Dynamic stability analysis of a soft viscoelastic dielectric elastomer

Dao Hong Hai[1*], Nguyen Tuan Dung[2]

[1]Institute of Defence Equipment, Academy of Military Science and Technology, 17 Hoang Sam, Nghia Do, Hanoi, Vietnam;
[2]Faculty of Aerospace Engineering, Le Quy Don Technical University, 236 Hoang Quoc Viet, Nghia Do, Hanoi, Vietnam.
[*]Corresponding author: dhhai1989@gmail.com

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ABSTRACT

Dielectric elastomers (DEs) exhibit pronounced electromechanical deformation under high-voltage excitation, a response actuation by their intrinsic hyperelastic properties. When exposed to alternating electric fields, DEs exhibit complex nonlinear vibrational dynamics, demonstrating their potential for dynamic electromechanical actuation and soft robotics applications. As is well known, the dynamic characteristics of vibrational systems, including dielectric elastomer systems, exhibit significant frequency-dependent behavior. In this study, an effective generalized rheological model is employed to characterize the electromechanical response behavior of dielectric elastomers. The dynamic stability evolution process is systematically investigated under alternating excitation voltages with varying frequencies. Based on current applications of dielectric elastomers, this study contributes an effective modeling approach for analyzing the dynamic behavior of DEs, providing valuable guidance for the design and practical implementation of dielectric elastomer-based soft actuators and robotic systems.

Keywords: Dielectric elastomers; Dynamic stability; Soft robot.

1. INTRODUCTION

Dielectric elastomers (DEs) feature a three-dimensional crosslinked network structure composed of interconnected long-chain polymers, representing an important class of electroactive polymer materials. When subjected to external stimuli, this unique network architecture enables substantial deformations due to its molecular reconfiguration capabilities. Figure 1 illustrates the working principle of dielectric elastomers. Prior to electrical excitation, compliant electrodes are applied to both surfaces of the material. Under high-voltage stimulation, an electric field is established within the material, inducing dipole formation between charges on the opposing electrodes. The resulting Maxwell stress causes thickness reduction along the electric field direction. Due to the material's incompressibility, this deformation leads to planar expansion in both lateral directions, producing significant areal strain. Researchers have explored a variety of dielectric elastomer actuator (DEA) structural configurations to achieve diverse actuation effects. This enables DEAs to mimic the motion characteristics of natural muscles, showcasing broad application prospects in the field of soft robotics [1-4].

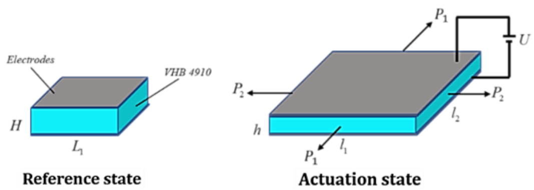


Figure 1. *Deformation of the dielectric elastomer.*

Under dynamic excitation, DEs exhibit complex nonlinear vibration behavior, demonstrating significant potential for dynamic electromechanical actuator applications. In vibratory systems incorporating DEs, geometric dimensions critically influence the system dynamics. As investigated by Zhang [5] during dynamic modeling development, the geometric parameters of DEs were systematically analyzed through comparative modeling and experimental studies of square and rectangular actuator configurations. For DE-based systems, diverse deformation modes can be achieved through the application of different voltage signals. Static or quasi-static deformations are obtained under DC voltage excitation, where the actuator maintains a steady equilibrium configuration. Over the past decade, significant research efforts have been devoted to developing nonlinear models and static analyses of DEs [6,7]. When functioning as dynamic electromechanical actuators under alternating loads, DEAs have found applications in bio-inspired flapping wings and micropumps, where inertial effects play a non-negligible role in system dynamics.

