving an object-to-image plane distance of 59.5 mm. The configuration consists of four singlet lenses, utilizing spherical surfaces in the design. The optical system achieves high image quality across the entire field of view, with MTF values exceeding 0.1 at a spatial frequency of 56 lp/mm.

Keywords: Optical system design; Lens-based image inversion system; Optical system optimization.

1. INTRODUCTION

The -1^x image-inverting optical assembly is an essential component in many optical systems such as microscopes, industrial cameras, and precision measurement devices. Designing this optical assembly requires balancing the physical size of the system with the quality of the output image [1, 2]. One of the key objectives is to shorten the distance between the object plane and the image plane, thereby reducing the overall size of the device while maintaining high accuracy [3, 4]. The challenge lies in adjusting the positions of the principal planes without altering the focal length or introducing image distortion [5]. Previous studies have proposed various methods of system adjustment by shifting either the front or rear principal plane to optimize the optical path length [6, 7]. This paper further develops that approach to achieve an optimal solution for the $-1^\times$ image-inverting optical assembly.

Adjusting the rear principal plane forward and shifting the front principal plane backward shortens the distance between the object plane and the image plane while maintaining stable magnification [8]. The optical system, designed with a focal length of 40.48 mm, achieved an ideal object-to-image distance of only 59.5 mm, significantly reducing the system size compared to traditional designs [1, 3]. This configuration consists of four singlet spherical lenses, carefully selected to ensure manufacturing simplicity and effective correction of optical aberrations. The use of singlet lenses also reduces cost and enhances feasibility for practical applications [4, 7]. The image quality obtained from the system was verified to exhibit high sharpness and uniform brightness across the entire field of view, meeting stringent requirements for resolution and contrast [2, 6]. This represents an important result that broadens the potential applications of the -1^x image-inverting optical assembly in compact optical devices.

Design and optimization strategies for image-inverting optical assemblies have been widely investigated over the past decades, especially with respect to focal length optimization and the suppression of optical aberrations [5, 9]. Striking a balance between physical constraints such as size and weight and the demand for high image quality has consistently been a significant challenge in applied optics [3]. In many cases, researchers emphasize the development of systems with -1x magnification to maintain the true object-to-image scale while avoiding distortion [6, 10].

Among the governing parameters, the spacing between the object plane and the image plane plays a decisive role, as it directly affects the total length of the optical assembly. In this work, a novel strategy is proposed to shift the principal planes, thereby achieving an optimized system size without compromising image quality. Experimental validation confirms both the practicality and the effectiveness of the proposed approach.

The use of a four-singlet spherical lens configuration in the design not only reduces optical aberrations but also streamlines fabrication and system alignment processes [1, 4]. A focal length of 40.48 mm was chosen as a compromise between compact system dimensions and the ability to deliver high image quality. The lenses were positioned carefully to minimize aberrations and preserve sharpness throughout the full field of view [7]. The system demonstrated excellent imaging performance with consistent sharpness and minimal distortion, which is particularly important for precision-demanding applications such as metrology and product quality inspection [8, 9]. In addition, shortening the separation between the object plane and the image plane facilitates the integration of the system into instruments with restricted space. Consequently, this work presents promising opportunities for broader adoption in the optical industry and medical instrumentation [10].

In this study, the paper presents the optimization of the size of a -1^x image-inverting optical assembly, with the objective of shortening the distance between the object plane and the image plane through adjustment of the principal plane positions. The system, configured with four singlet spherical lenses and a focal length of 40.48 mm, achieved an object-to-image distance of 59.5 mm while maintaining high image quality across the entire field of view. This result is significant for the design of compact and efficient optical systems that meet the increasing demands of modern optical devices. The study also provides both theoretical and experimental foundations for practical applications in various industrial and medical fields. In the next steps, this method can be further extended and optimized to adapt to different types of optical systems, thereby contributing to the advancement and diversification of optical technology.

