Smooth robust adaptive sliding mode control of 2-DOF helicopter system

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ABSTRACT

This paper investigates the development of a tracking control system for a 2-degree-of-freedom (2-DOF) helicopter under parameter uncertainties and time-varying external disturbances. A nonlinear mathematical model of the 2-DOF helicopter system is first established. Subsequently, a conventional sliding mode controller (SMC) is designed, and its limitations in handling parameter uncertainties and external disturbances are analyzed. To address these limitations, the paper proposes a smooth, robust adaptive sliding mode controller for the 2-DOF helicopter system, ensuring convergence of closed-loop signals. The robust stability of the proposed controller is rigorously proven using Lyapunov stability theory. The system is simulated in MATLAB/Simulink with both the conventional and proposed controllers. Comparative simulation results demonstrate that the proposed adaptive robust controller outperforms the conventional sliding mode controller and fully meets the control requirements.

Keywords: 2-DOF helicopter; Nonlinear system; Robust adaptive sliding mode control; Smooth function; Parameter uncertainty; Time-varying disturbances.

1. INTRODUCTION

Helicopters are critical in transportation, logistics, firefighting, rescue, and geological surveys due to their vertical takeoff/landing capability, hovering stability, cost-effectiveness, and runway independence [1]. However, helicopter control presents significant challenges: strong nonlinear dynamics with cross-channel coupling, complex MIMO structure, and performance degradation from parameter uncertainties and external disturbances. These factors motivate extensive research in robust adaptive control for uncertain nonlinear helicopter systems.

Various control strategies such as sliding mode, backstepping, adaptive, robust nonlinear, and disturbance observer-based have been developed to mitigate uncertainties while ensuring stability and tracking accuracy. In Vietnam, researchers have contributed notably: adaptive sliding mode with fuzzy neural networks for 2-DOF helicopters [2]; sliding mode guidance achieving superior performance over PI controllers [3]; robust H∞ control for linearized models [4]; and invariant manifold-based nonlinear control outperforming classical backstepping [5]. International contributions include adaptive backstepping with neural networks ensuring bounded signals under input constraints [6, 7]; adaptive sliding mode demonstrating robustness under uncertainties and saturation [8, 9]; nonlinear sliding mode with observers providing asymptotic tracking superior to PID [10].

Current approaches typically rely on either linearized models or precisely known parameters. Adaptive methods address only partial uncertainties or employ computationally intensive neural network approximations, while many ignore chattering phenomena. To address these limitations, this paper proposes a smooth robust adaptive sliding mode controller. The method transforms the helicopter model into pseudo-linear parameterized form, assuming slowly time-varying uncertain parameters consistent with practical systems. Key advantages include: resolving design limitations in existing studies without complex neural network approximations (significantly reducing computational burden), mitigating chattering effects, and maintaining required control accuracy.

The controller compensates for time-varying disturbances by selecting a sliding gain exceeding the disturbance upper bound, functioning as a robust compensator in the adaptive law. Potential chattering from large gains is eliminated via smooth switching functions.

2. DESCRIPTION OF THE 2-DOF HELICOPTER SYSTEM

The 2-DOF helicopter model consists of a rigid beam mounted on a support base, enabling free rotation in both the horizontal (yaw) and vertical (pitch) planes. The system features two rotors (main rotor and tail rotor) positioned symmetrically at opposite ends of the beam, with each rotor independently driven by a variable-speed DC motor. When the pitch motor is supplied with voltage V_p, the rotational speed generates a force F_p acting perpendicular to the body at a distance r_p from the pitch axis. Consequently, the pitch angle θ changes about the y-axis. Similarly, the yaw angle ψ changes about the z-axis when voltage V_y is supplied to the yaw motor [8]. The structural diagram of the 2-DOF helicopter is shown in figure 1.

