Adaptive neural network controller for trajectory tracking of wheel mobile robots with velocity constraints

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ABSTRACT

This paper presents an adaptive neural control scheme for trajectory tracking of a nonholonomic wheeled mobile robot (WMR) under velocity constraints. The control objective is to ensure stable motion tracking despite model uncertainties and external disturbances. An adaptive neural network (ANN) is employed to approximate unknown nonlinearities in the robot dynamics, while an auxiliary control law compensates for velocity limitations and ensures safe operation within the admissible region. The proposed design combines model-based feedback with data-driven adaptation to improve tracking accuracy and robustness. The stability of the overall closed-loop system is demonstrated through Lyapunov analysis, guaranteeing the convergence of the tracking errors. Simulation results validate that the proposed ANN-based controller achieves faster convergence, smaller steady-state errors, and stronger robustness compared to conventional model-based controllers.

Keywords: Mobile robot; Adaptive control; Velocity constraint; Neural network; Backstepping; Trajectory tracking.

1. INTRODUCTION

Wheeled mobile robots (WMRs) have attracted significant attention in the fields of logistics, industrial automation, and service robotics because of their high maneuverability, flexibility, and ability to perform autonomous navigation tasks [1, 2]. However, the design of trajectory tracking controllers for such robots remains challenging due to the presence of nonholonomic motion constraints, which prevent direct linearization of the system dynamics [3, 4].

In the past decades, numerous control techniques have been proposed to address trajectory tracking problems for WMRs, such as kinematic control, backstepping, sliding mode control (SMC), and adaptive control approaches [5, 6]. These methods have demonstrated improved robustness and tracking performance in ideal conditions; nevertheless, their effectiveness may deteriorate when model uncertainties, parameter variations, or external disturbances occur [7, 8].

To overcome such limitations, intelligent control algorithms have been developed by incorporating learning-based mechanisms like fuzzy logic systems and artificial neural networks (ANNs), which are capable of approximating unknown nonlinearities in robotic systems [9, 10]. Compared to traditional model-based controllers, ANN-based schemes provide stronger adaptability and generalization capabilities. However, some of these designs still exhibit slow adaptation rates or approximation inaccuracies, especially when the robot operates under time-varying environments or uncertain load conditions [11, 12].

Moreover, during practical operation, the velocity and input constraints caused by actuator saturation and mechanical limits must be carefully considered [13, 14]. Neglecting such constraints can result in excessive control effort, leading to performance degradation or even system instability [15, 16]. Consequently, developing a control method that simultaneously manages nonlinear dynamics, velocity constraints, and external disturbances is of both theoretical and practical significance.

In recent years, researchers have combined adaptive neural networks with auxiliary dynamic systems or barrier Lyapunov functions to ensure constraint satisfaction and system stability [17, 18]. These approaches demonstrate that integrating adaptive learning with constraint handling can enhance tracking precision and maintain safe operation. However, most existing methods either assume ideal actuator behavior or do not fully address the coupling between velocity saturation and nonlinear uncertainties [19, 20].

Motivated by these observations, this paper presents an adaptive neural control strategy for trajectory tracking of a nonholonomic wheeled mobile robot subject to velocity constraints. The neural network module is designed to estimate unmodeled dynamics and compensate for unknown disturbances, while an auxiliary control term is introduced to prevent velocity violation and ensure bounded control inputs [21]. The overall stability of the closed-loop system is rigorously analyzed based on Lyapunov theory, proving that the tracking errors converge asymptotically to a small neighborhood around zero. Finally, simulation experiments are conducted to verify the effectiveness, robustness, and superior tracking performance of the proposed control scheme in comparison with existing methods.

2. KINEMATIC AND DYNAMIC MODELING OF THE WHEELED MOBILE ROBOT

Consider the configuration of the mobile robot as shown in Figure 1. The position of the robot is defined by the vector $q=[x,y,\theta]^T$ where x and y denote the coordinates of point C (the center of mass of the robot) in the global coordinate frame OX_oY_o and θ represents the orientation angle of the local coordinate frame CX_CY_C attached to the WMR with respect to the global frame OX_oY_o. The distance between the two driving wheels is *2a*, and the wheel radius is denoted by *r*.