2. METHOD

In this paper, we study and design an image-inverting optical system with a magnification of -1^x, aiming to reduce the longitudinal size. The technical specifications of the image-inverting optical system are presented in table 1.

Design concept: Typically, an image-inverting system with a magnification of –1x and a focal length of 40.48 mm will have an object-to-image distance equal to four times the focal length, i.e., 161.92 mm, under the ideal condition when the two principal planes coincide. However, in practice, the use of real lenses instead of ideal thin lenses usually increases this distance, thereby enlarging the axial length of the optical system. This becomes disadvantageous in applications that require compact and space-efficient optical assemblies. In this paper, we propose a design solution for the image-inverting system by adjusting the positions of the principal planes. Specifically, the rear principal plane is shifted forward, while the front principal plane is moved backward. This configuration significantly reduces the object-to-image distance. As a result, the axial size of the system employing the lens-based image inverter is minimized. This solution enhances the feasibility of integrating the image-inverting assembly into modern optical devices with limited space.

Table 1. *Technical specifications of the magnification image-inverting optical assembly.*

Focal length of an image-inverting lens group	40,48 mm
Magnification	-1^x
Operating spectral range	0.48-0.65 μm
Image size	4.017 mm

Design: The design process was carried out in the following sequence. First, an initial optical system was constructed to satisfy the basic requirements of field of view, resolution, and operational wavelength range. Next, the designer selected an appropriate merit function for optimization and defined the design variables, including surface curvatures, inter-surface distances, and lens materials. The optimization process was then guided by suitable algorithms to enhance the system's performance. The outcome of this process is an optical system with final structural parameters that meet the technical requirements, such as focal length, axial size, and image quality. The detailed steps of the design procedure will be presented in the following sections.

In a $-1^{\times}$ magnification image-inverting optical system, materials are strategically distributed to ensure high optical performance while minimizing the axial length. The system primarily employs two types of glass: LZ_TF10 and LZ_K8**.** LZ_TF10, a high-refractive-index and low-dispersion glass, is positioned at both ends of the system to accurately guide rays and shape the light beam. In contrast, LZ_K8, which exhibits higher dispersion, is placed in the central group to balance chromatic aberrations. The alternating use of these two materials not only provides effective chromatic aberration correction but also enables a more compact optical design. Such material distribution optimizes image quality across the entire field of view without increasing the number of lenses, representing a key strategy for achieving an efficient and structurally compact image-inverting system.

During the refinement of the optical system, two key parameters were selected for optimization: the curvature of optical surfaces and the spacing between elements. Adjusting the curvature of each lens surface allows precise control of light convergence, thereby significantly improving the correction of aberrations such as spherical aberration, astigmatism, and image distortion. At the same time, controlling the spacing between lens surfaces influences the position of the focal plane and the point spread function, enabling more effective ray path coordination. Both variables were incorporated into the optimization algorithm and processed automatically by the software to minimize overall error. Practical manufacturing constraints, such as the maximum achievable curvature or the available physical space between lens groups, were also integrated as boundary conditions in the computation. Proper identification of these critical design variables not only shortens the calculation time but also ensures that the optical system configuration operates efficiently.

When establishing the merit function for the optimization process, assigning appropriate weights to each criterion plays a crucial role in the resulting image quality. Fields of view that are more sensitive to errors or directly affect sharpness are typically assigned greater weights to prioritize correction. These weights can be adjusted over multiple iterations to accurately reflect the relative importance of each parameter at different optimization stages. Instead of maintaining the initial configuration, the software guides adjustment steps toward minimizing the dominant aberrations. Iteratively refining the weighting scheme in combination with modifying the key design variables gradually brings the system closer to an ideal optical model. Each computational cycle contributes incremental improvements to the system structure, leading to stable and rapid convergence. With this method, the optical system not only achieves high resolution but also maintains uniform image quality across the entire field of view.