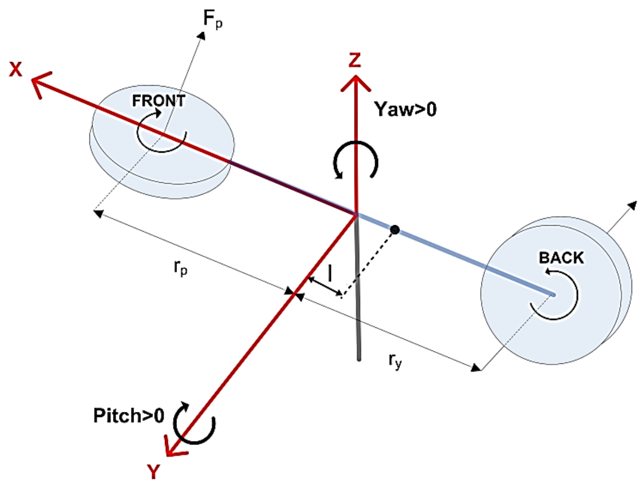


Figure 1. *Structural diagram of the 2-DOF helicopter [8].*

The 2-DOF helicopter dynamics, derived via Lagrange's principle, are described by the following fourth-order nonlinear system [6, 8]:

$$\begin{cases} \ddot{\theta} = \dfrac{K_{pp}V_p + K_{py}V_y - mgl\cos(\theta) - D_p\dot{\theta} - ml^2\dot{\psi}^2\sin(\theta)\cos(\theta)}{\left(J_p + ml^2\right)} + d_\theta \\ \ddot{\psi} = \dfrac{K_{yp}V_p + K_{yy}V_y - D_y\dot{\psi} + 2ml^2\dot{\psi}\dot{\theta}\sin(\theta)\cos(\theta)}{\left(J_y + ml^2\cos^2(\theta)\right)} + d_\psi \end{cases} \tag{1}$$

where: m – Helicopter mass; g – Gravitational acceleration; l – Distance from center of gravity to coordinate origin; J_p, D_p – Moment of inertia and damping coefficient of pitch axis; J_y, D_y – Moment of inertia and damping coefficient of yaw axis; K_{py} – Cross moment force coefficient of pitch axis from yaw rotor; K_{pp} – Moment force coefficient from pitch rotor; K_{yp} – Cross moment force coefficient of yaw axis from pitch rotor; K_{yy} – Moment force coefficient from yaw rotor; d_θ, d_ψ – Disturbances acting on pitch and yaw channels.

In the mathematical model, it is assumed that motor delay and actuator saturation are neglected.

Additionally, the current control loop dynamics are ignored due to their very small time constant. The friction effects on the system are incorporated into the disturbance term. The matrix-vector form is:

$$\ddot{\mathbf{x}} = \mathbf{f}_0\left(\mathbf{x},\dot{\mathbf{x}}\right) + \mathbf{g}_0\left(\mathbf{x},\dot{\mathbf{x}}\right)\mathbf{u} + \mathbf{d}\left(t\right), \tag{2}$$

Where:

$$\mathbf{x} = \begin{bmatrix}\theta & \psi\end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{2\times1};\ \mathbf{u} = \begin{bmatrix}V_p & V_y\end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{2\times1}$$

$$\mathbf{d}\left(t\right) = \begin{bmatrix}d_\theta\left(t\right) & d_\psi\left(t\right)\end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{2\times1}$$

$$\mathbf{f}_0\left(\mathbf{x},\dot{\mathbf{x}}\right) = \begin{bmatrix}\dfrac{-mgl\cos\left(\theta\right) - D_p\dot{\theta} - ml^2\dot{\psi}^2\sin\left(\theta\right)\cos\left(\theta\right)}{J_p + ml^2} & \dfrac{-D_y\dot{\psi} + 2ml^2\dot{\psi}\dot{\theta}\sin\left(\theta\right)\cos\left(\theta\right)}{J_y + ml^2\cos^2\left(\theta\right)}\end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{2\times1}$$

$$\mathbf{g}_0\left(\mathbf{x},\dot{\mathbf{x}}\right) = \begin{bmatrix}\dfrac{K_{pp}}{J_p + ml^2} & \dfrac{K_{py}}{J_p + ml^2} \\ \dfrac{K_{yp}}{J_y + ml^2\cos^2\left(\theta\right)} & \dfrac{K_{yy}}{J_y + ml^2\cos^2\left(\theta\right)}\end{bmatrix} \in \mathbf{R}^{2\times2}$$

$$\left\|\mathbf{d}\left(t\right)\right\| \le D \ \text{– Positive constant.}$$

The control objective is to design a controller such that $\mathbf{x}$ converges to the desired value $\mathbf{x}_d$ under parameter uncertainties and time-varying external disturbances.