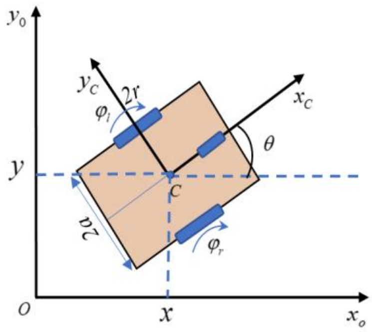

Figure 1. *Coordinates of the autonomous vehicle system.*

The non-holonomic constraints of WMR are: $\dot{x}\cos(\theta)+\dot{y}\sin(\theta)=0.$ (1)

The nonholonomic constraint can be expressed as $A(q)\dot{q}=0.$ The matrix $H(q)$ is formed by a set of linearly independent vectors that span the null space of $A(q),$ that is $H^T(q)A(q)=0$. Accordingly, the **kinematic model** of the WMR can be represented as follows:

$$\dot{q}=H(q)v\text{ ; with } H(q)=\begin{bmatrix}\cos(\theta) & 0\\ \sin(\theta) & 0\\ 0 & 1\end{bmatrix} \tag{2}$$

And the constraint coefficient matrix is given by: $A(q)=\begin{bmatrix}\cos(\theta) & \sin(\theta) & 0\end{bmatrix}$ (3)

Here, wheel slip is not taken into account. Considering the three degrees-of-freedom WMR models [17, 18], the nonlinear dynamic model of the WMR is expressed as follows:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+F(\dot{q})=B(q)\tau-A(q)\lambda \tag{4}$$

With: $M(q)=\begin{bmatrix}m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I\end{bmatrix}$; $C(q,\dot{q})=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$; $F(\dot{q})=\begin{bmatrix}\mu N\sin\theta\\ \mu N\cos\theta\\ 0\end{bmatrix}$; $B(q)=\dfrac{1}{r}\begin{bmatrix}-\sin(\theta) & -\sin(\theta)\\ \cos(\theta) & \cos(\theta)\\ a & -a\end{bmatrix}$

where m is the mass of the WMR, I is the moment of inertia of the WMR, $C(q,\dot{q})$ is the centripetal and Coriolis matrix, $B(q)$ is the input transformation matrix, τ is the input torque vector, $F(\dot{q})$ represents the friction force vector, and $A(q)\lambda$ denotes the nonholonomic constraint forces.

From (2) and (3), substituting into (4), we obtain:

$$M_1(q)\dot{v} + C_1(q,\dot{q})v + F_1(q,\dot{q}) = B_1(q)\tau \tag{5}$$

with $M_1(q) = H^T(q)M(q)H(q),\ C_1(q,\dot{q}) = H^T(q)\left[M(q)H(q) + C(q,\dot{q})H(q)\right],\ F_1(q,\dot{q}) = H^T(q)F(\dot{q})$

and $B_1(q) = H^T(q)B(q)$, when $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ \omega \end{bmatrix}$ (6)

where v_1 and ω denote the linear and angular velocities, respectively. With $H(q)$, we have $C_1 = 0$ and $M_1 = diag[M_{111}, M_{122}]^T = diag[m, I]^T$, where M_1 is a positive definite diagonal matrix. The velocity constraints are given by $|v_i| < v_{u1i}, i = 1,2,$ and $v_{u1} = \left[v_{u11}, v_{u12}\right]^T$. Within the scope of this paper, the model is implemented using the following system parameters:

3. ADAPTIVE NEURAL NETWORK CONTROLLER FOR WMR

3.1. Trajectory tracking control law for WMR based on the model

The model-based control law is derived directly from the system's mathematical dynamics, capturing the physical relationships among variables within a unified framework. Using analytical transformations and Lyapunov stability theory, the controller ensures accurate trajectory tracking while meeting velocity constraints. The following section describes the controller design and simulation results for validation.

3.1.1. Trajectory tracking control using auxiliary input velocity

The mobile robot is a nonholonomic system; therefore, the auxiliary input $H(q)$ is introduced to decouple the model and design a cascaded control structure, as illustrated in figure 2.