The optical system in table 2 was designed with a magnification of -1^{x}, indicating the goal of transferring a true-to-scale image from the object to the image plane. The specification table consists of nine optical surfaces, comprising two main lens groups that employ LZ_TF10 and LZ_K8 materials for chromatic aberration correction and image quality optimization. The first group consists of a single lens, while the second group comprises three lenses. The first group functions to narrow the light beam, whereas the second group works in conjunction with the first lens to correct aberrations, thereby achieving a high-quality optical system. The radii of curvature range from ± 10 mm to ± 86 mm, reflecting the combination of converging and diverging lenses. The effective diameters, ranging from 4 mm to 10 mm, indicate a medium aperture suitable for precision applications such as microscopy or digital imaging modules. The final surface has an infinite radius of curvature, representing a flat image plane, while the last distance of 17.5 mm serves as the back focal length. This design ensures accurate image transfer and reduces optical aberrations within a short working distance at unit magnification. Figure 1 illustrates the layout of the optimized image-inverting optical system.

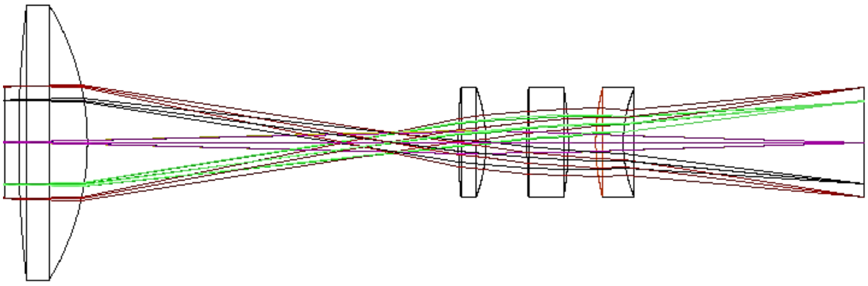


Figure 1. *Optimized image-inverting optical system.*

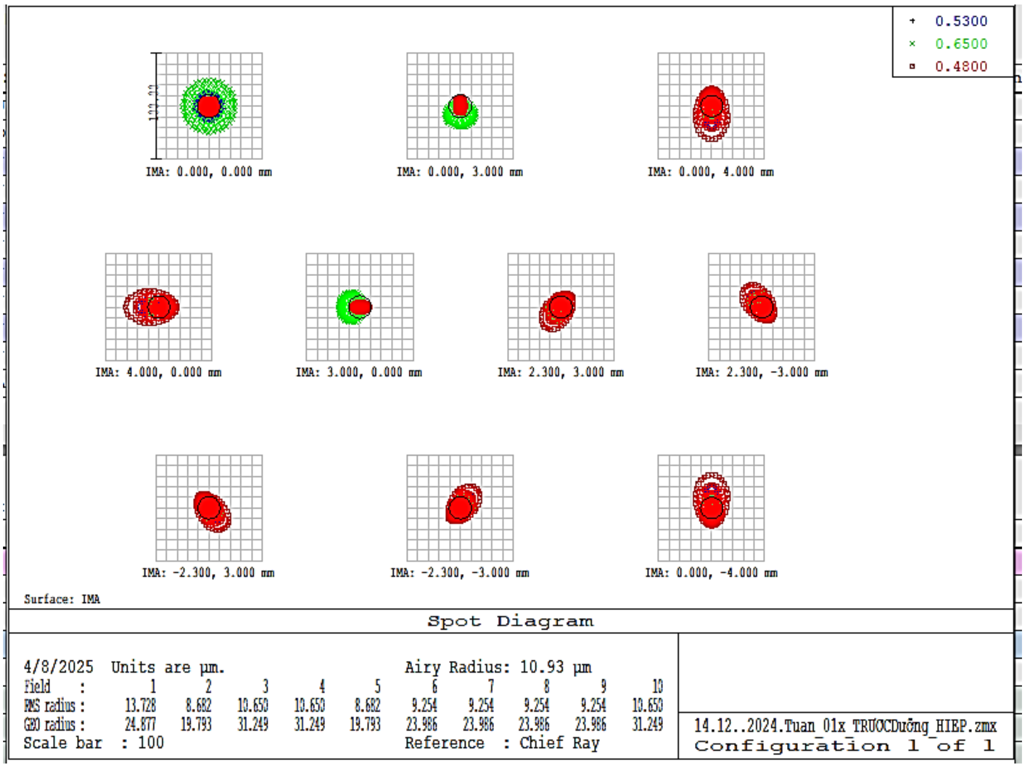
Table 2. *Structural parameters of the image-inverting optical system.*