3. SYNTHESIS OF 2-DOF HELICOPTER CONTROL SYSTEM

3.1. Design of conventional sliding mode control

Let $\mathbf{x}_d = \begin{bmatrix}\theta_d & \psi_d\end{bmatrix}^{\mathrm{T}}$ – Desired output vector; $\mathbf{e} = \mathbf{x} - \mathbf{x}_d = \begin{bmatrix}e_\theta & e_\psi\end{bmatrix}^{\mathrm{T}}$ – Tracking error vector; $\mathbf{s} = \mathbf{ce} + \dot{\mathbf{e}} = \begin{bmatrix}s_\theta & s_\psi\end{bmatrix}^{\mathrm{T}}$ – Sliding surface function; $\mathbf{c} = \mathrm{diag}\left(\begin{bmatrix}c_\theta & c_\psi\end{bmatrix}\right) > 0$ – Positive definite diagonal matrix. From (2) we obtain:

$$\dot{\mathbf{s}} = \mathbf{c}\dot{\mathbf{e}} + \ddot{\mathbf{e}} = \mathbf{f}_0 + \mathbf{g}_0\mathbf{u} + \mathbf{d} + \mathbf{c}\dot{\mathbf{e}} - \ddot{\mathbf{x}}_d \tag{3}$$

For a system with known parameters, the sliding mode control law is designed:

$$\mathbf{u} = -\mathbf{g}_0^{-1}\left(\mathbf{f}_0 + \mathbf{c}\dot{\mathbf{e}} - \ddot{\mathbf{x}}_d + \mathbf{Ks} + \eta\,\mathrm{sign}\left(\mathbf{s}\right)\right) \tag{4}$$

where: $\mathbf{K} = \mathbf{K}^{\mathrm{T}} \in \mathbf{R}^{2\times2}$ – Positive definite symmetric matrix; $\eta > D > 0$ – Positive constants; $\mathrm{sign}\left(\mathbf{s}\right) = \begin{bmatrix}\mathrm{sign}\left(s_\theta\right) & \mathrm{sign}\left(s_\psi\right)\end{bmatrix}^{\mathrm{T}}$.

Consider Lyapunov function: $V_1 = 0.5\mathbf{s}^{\mathrm{T}}\mathbf{s} > 0$. Using (2)-(4), we obtain:

$$\dot{V}_1 = \mathbf{s}^{\mathrm{T}}\dot{\mathbf{s}} = -\mathbf{s}^{\mathrm{T}}\mathbf{Ks} - \eta\left\|\mathbf{s}\right\| + \mathbf{s}^{\mathrm{T}}\mathbf{d} \le -\lambda_{\min}\left(\mathbf{K}\right)\left\|\mathbf{s}\right\|^2 - \eta\left\|\mathbf{s}\right\| + D\left\|\mathbf{s}\right\| \le -\lambda_{\min}\left(\mathbf{K}\right)\left\|\mathbf{s}\right\|^2 < 0$$

where $\lambda_{\min}\left(\mathbf{K}\right)$ is the smallest eigenvalue of matrix $\mathbf{K}$.

The Lyapunov analysis yields $\mathbf{s} \to 0$ or $\mathbf{e},\dot{\mathbf{e}} \to 0$ as $t \to \infty$, confirming asymptotic stability. However, conventional sliding mode control fails under parameter uncertainties and external disturbances, potentially destabilizing the system when actual parameters deviate significantly from nominal values. Furthermore, the inherent chattering phenomenon degrades system performance [11]. A smooth, robust adaptive sliding mode controller is proposed to mitigate these limitations.