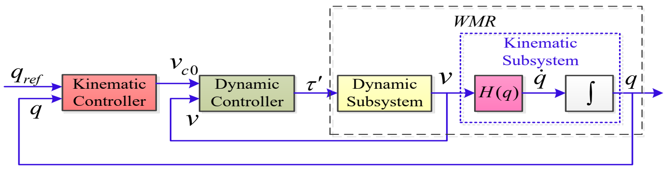

Figure 2. *Cascaded control diagram for the WMR.*

The objective is to determine the control input values τ' such that the actual trajectory of the WMR follows the desired trajectory under the influence of system uncertainties and external disturbances. Therefore, the design of the outer kinematic control loop is proposed. The goal is to find v_{c0} such that $\{q_{ref}(t) - q(t)\} \to 0$ as $t \to \infty$ where $q_{ref}(t) = \begin{bmatrix} x_{ref} & y_{ref} & \theta_{ref} \end{bmatrix}^T$ satisfies $A(q)\dot{q} = 0$. In this case, $A(q_{ref})\dot{q}_{ref} = 0$, which leads to the condition $\dot{x}_{ref}\cos(\theta) + \dot{y}_{ref}\sin(\theta) = 0$. This makes it difficult to construct a suitable Lyapunov function. According to [18], the tracking error can be transformed using a coordinate transformation matrix as follows:

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(q_{ref} - q\right) \Rightarrow \dot{e} = \frac{de}{dt} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega e_2 - v + v_{ref}\cos e_3 \\ -\omega e_1 - v_{ref}\sin e_3 \\ \omega_{ref} - \omega \end{bmatrix} \tag{7}$$

According to [18], the output of the controller is determined as an auxiliary velocity that ensures the tracking errors of q are asymptotically stable. The proposed control algorithm can thus be expressed by equation (8):

$$v_{c0} = \begin{bmatrix} v_{c01} \\ v_{c02} \end{bmatrix} = \begin{bmatrix} v_{1d}\cos e_3 + k_1 e_1 \\ v_{2d} + k_2 v_{1d} e_2 + k_3 \sin e_3 \end{bmatrix} \tag{8}$$

where k_1, k_2 and k_3 are positive constants with $k_2 \geq 1$. v1dv_{1d} v_{1d} and v_{2d} denote the desired velocities, satisfying $0 < v_{1d\min} < v_{1d} < v_{1d\max} < v_{u11}$.

To design a controller that enables trajectory tracking while satisfying the velocity constraints, the deviation between the actual velocity v and the auxiliary velocity v_{c0} (introduced to ensure stable tracking error convergence) is considered. Accordingly, we have:

$$z = \left[z_1, z_2\right]^T = v - v_{c0} \tag{9}$$

We select the following Lyapunov function for the constrained autonomous-vehicle model:

$$V_1 = \frac{1}{2} z^T z + f\left(v\right) + g(v_{c0}) \tag{10}$$

where the function $f\left(v\right)$ can be decomposed into two components corresponding to the linear (longitudinal) and angular velocities.

$$f(v) = f_1(v) + f_2(v) \tag{11}$$

With:

$$f_i(v) = \begin{cases} -\ln \dfrac{v_{u1i} - |v_i|}{v_{u1i} - v_{u2i}}, & |v_i| > v_{u2i} \\ 0, & others \end{cases} \quad \text{and}$$
$$g(\dot{v}_{c0}) = -\left\{\frac{1}{2}(v_{c1} - v_{c0})^T (v_{c1} - v_{c0}) + f(v_{c1})\right\} \tag{12}$$

Where: $v_{u2} = \left[v_{u21}, v_{u22}\right]^T$ is a positive-definite matrix satisfying $0 < v_{u2i} < v_{u1i}$; $v_{c1} = \left[v_{c11}, v_{c12}\right]^T$ denotes the minimizer of the following function $\left\{\frac{1}{2} z^T z + f(v)\right\}$ (13)

V_1 is a continuous positive-definite function when the velocity constraints are present, since $f\left(v\right)$ is continuous, and $g\left(v_{c0}\right)$ is a continuously differentiable function of v, where $-g\left(v_{c0}\right)$ corresponds to the minimum value of $\frac{1}{2} z^T z + f\left(v\right)$. For the velocity constraint $\left|v_i\right| < v_{u1i}, i = 1, 2,$ if $v_i \to v_{u1i}$ than $V_1 \to \infty$, this indicates that the Lyapunov function (10) can be effectively used to design a controller that satisfies the imposed velocity constraints.

It is noted that the Barrier Lyapunov Function (BLF) [18] used in other WMR control approaches requires the desired values to remain within the predefined constraints. However, in this study, the desired value of the BLF corresponds to the auxiliary velocity input, which may exceed the constraint limits when the WMR is far from the desired position. Therefore, the Lyapunov function (10) is specifically designed to handle this situation effectively.