Radius	Thickness	Glass	Semi-Diameter
86.535	5.000	LZ_TF10	10.00
-19.30	27.000		10.00
42.10	2.000	LZ_K8	4.000
-11.67	3.000		4.000
79.80	3.000	LZ_K8	4.000
-23.00	2.000		4.000
16.261	2.000	LZ_TF10	4.000
10.28	17.500		4.000

3. RESULTS AND DISCUSSION

The spot size diagram of the -1^x magnification relay optical assembly is shown in figure 2. As observed in figure 2, the root mean square (RMS) spot sizes corresponding to different field points range from 8.6 to 13.7 μm, meaning that the largest spot size is only about 1.5 times larger than the Airy disk. For the requirement of transferring the image formed by the microscope objective onto the CCD detector plane, the imaging quality of the relay assembly must approach the diffraction limit to ensure that the image quality provided by the objective lens is not significantly degraded after passing through this relay system. Therefore, with the obtained average spot size, the system basically meets the specified requirements.

As shown in figure 3, the optimized relay optical assembly with a magnification of -1^x demonstrates a relatively high capability of transferring modulation from the object to the image. Within the spatial frequency range from 0 to 56 line pairs/mm, the MTF values remain above 0.1 at almost all field points. With such an MTF performance, the resolution capability of the relay optical system is relatively high and essentially meets the design requirements. Furthermore, as indicated by the plot in figure 3, the MTF of the optimized 1x relay optical system is quite close to that of the diffraction-limited optical system.



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4/8/2025  Units are µm.                     Airy Radius: 10.93 µm
Field      :      1       2       3       4       5       6       7       8       9      10
RMS radius :   13.728   8.682  10.650  10.650   8.682   9.254   9.254   9.254   9.254  10.650
GEO radius :   24.877  19.793  31.249  31.249  19.793  23.986  23.986  23.986  23.986  31.249
Scale bar  : 100                             Reference  : Chief Ray
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Figure 2. *Spot sizes corresponding to different field points range from 8.6 to 13.7 µm.*