3.2. Design of smooth robust adaptive sliding mode control

First, we transform the mathematical model of system (2) into the following form:

$$\ddot{\mathbf{x}} = \mathbf{W}^{\mathrm{T}}\boldsymbol{\varphi}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{g}_0(\mathbf{x},\dot{\mathbf{x}})\mathbf{u} + \mathbf{d}(t) \tag{5}$$

where:

$$\mathbf{W} = \begin{bmatrix} \dfrac{-mgl}{J_p + ml^2} & \dfrac{-D_p}{J_p + ml^2} & \dfrac{-ml^2}{J_p + ml^2} & 0 & 0 \\ 0 & 0 & 0 & \dfrac{-D_y}{J_y + ml^2\cos^2(\theta)} & \dfrac{2ml^2}{J_y + ml^2\cos^2(\theta)} \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{5\times 2}$$

$$\boldsymbol{\varphi}(\mathbf{x},\dot{\mathbf{x}}) = \begin{bmatrix} \cos(\theta) & \dot{\theta} & \dot{\psi}^2\sin(\theta)\cos(\theta) & \dot{\psi} & \dot{\psi}\dot{\theta}\sin(\theta)\cos(\theta) \end{bmatrix}^{\mathrm{T}} \in \mathbf{R}^{5\times 1}$$

In practice, the value of $\cos^2(\theta)$ is small and changes quite slowly, so we can consider $\mathbf{W}$ and $\mathbf{g}_0$ as constant. Let $\hat{\mathbf{W}} \in \mathbf{R}^{5\times 2}$ and $\hat{\mathbf{g}}_0 \in \mathbf{R}^{2\times 2}$ be estimates of $\mathbf{W}$ and $\mathbf{g}_0$ respectively, $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$ và $\tilde{\mathbf{g}}_0 = \mathbf{g}_0 - \hat{\mathbf{g}}_0$ are estimation errors. Hence

$$\begin{cases} \mathbf{u} = -\hat{\mathbf{g}}_0^{-1}\left(\hat{\mathbf{W}}^{\mathrm{T}}\boldsymbol{\varphi} + \mathbf{c}\dot{\mathbf{e}} - \ddot{\mathbf{x}}_d + \mathbf{K}\mathbf{s} + \eta\tanh(\mathbf{s}/\varepsilon)\right) \\ \dot{\hat{\mathbf{W}}} = \boldsymbol{\Gamma}_1\boldsymbol{\varphi}\mathbf{s}^{\mathrm{T}};\ \dot{\hat{\mathbf{g}}}_0 = \boldsymbol{\Gamma}_2\mathbf{u}\mathbf{s}^{\mathrm{T}} \end{cases} \tag{6}$$

where $\mathbf{K} = \mathbf{K}^{\mathrm{T}} \in \mathbf{R}^{2\times 2}$, $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_1^{\mathrm{T}} \in \mathbf{R}^{5\times 5}$, $\boldsymbol{\Gamma}_2 = \boldsymbol{\Gamma}_2^{\mathrm{T}} \in \mathbf{R}^{2\times 2}$ – Positive definite symmetric matrices; $\tanh(\mathbf{s}) = \begin{bmatrix} \tanh(s_\theta) & \tanh(s_\psi) \end{bmatrix}^{\mathrm{T}}$; $\eta > D > 0,\ \varepsilon > 0$ – Positive constants.

System (6) maintains bounded tracking error and smooth control under parameter uncertainties. Define the Lyapunov function

$$V_2 = 0.5\left[\mathbf{s}^{\mathrm{T}}\mathbf{s} + \boldsymbol{\Gamma}_1^{-1}\mathrm{Tr}\left(\tilde{\mathbf{W}}^{\mathrm{T}}\tilde{\mathbf{W}}\right) + \boldsymbol{\Gamma}_2^{-1}\mathrm{Tr}\left(\tilde{\mathbf{g}}_0^{\mathrm{T}}\tilde{\mathbf{g}}_0\right)\right] > 0 \tag{7}$$