Recent advances in dielectric elastomer actuator (DEA) modeling have yielded various dynamic control equations through different analytical approaches, predominantly employing the principle of virtual work [8-10] and Euler-Lagrange formulations [11,12]. While conventional models simplify analysis by assuming constant membrane thickness and neglecting thickness-direction inertia - valid approximations for large-area DEs - Zhang's work [5] critically addresses these limitations by incorporating both geometric dependencies and through-thickness inertial effects, particularly crucial for small-area configurations. Current modeling frameworks remain incomplete due to three significant oversights: (1) the exclusion of viscous damping's energy dissipation, (2) inadequate consideration of geometry's profound influence on dynamic characteristics (resonance frequency, vibration amplitude, and periodicity), and (3) simplified treatment of viscoelasticity. The time-dependent viscoelastic behavior inherent to electroactive polymers introduces substantial complexities, including mechanical energy loss, vibration attenuation, and problematic creep phenomena in highly viscoelastic materials [13,14] - all contributing to observable performance degradation through drift and hysteresis effects. These unresolved challenges underscore the pressing need for comprehensive theoretical models that simultaneously account for geometric nonlinearities, three-dimensional inertial coupling, and viscoelastic constitutive behavior to enable accurate performance prediction across Dielectric elastomer actuator (DEA) applications.

As a contribution, this study employs an effective rheological model [15] as the constitutive model to accurately characterize the electromechanical response of dielectric elastomers (DEs) and investigate their dynamic stability evolution. The findings provide critical design insights for dielectric elastomer actuators (DEAs) and soft robotics, significantly enhancing the practical application of DE-based systems.

2. CONSTITUTIVE MODELING OF DIELECTRIC ELASTOMERS

The dynamic electromechanical response of dielectric elastomers (DEs) under high-voltage excitation is governed by competing hyperelastic and viscoelastic effects. Our constitutive modeling reveals that: (i) dynamic operation enables larger deformations than quasi-static conditions, (ii) decreasing frequency induces aperiodic-to-periodic stability transitions, and (iii) response modes evolve through distinct periodic regimes with frequency-dependent amplitude modulation. These findings demonstrate that frequency control represents a critical design parameter for DE actuators, with dynamic excitation offering performance advantages over static operation. Subsequent analysis employs frequency-varied loading to fully characterize these stability transitions.

Due to their inherent nonlinear viscoelastic characteristics, dielectric elastomers (DEs) have been extensively studied, with numerous models proposed to characterize their viscoelastic behavior [16-18]. Among these, generalized rheological models have been established as an effective approach for describing complex nonlinear viscoelasticity. In this study, we employ the KV-GM constitutive model developed in [15], as shown in figure 2, where the constitutive relation for our dynamic stability analysis is expressed as:

$$\begin{aligned}&\frac{2\rho L^2}{3}\frac{d^2\lambda}{dt^2}+\frac{\eta_\alpha}{H}\frac{d\lambda}{dt}-\varepsilon\left(\frac{\phi}{H}\right)^2\lambda^3+\frac{2\mu_\alpha\left(\lambda-\lambda^{-5}\right)}{1-\left(2\lambda^2+\lambda^{-4}-3\right)/J^\alpha}\\&+\sum_{i=1}^{3}\frac{2\mu_\beta\left(\lambda\xi_I^{-2}-\lambda^{-5}\xi_I^4\right)}{1-\left(2\lambda^2\xi^{-2}+\lambda^{-4}\xi_I^4-3\right)/J^\beta}-2\sigma=0\end{aligned} \quad (1)$$

$$\frac{d\xi_i}{dt}=\frac{2H\mu_{\beta I}}{\eta_{\beta I}}\frac{\lambda^2\xi_I^{-3}-\lambda^{-4}\xi_I^3}{1-\left(2\lambda^2\xi^{-2}+\lambda^{-4}\xi_I^4-3\right)/J^\beta},\ (i=1,2,3) \quad (2)$$

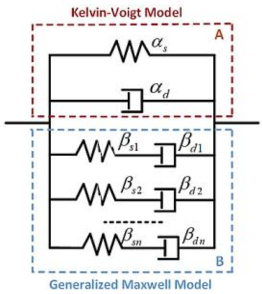


Figure 2. *The Kelvin-Voigt-Generalized Maxwell (KV-GM) model [15].*

Table 1. *Dynamic equation solution process of the dielectric elastomer system.*

Algorithm
Input
1. Input the mechanic parameter of the spring $\mu_\alpha, J_{\lim}^\alpha,\ \mu_\beta,\ J_{\lim}^\beta$
2. Input the viscous damping of the dashpots $\eta_\alpha,\ \eta_\beta$
3. Input the dielectric permittivity of the DE membrane ε
4. Input the geometry parameter of the membrane $H,\ L_1,\ L_2$
5. Input the pre-strain of the DE membrane
6. Input the Voltage signal $U(t)$
Solve
7. Initialize the variables $\ddot{\lambda},\ \dot{\lambda},\ \lambda,\ \dot{\xi}_i$
8. Call the MATLAB ode45 function for the time interval t
Output
9. Get the voltage $U(t)$ at the current time
10. Solve $d^2\lambda/dt^2$, $d\lambda/dt,\ d\xi_i/dt$
11. Solve $\lambda(t),\ \xi_i(t)$