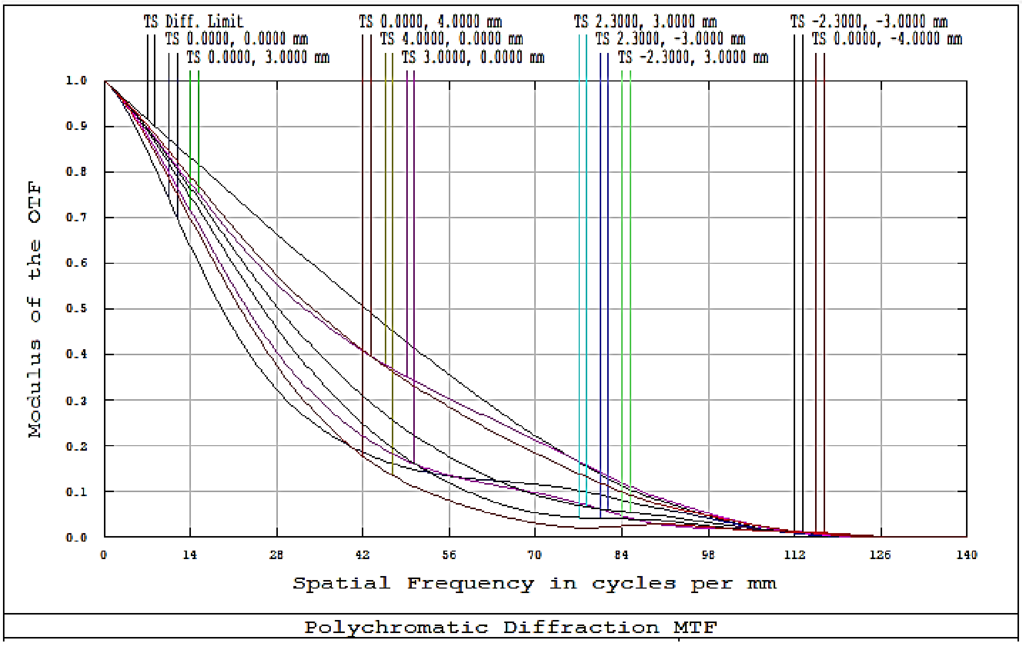

Figure 3. *Optimized relay optical assembly with a magnification of -1^x.*

The energy distribution diagram in the image spot provides information on how the energy is distributed within the spot size across different field points. From figure 4, it can be observed that 70% of the image energy is concentrated within a radius of 12 µm or larger for most object points within the field of view of the optical system. When compared with the pixel size of the detector used for image acquisition, such spot sizes are sufficient to ensure the required imaging quality.

Furthermore, as shown in figure 4, the energy concentration distribution of the initial -1^x relay optical system remains relatively close to that of a diffraction-limited optical system.