The time derivative of function (7) is determined as follows:

$$\dot{V}_2 = \mathbf{s}^{\mathrm{T}}\dot{\mathbf{s}} + \boldsymbol{\Gamma}_1^{-1}\mathrm{Tr}\left(\tilde{\mathbf{W}}^{\mathrm{T}}\dot{\tilde{\mathbf{W}}}\right) + \boldsymbol{\Gamma}_2^{-1}\mathrm{Tr}\left(\tilde{\mathbf{g}}_0^{\mathrm{T}}\dot{\tilde{\mathbf{g}}}_0\right)$$

$$\dot{V}_2 = \mathbf{s}^{\mathrm{T}}\left(\mathbf{W}^{\mathrm{T}}\boldsymbol{\varphi} + \mathbf{g}_0\mathbf{u} + \mathbf{d} + \mathbf{c}\dot{\mathbf{e}} - \ddot{\mathbf{x}}_d\right) - \boldsymbol{\Gamma}_1^{-1}\mathrm{Tr}\left(\tilde{\mathbf{W}}^{\mathrm{T}}\dot{\hat{\mathbf{W}}}\right) - \boldsymbol{\Gamma}_2^{-1}\mathrm{Tr}\left(\tilde{\mathbf{g}}_0^{\mathrm{T}}\dot{\hat{\mathbf{g}}}_0\right) \tag{8}$$

Substituting (6) into (8):

$$\dot{V}_2 = -\mathbf{s}^{\mathrm{T}}\mathbf{K}\mathbf{s} - \eta\mathbf{s}^{\mathrm{T}}\tanh(\mathbf{s}/\varepsilon) + \mathbf{s}^{\mathrm{T}}\mathbf{d} \tag{9}$$

According to [12], we have: $0 \le |s_i| - s_i\tanh(s_i/\varepsilon) \le \mu\varepsilon$, where $\mu = 0.2785$; $i = \{\theta;\psi\}$. Thus, $0 \le \|\mathbf{s}\| - \mathbf{s}^{\mathrm{T}}\tanh(\mathbf{s}/\varepsilon) \le 2\mu\varepsilon \Rightarrow -\eta\mathbf{s}^{\mathrm{T}}\tanh(\mathbf{s}/\varepsilon) \le -\eta\|\mathbf{s}\| + 2\eta\mu\varepsilon$. Substituting into (9):

$$\dot{V}_2 \le -\mathbf{s}^{\mathrm{T}}\mathbf{K}\mathbf{s} - \left(\eta\|\mathbf{s}\| - \mathbf{s}^{\mathrm{T}}\mathbf{d}\right) + 2\eta\mu\varepsilon \le -\lambda_{\min}(\mathbf{K})\|\mathbf{s}\|^2 + 2\eta\mu\varepsilon \tag{10}$$

From (10), if $\|\mathbf{s}\| \ge \sqrt{2\eta\mu\varepsilon/\lambda_{\min}(\mathbf{K})}$, then $\dot{V}_2 \le 0$, indicating that $\|\mathbf{s}\|$ remains bounded and converges to the largest invariant set with magnitude $\sqrt{2\eta\mu\varepsilon/\lambda_{\min}(\mathbf{K})}$. Therefore, if the matrix $\mathbf{K}$ is selected with sufficiently large eigenvalues and the parameters, η and ε are sufficiently small, then $\|\mathbf{s}\|$ will approach a neighborhood of zero, or equivalently $\mathbf{e}$ and $\dot{\mathbf{e}}$ will approach a neighborhood of zero as $t \to \infty$.

Furthermore, when $\dot{V}_2 \le 0$, V_2 is bounded, implying that $\tilde{\mathbf{W}}$ and $\tilde{\mathbf{g}}_0$ are also bounded. However, when $0 < \|\mathbf{s}\| \le \sqrt{2\eta\mu\varepsilon/\lambda_{\min}(\mathbf{K})}$, then $\dot{V}_2 > 0$, and the controller cannot guarantee bounds for variables $\hat{\mathbf{W}}$ and $\hat{\mathbf{g}}_0$. This implies that when $\dot{V}_2 > 0$, variables $\hat{\mathbf{W}}$, $\hat{\mathbf{g}}_0$ and $\mathbf{u}$ may increase beyond control thresholds, leading to risks of system malfunction or instability. To address this, the modified controller is