For the numerical solution procedure, MATLAB was employed as the computational platform. The constitutive equations (1) and (2) governing static stability analysis were solved using the fsolve function, while the dynamic stability evolution was investigated through numerical integration of the governing equations via the Dormand-Prince method (ode45) from the Runge-Kutta ODE family. The dielectric elastomer membrane actuator is characterized by an in-plane dimension $L_1=L_2=L=0.01\ m$ and thickness $H=0.001\ m$, where the thickness of the compliant electrodes is neglected. The material selected for the dielectric elastomer is VHB4910, with a density of $\rho=960kg/m^3$ and relative permittivity of $\varepsilon=4.11*10^{-11}\ F/m$. For the numerical solution of the nonlinear differential constitutive equations, the initial condition is set to $\lambda(0)=3$,

$d\lambda(0)/dt = 0$, $d^2\lambda(0)/dt^2 = 0$, $d\xi_i(0)/dt = 3$ under the stress-free state. The computational procedure for solving the constitutive equations is summarized in table 1.

3. FREQUENCY DEPENDENT ENERGY DISSIPATION IN DIELECTRIC ELASTOMERS

Dielectric elastomer (DE) systems exhibit strongly nonlinear behavior, leading to complex dynamic responses under varying excitation voltages. In this study, we investigate the frequency-dependent stability evolution of DE materials by applying excitation frequencies ranging from $0 \sim 10\,Hz$, systematically analyzing how the dynamic stability transitions with actuation frequency.