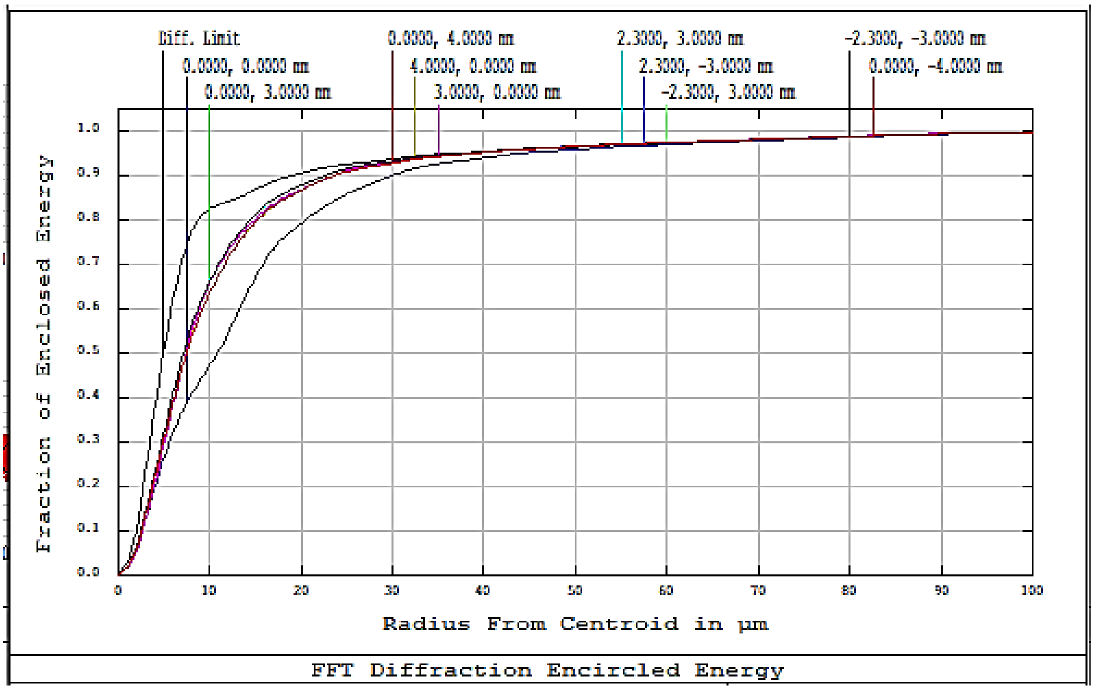

Figure 4. *FFT diffraction encircled energy.*

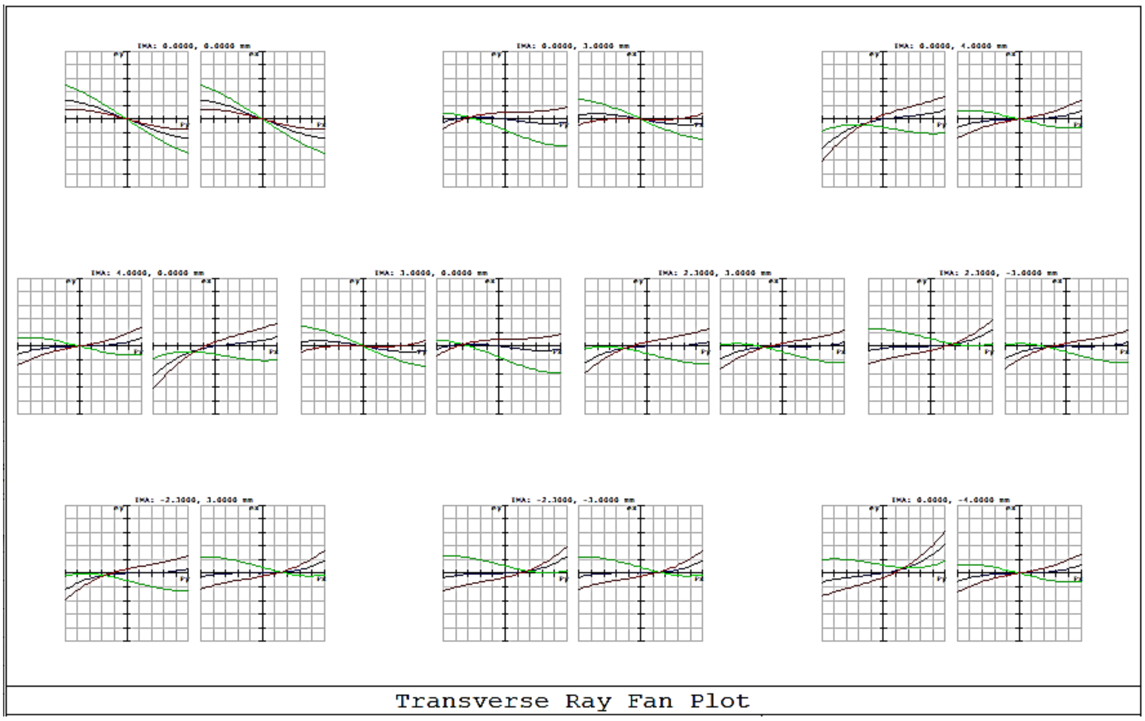

Figure 5. *Transverse ray fan plot.*

Figure 5 illustrates the ray aberration diagram of the optimized relay optical system with a magnification of -1^x. It is observed that the ray aberrations at different field points remain within the interval of 20 to 30 µm, thereby satisfying the imaging requirements of the optical configuration. The diagrams of field curvature and distortion are also crucial for assessing the imaging capability of the system, especially for optical designs that project images onto detectors with a flat photosensitive surface, such as CCD or CMOS sensors.

Figure 6 illustrates the field curvature and distortion of the optimized relay optical system with a magnification of -1^x. It can be observed that the field curvature of the optimized system is relatively small (on the order of 1 mm), which is compatible with a flat image plane. The distortion level is also very low, around 1%, ensuring a high degree of uniformity between the image captured through the objective lens and the image displayed on the screen.