$$\begin{cases} \mathbf{u} = -\hat{\mathbf{g}}_0^{-1}\left(\hat{\mathbf{W}}^{\mathrm{T}}\boldsymbol{\varphi} + \mathbf{c}\dot{\mathbf{e}} - \ddot{\mathbf{x}}_d + \mathbf{K}\mathbf{s} + \eta\tanh(\mathbf{s}/\varepsilon)\right) \\ \dot{\hat{\mathbf{W}}} = \begin{cases} \boldsymbol{\Gamma}_1\boldsymbol{\varphi}\mathbf{s}^{\mathrm{T}} & \text{khi } \|\mathbf{s}\| \ge \sigma \\ 0 & \text{khi } \|\mathbf{s}\| < \sigma \end{cases};\ \dot{\hat{\mathbf{g}}}_0 = \begin{cases} \boldsymbol{\Gamma}_2\mathbf{u}\mathbf{s}^{\mathrm{T}} & \text{khi } \|\mathbf{s}\| \ge \sigma \\ 0 & \text{khi } \|\mathbf{s}\| < \sigma \end{cases} \end{cases} \tag{11}$$

where $\sigma > 0$ is a positive number to be chosen (we can choose $\sigma \ge \sqrt{2\eta\mu\varepsilon/\lambda_{\min}(\mathbf{K})}$).

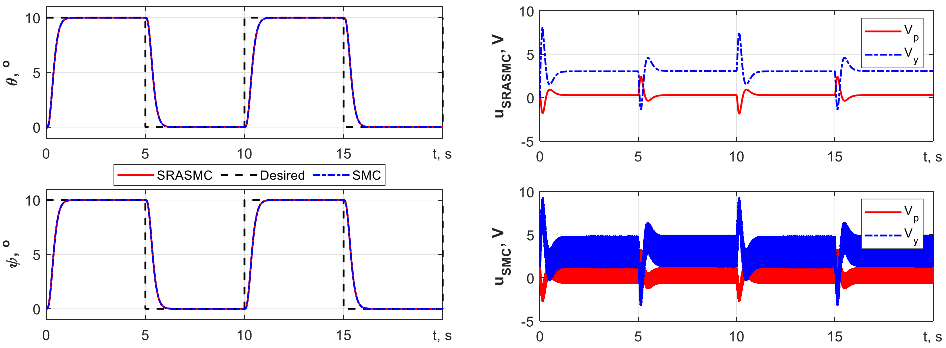
Clearly, when $\|\mathbf{s}\| \ge \sigma$ then $\hat{\mathbf{W}}$ and $\hat{\mathbf{g}}_0$ are continuously adapted but remain bounded as analyzed above, whereas when $\|\mathbf{s}\| < \sigma$, the adaptation process stops, and the parameters $\hat{\mathbf{W}}$ and $\hat{\mathbf{g}}_0$ maintain constant values. This mechanism prevents parameter drift, enhancing control reliability. Controller (11) guarantees bounded closed-loop signals, asymptotic tracking convergence, and chattering-free operation.

4. COMPARATIVE SIMULATION RESULTS

This section presents simulation results of the proposed controller applied to the Quanser AERO 2-DOF helicopter system model with nominal parameters as follows [6]: $m = 1.075$ kg; $g = 9.8\ \mathrm{m/s^2}$; $l = 0.0071$ m; $J_p = 0.0215\ \mathrm{kg \cdot m^2}$; $D_p = 0.0071$ N/V; $J_y = 0.0237\ \mathrm{kg \cdot m^2}$; $D_y = 0.0220$ N/V; $K_{py} = 0.0221$ N·m/V; $K_{pp} = 0.022$ N·m/V; $K_{yp} = -0.0227$ N·m/V; $K_{yy} = 0.0022$ N·m/V; $V_p, V_y \in [-24\quad 24](V)$. The external disturbances have the form $d_\theta = d_\psi = 0.75\sin(2.5t)$. The controller parameters are chosen as: $\mathbf{c} = \mathrm{diag}([10\quad 10])$; $\mathbf{K} = \mathrm{diag}([20\quad 20])$; $\boldsymbol{\Gamma}_1 = 10^3 \cdot \mathrm{diag}([1\quad 1])$; $\boldsymbol{\Gamma}_2 = \mathrm{diag}([2.5\quad 2.5])$; $\eta = 1$; $\varepsilon = 0.01$. Performance validation employs four test scenarios comparing the proposed and conventional SMC.