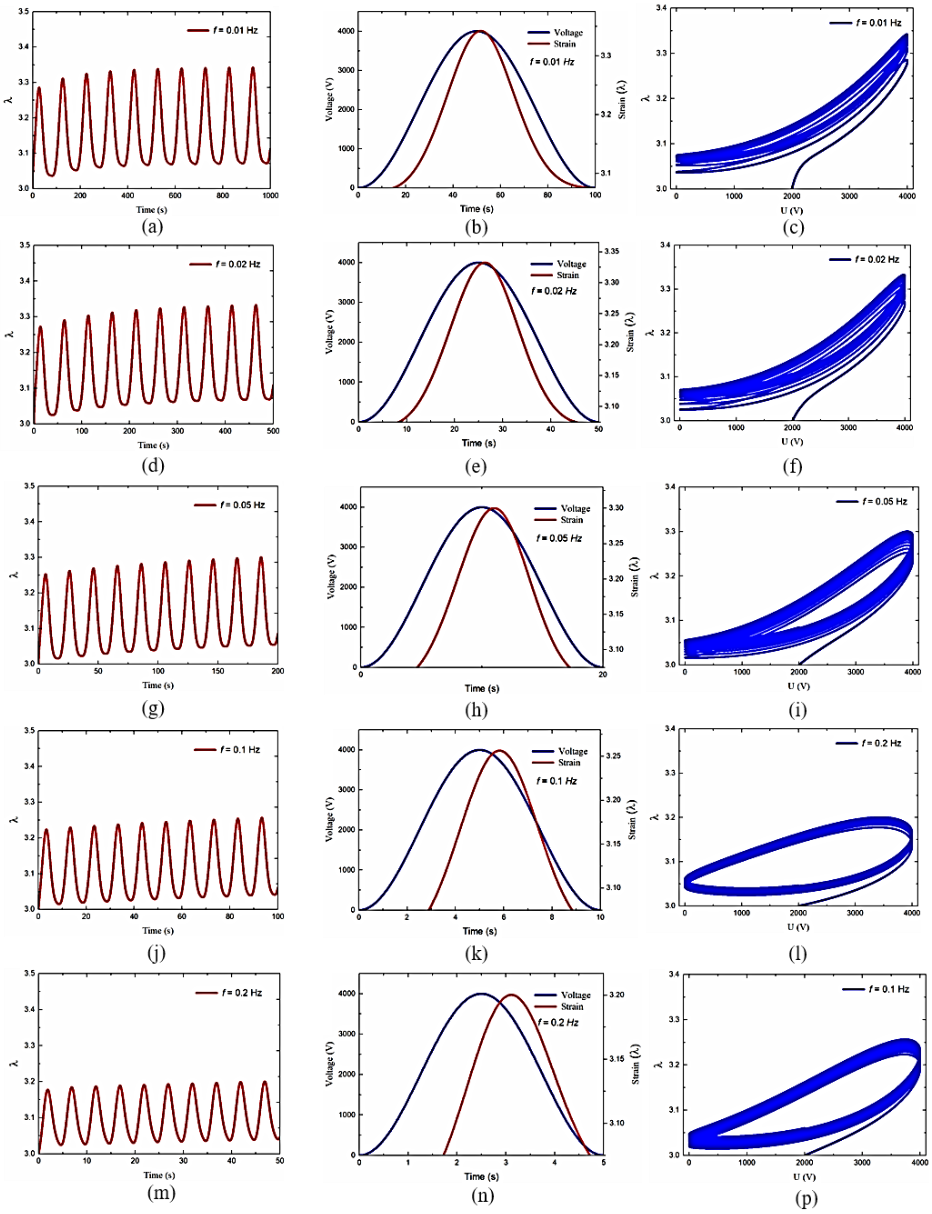
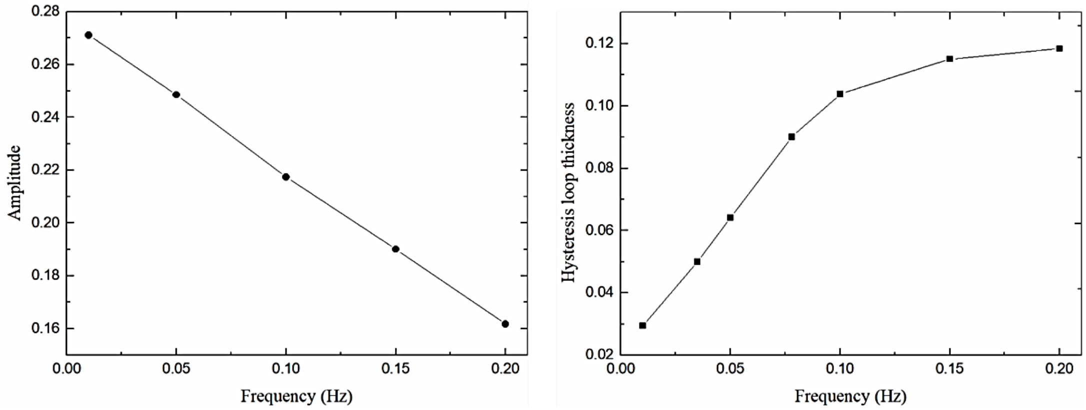


Figure 3. *Dynamic behavior of dielectric elastomer under different excitation frequencies (a, d, g, j, m), response strain (b, e, h, k, n), phase difference between excitation voltage and response strain (c, f, i, l, p), and hysteresis loop.*

Figure 3 presents the strain response and voltage-strain hysteresis loops of the dielectric elastomer system under different excitation frequencies. To investigate how frequency-dependent material properties influence energy dissipation characteristics, we applied a sinusoidal voltage signal $U = U_0 + U_0 * \sin(2\pi ft)$ (V), $U_0 = 2kV$, $\left(f = 0.01, 0.02, 0.05, 0.1, 0.2\ Hz\right)$ to the DE membrane. As shown in figures 3(a, d, g, j, m), the strain amplitude exhibits progressive attenuation with increasing frequency, demonstrating significant rate-dependent viscoelastic behavior.