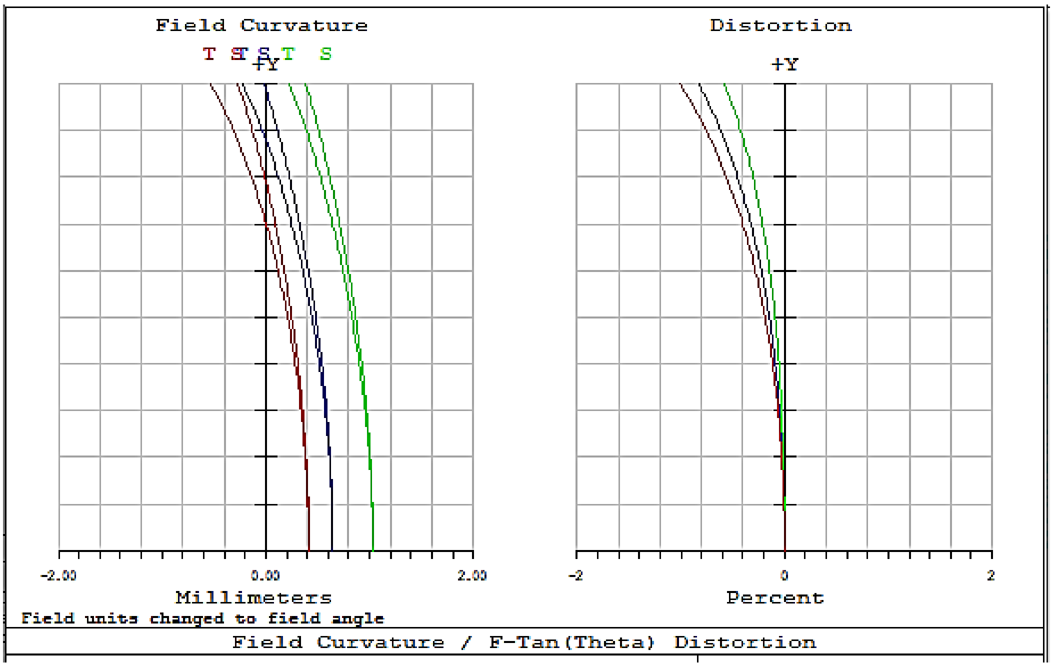

Figure 6. *Field curvature and distortion of the optimized relay optical system.*

4. CONCLUSIONS

This paper presented the optimization of a -1^x relay optical system with the objective of reducing the axial length, thereby enhancing compactness and integrability into optical devices. By adjusting the positions of the principal planes - shifting the rear principal plane forward and the front principal plane backward, the distance between the object and image planes was shortened to 59.5 mm, compared to the system focal length of 40.48 mm. The optical configuration employs four singlet spherical lenses, ensuring simplicity in both fabrication and assembly. The optimization results demonstrate that the system achieves high image quality across the full field of view, meeting stringent requirements for resolution and image fidelity. This design provides an effective approach for applications demanding compact, precise, and practically deployable relay optics.

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TÓM TẮT

Thiết kế và tối ưu hóa chiều dài dọc trục trong hệ thống quang học đảo ảnh có độ phóng đại -1^x

Bài báo này tập trung nghiên cứu việc tối ưu kích thước dọc trục của hệ đảo ảnh có độ phóng đại -1^x*. Mục tiêu là rút ngắn khoảng cách từ mặt phẳng vật đến mặt phẳng ảnh. Điều này được thực hiện bằng cách điều chỉnh mặt phẳng chính sau tiến lên và mặt phẳng chính trước lùi lại. Hệ thống thấu kính đảo ảnh với tiêu cự 40,48 mm đã được thiết kế, đạt khoảng cách từ mặt phẳng ảnh đến mặt phẳng vật là 59,5 mm. Cấu hình gồm bốn thấu kính đơn, và sử dụng các bề mặt cầu để thiết kế. Hệ thống quang học đạt chất lượng tốt trên toàn thị giới, giá trị hàm MTF tại 56 vạch/mm đều lớn hơn 0.1.*

Từ khóa: Thiết kế hệ thống quang học; Hệ đảo ảnh thấu kính; Tối ưu hóa hệ quang.