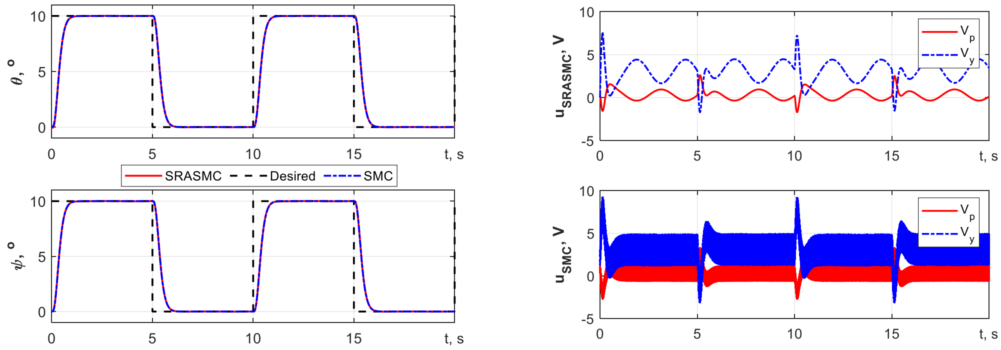
- **Scenario 1 - Basic scenario**: System operates with nominal parameters and no external disturbances;
- **Scenario 2 - Disturbance scenario**: System operates with nominal parameters but under external disturbance effects;
- **Scenario 3 - Parameter variation scenario**: System parameters change (m, J_p, J_y increase by 100%, $K_{py}, K_{pp}, K_{yp}, K_{yy}$ decrease by 20% from nominal values), with no external disturbances;
- **Scenario 4 - Comprehensive scenario**: System has both parameter variations (same as scenario 3) and external disturbances.

Simulation results are shown in figures 2-5, where "Desired" – Reference signal; "SRASMC" – Robust adaptive sliding control; "SMC" – Conventional sliding mode control.



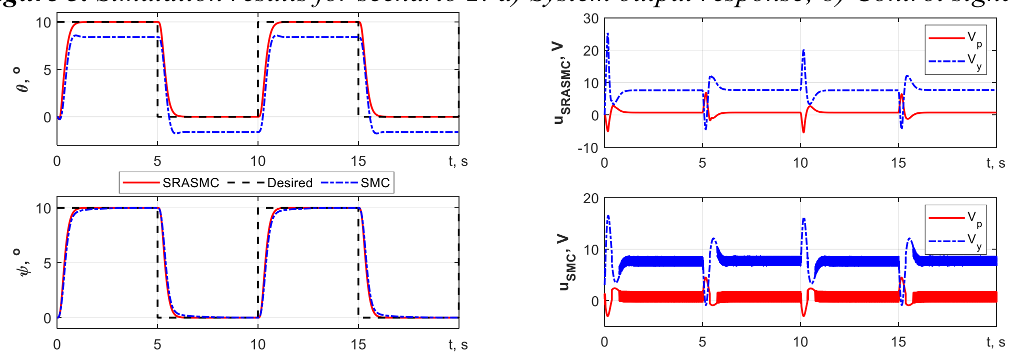
a) *b)*

Figure 2. *Simulation results for scenario 1: a) System output response; b) Control signals.*