The simulation results are in perfect agreement with experimental data obtained in the previous work [15]. This phenomenon is primarily attributed to the inherent viscoelasticity of the material, causing the film to enter the response of the next cycle before fully deforming in the preceding one. As frequency increases, the material's response cycle is shortened, restricting its deformation and leading to a decrease in vibration amplitude that is dependent on frequency. To investigate the energy dissipation characteristics of dielectric elastomers in a steady state, figure 3 (b, e, h, k, n) illustrates a phase shift phenomenon in the system's response during the last cycle of the steady state, occurring under the same alternating voltage amplitude as the loading frequency increases. It is evident that the response strain of the dielectric elastomer system exhibits a rightward shift, resulting in an increasing phase difference between the input voltage signal and the output strain with increasing frequency. Figures 3 (c, f, i, l, p) display the hysteresis loops of the dielectric elastomer film at different alternating voltage excitation frequencies; these results also demonstrate the frequency-dependent characteristics of dielectric elastomeric materials, where the width of the hysteresis loop increases with excitation frequency, confirming the energy dissipation process during material actuation. This phenomenon is explained by the fact that the viscoelastic relaxation time is faster than the characteristic relaxation time, leading to incomplete relaxation of the driven strain in each actuation cycle when the excitation frequency increases, severely affecting the deformation capability of the dielectric elastomer film and thus inducing a frequency-dependent energy dissipation process.



(a) (b)

Figure 4. *The dynamic behavior of the dielectric elastomers depends on the excitation frequency: (a) Vibration amplitude; (b) The width of the hysteresis loop.*

4. THE EVOLUTION OF DYNAMIC STABILITY IN DIELECTRIC ELASTOMER

Further research into the dynamic stability evolution of dielectric elastomer material systems under different loading frequencies is crucial. Due to the system's pronounced nonlinear characteristics, the response of dielectric elastomers to alternating voltage is highly complex. In this section, we consider various levels of loading frequencies and investigate the system's frequency-dependent periodicity using hysteresis loops and phase diagrams of the system's response.

As illustrated in figure 5, we consider the dielectric elastomer system under a relatively high excitation voltage frequency $f = 10\ Hz$. From figure 5 (a), it is observed that the film achieves a comparatively small vibration amplitude in the steady state. The system's phase portrait in figure 5 (b) is not a closed loop at the initial moment, instead exhibiting a staggered curve, which indicates that the system is aperiodic and unstable. After several cycles, the system demonstrates a clear closed-loop phase portrait, signifying that the system is in a quasiperiodic vibration state. As indicated above, the amplitude of the system at this stage is relatively small; thus, this quasiperiodic vibration state can almost be considered static. Figure 5(c) reveals a relatively complex creep phenomenon in the system, and the results show no distinct limit hysteresis loop. This phenomenon suggests that the DEs system does not exhibit significant hysteresis under high-frequency excitation.

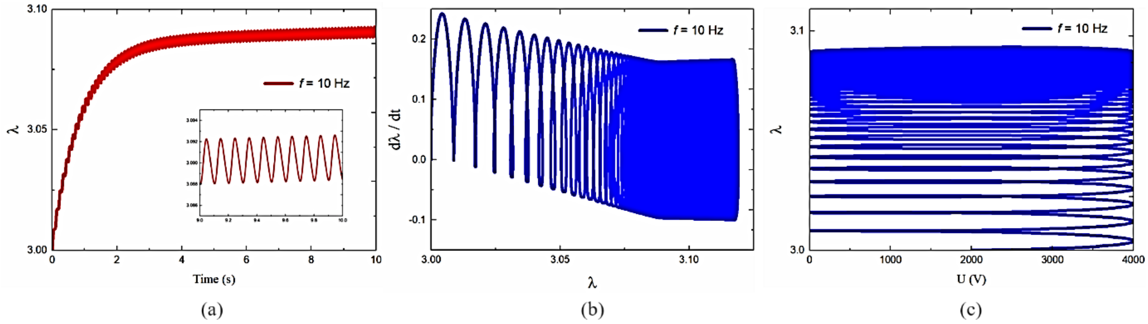


Figure 5. *Dynamic behavior of dielectric elastomer at excitation frequency f = 10 Hz: (a) Response strain; (b) Phase diagram; (c) Hysteresis loop.*

When an excitation voltage with a frequency of $f = 1\ Hz$ is applied, the dynamic characteristics of the system are depicted in figure 6. Figure 6 (a) illustrates the response strain of the dielectric elastomer system, showing an initial abrupt change followed by a gradual transition into a steady state. The system's phase portrait, presented in figure 6 (b), indicates that the system reaches stable oscillation only after several cycles, exhibiting a closed-loop phase portrait in the steady state. This result suggests that the system undergoes nonlinear quasiperiodic oscillation. Furthermore, the hysteresis loop observed in figure 6 (c) clearly demonstrates energy dissipation within the system.