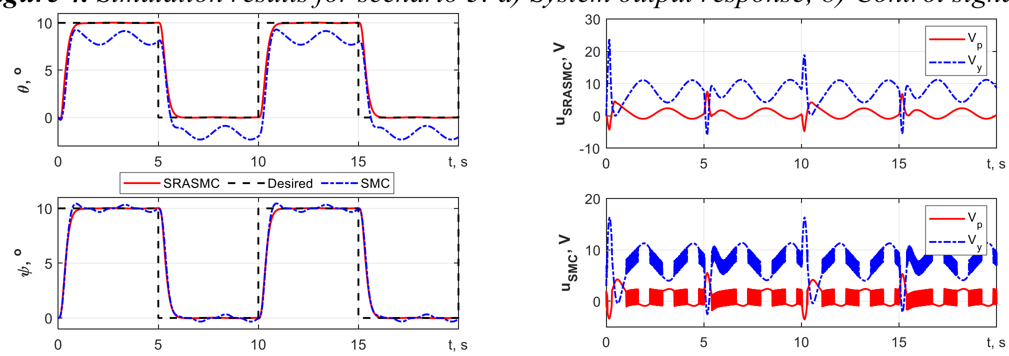
a) *b)*

Figure 3. *Simulation results for scenario 2: a) System output response; b) Control signals.*



a) *b)*

Figure 4. *Simulation results for scenario 3: a) System output response; b) Control signals.*



a) *b)*

Figure 5. *Simulation results for scenario 4: a) System output response; b) Control signals.*

Analysis of simulation result in figures 2-5 and table 1:

Under nominal conditions (scenarios 1-2), both controllers achieve stable operation with 0.8 s settling time, < 0.4% tracking error, and zero overshoot (figures 2(a), 3(a)). With parameter variations only (scenario 3), the proposed controller maintains nominal performance while conventional SMC degrades significantly (tracking error > 16%, figure 4(a)). Under combined uncertainties and disturbances (scenario 4), the proposed controller preserves stability and performance, whereas conventional SMC exhibits instability (figure 5(a)). The proposed controller produces smooth control signals throughout all scenarios, eliminating the chattering observed in conventional SMC (figures 2(b)–5(b)). These results confirm the proposed controller's practical feasibility over conventional SMC.

5. CONCLUSIONS

A smooth, robust adaptive SMC for 2-DOF helicopters under parameter uncertainties and time-varying disturbances is presented. The controller achieves structural simplicity, guaranteed stability via Lyapunov analysis, high tracking accuracy, and chattering-free operation through smooth functions. Simulations confirm superior performance over conventional SMC under uncertainties and disturbances. The proposed approach is suitable for practical implementation and extensible to other nonlinear systems. Future work will focus on addressing practical implementation challenges, including actuator saturation, time delays, and measurement noise. The proposed control framework will be validated experimentally on a 6-DOF UAV platform. Subsequently, adaptive control mechanisms and disturbance observer techniques will be integrated to achieve complete rejection of unknown external disturbances with uncertain bounds and time-varying characteristics.

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TÓM TẮT

Điều khiển trượt bền vững thích nghi với hàm trơn cho hệ thống trực thăng hai bậc tự do

Bài báo xem xét vấn đề phát triển hệ thống điều khiển bám cho trực thăng hai bậc tự do trong điều kiện bất định tham số và nhiễu ngoài biến thiên theo thời gian. Mô hình toán học phi tuyến của hệ thống trực thăng hai bậc tự do được xây dựng. Tiếp theo, bộ điều khiển trượt thông thường được tổng hợp, đồng thời tiến hành phân tích nhược điểm của nó trong trong điều kiện bất định tham số và nhiễu ngoài. Để khắc phục các nhược điểm trên của bộ điều khiển trượt thông thường, bài báo đề xuất bộ điều khiển trượt thích nghi bền vững với hàm trơn cho hệ thống trực thăng hai bậc tự do, đảm bảo sự hội tụ của các tín hiệu của hệ thống kín. Tính ổn định bền vững của bộ điều khiển được khảo sát dựa trên lý thuyết ổn định Lyapunov. Mô phỏng hệ thống với hai bộ điều khiển tổng hợp trong bài báo được tiến hành trên MATLAB/Simulink. Kết quả mô phỏng so sánh cho thấy bộ điều khiển bền vững thích nghi đề xuất trong bài báo hiệu quả hơn bộ điều khiển trượt phi tuyến thông thường và hoàn toàn đáp ứng được các yêu cầu điều khiển đặt ra.

Từ khóa: Trực thăng hai bậc tự do; Hệ thống phi tuyến; Điều khiển trượt bền vững thích nghi; Hàm trơn; Tham số bất định; Nhiễu ngoài biến thiên theo thời gian.