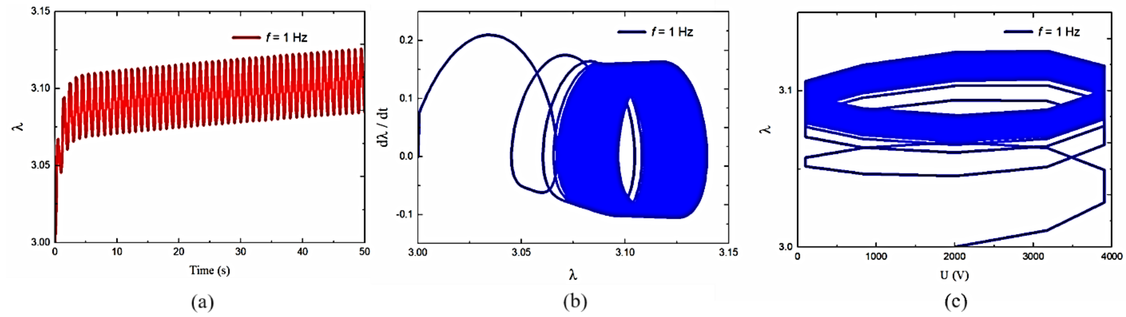


Figure 6. *Dynamic behavior of dielectric elastomer at excitation frequency f = 1 Hz: (a) Response strain; (b) Phase diagram; (c) Hysteresis loop.*

Figure 7 illustrates the dynamic behavior of the dielectric elastomer material system under an excitation load frequency of $f = 0.1\ Hz$. As shown in figure 7(a), the system's response strain indicates a notable increase in vibration amplitude, quickly reaching a steady oscillation. This finding is further supported by the system's phase portrait in figure 7(b), where a closed loop confirms the oscillatory steady state. Furthermore, the hysteresis loop in figure 7(c) clearly demonstrates significant energy dissipation.

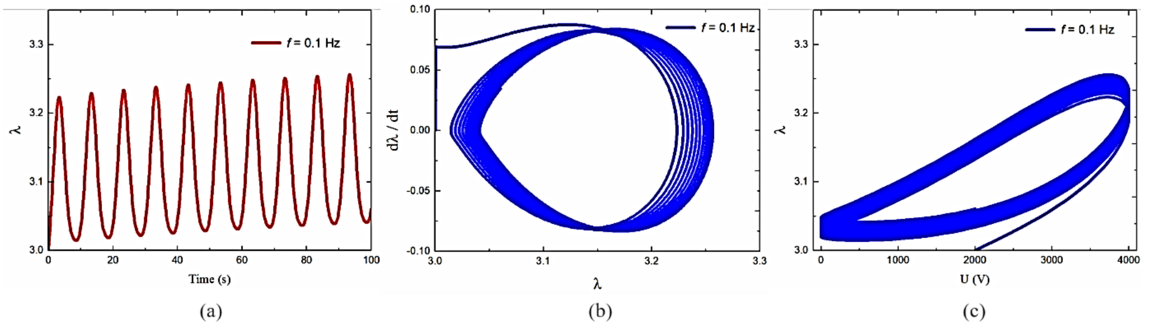


Figure 7. *Dynamic behavior of dielectric elastomer at excitation frequency f = 0.1 Hz: (a) Response strain; (b) Phase diagram; (c) Hysteresis loop.*

Figure 8 illustrates the dynamic behavior of the dielectric elastomer system at an excitation load frequency of $f = 0.01\ Hz$. From the response strain plot in figure 8(a) and the phase portrait in figure 8(b), it is evident that the system has reached a steady-state oscillation. The only energy dissipation observed is due to nonlinear viscoelastic creep, where the system requires several cycles to achieve a stable state. As shown in the phase portraits of figure 7(b) and figure 8(b), it is clear that the system has nearly entered a quasiperiodic oscillation state, which begins to appear only within a certain range of excitation frequencies. A prominent characteristic of this behavior is the rapid formation of a closed-loop phase portrait, with its repetition indicating the system's periodic response. Figure 8(c) displays the hysteresis loop of the system's response. Based on the assessment of the system's energy dissipation characteristics, the width of the hysteresis loop decreases as the excitation frequency decreases, implying minimal energy dissipation when the excitation load frequency is reduced. This phenomenon is also attributed to the inherent viscoelasticity of the material, as a lower frequency allows the material more time to complete its relaxation process during each response cycle. In practical applications, particularly in robot actuation, utilizing alternating load excitation is currently an effective method for achieving reciprocal motion in robots. Therefore, the selection of driving materials and excitation loads remains a critical and promising area for dynamic research.

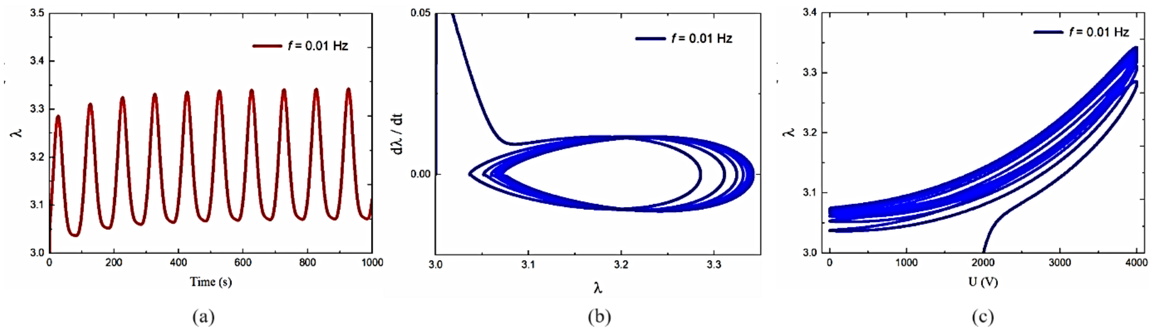


Figure 8. *Dynamic behavior of dielectric elastomer at excitation frequency f = 0.01 Hz: (a) Response strain; (b) Phase diagram; (c) Hysteresis loop.*

5. CONCLUSIONS

For actuators based on electroactive materials, optimizing the effectiveness of the actuation process through appropriate excitation loads is a pressing research question. In this chapter, we investigate the response of dielectric elastomer films under sinusoidal voltage excitation, focusing on their dynamic behavior at varying excitation frequencies. This includes exploring the frequency-dependent evolution of energy dissipation, a phenomenon rooted in the material's inherent viscoelastic properties. Recognizing the practical need for potentially high excitation load frequencies, we employ the phase portrait method to assess the periodicity of the system's response. Our findings indicate that at higher excitation frequencies, the material enters an aperiodic oscillation regime. Conversely, as the excitation load frequency is progressively reduced, we observe that the system exhibits periodic oscillatory behavior within a specific frequency range, which can even lead to a reduction in the material's energy dissipation. The contributions of this paper offer theoretical guidance for dielectric elastomer systems, particularly in the fields of soft actuators and soft robotics. By selecting suitable pre-treatment methods and load characteristics, our research aims to enhance the actuation properties of these materials.

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TÓM TẮT

Giải pháp phân tích ổn định động của vật liệu điện môi đàn nhớt mềm

Các vật liệu đàn hồi điện môi (DEs) thể hiện biến dạng cơ điện rõ rệt dưới kích thích điện áp cao, một phản ứng được kích hoạt bởi các tính chất siêu đàn hồi nội tại của chúng. Khi tiếp xúc với các trường điện xoay chiều, DEs thể hiện động lực học rung động phi tuyến phức tạp, chứng tỏ tiềm năng của chúng trong các ứng dụng kích hoạt cơ điện động và robot mềm. Như đã biết rõ, các đặc tính động của các hệ thống rung động, bao gồm cả hệ thống vật liệu đàn hồi điện môi, thể hiện hành vi phụ thuộc tần số của vật liệu. Trong nghiên cứu này, một mô hình lưu biến tổng quát hiệu quả được sử dụng để đặc trưng hóa hành vi phản ứng cơ điện của vật liệu đàn hồi điện môi. Quá trình tiến hóa ổn định động được nghiên cứu một cách có hệ thống dưới các điện áp kích thích xoay chiều với tần số khác nhau. Dựa trên các ứng dụng hiện tại của vật liệu đàn hồi điện môi, nghiên cứu này đóng góp một phương pháp mô hình hóa hiệu quả để phân tích hành vi động của DEs, cung cấp hướng dẫn có giá trị cho việc thiết kế và triển khai thực tế các bộ truyền động mềm và hệ thống robot dựa trên vật liệu đàn hồi điện môi.

Từ khoá: Điện môi đàn hồi; Ổn định động; Robot mềm.