The Lyapunov function (10) is constructed from three components: $\frac{1}{2} z^T z$ is used to reduce is used to reduce $\|v - v_{c0}\|$, $f(v)$ is employed to enforce the velocity constraints, and $g(v_{c0})$ is introduced to ensure that the minimum value of V_1 is zero. The Lyapunov function V_1 can be decomposed into two parts corresponding to v_1 and v_2 : $V_1 = V_{11} + V_{12}$ (14)

Where: $V_{1i} = \frac{1}{2}(v_i - v_{c0i})^2 + f_i(v_i) - \frac{1}{2}(v_{c1i} - v_{c0i})^2 - f_i(v_{c1i}).$ (15)

According to [32], the following control model is proposed:

$$\tau' = B_1^{-1}\left\{M_1\left\{\begin{bmatrix} A_1' \\ A_2' \end{bmatrix}(-\rho\begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} + (v - v_{c1})\dot{v}_{c0})\right\} + F_1\right\} \tag{16}$$

where ρ denotes a positive constant: $A_i' = \begin{cases} \frac{1}{A_i}, A_i \neq 0 \\ 0, A_i = 0 \end{cases}, i = 1,2,$ (17)

$A = [A_1, A_2]^T$ is defined as follows: $A = v - v_{c0} + \left[\frac{\mathrm{sgn}(v_1)h_1(v_1)}{v_{u11} - |v_1|}, \frac{\mathrm{sgn}(v_2)h_2(v_2)}{v_{u12} - |v_2|}\right]^T$ (18)

Where $h_i(x), i = 1,2$ is defined as follows: $h_i(x) = \begin{cases} 1, & |x| > v_{u21} \\ 0, & others \end{cases}$ (19)

✓ ***Stability analysis of the control law based on the model***

<u>Theorem 1:</u> According to [19], for the system described by (14) together with the control law (16), if the initial states satisfy the constraints $|v_i| < v_{u1i}, i = 1,2,$ then the system states will not violate the imposed constraints. Moreover, when $|v_{c0i}| < v_{u2i}, i = 1,2$ the velocity error signals z and the position tracking errors e converge to zero. Consequently, the closed-loop system is asymptotically stable.

<u>Proof:</u> Taking the time derivative V_1 of (19) with respect to time yields:

$$\dot{V}_1 = z\dot{z} + f'(v) + g'(v_{c0}) \tag{20}$$

Where: $f'(v) = \sum_{i=1}^{2}\begin{cases} \frac{\mathrm{sgn}(v_i)}{v_{u1i} - |v_i|}, & |v_i| > v_{u2i} \\ 0, & others \end{cases}$ (21)

$$g'(v_{c0}) = \{-z\dot{z} - f'(v)\}\big|_{v = v_{c1}} - (v_{c1} - v_{c0})^T(\dot{v}_{c1} - \dot{v}_{c0}) - \sum_{i=1}^{2}\begin{cases} \frac{\mathrm{sgn}(v_{c1i})\dot{v}_{c1i}}{v_{u1i} - |v_{c1i}|}, & |v_{c1i}| > v_{u2i} \\ 0, & others \end{cases} \tag{22}$$

we have: $(v_{c1} - v_{c0})^T\dot{v}_{c1} + \sum_{i=1}^{2}\begin{cases} \frac{\mathrm{sgn}(v_{c1i})\dot{v}_{c1i}}{v_{u1i} - |v_{c1i}|}, & |v_{c1i}| > v_{u2i} \\ 0, & others \end{cases} = 0.$ (23)

Then, substituting (23) into (22), we obtain: $g'(v_{c0}) = -(v_{c1} - v_{c0})^T\dot{v}_{c0}$ (24)

Substituting (17), (21), and (24) into (20), we have: $\dot{V}_1 = A^T\dot{v} - (v - v_{c1})^T\dot{v}_{c0}$ (25)

Split $\dot{V}_1$ into two components corresponding to v_1 and v_2 we have:

$$\dot{V}_1 = \dot{V}_{11} + \dot{V}_{12} \Leftrightarrow \begin{bmatrix} \dot{V}_{11} \\ \dot{V}_{12} \end{bmatrix} = A\dot{v} - (v - v_{c1})\dot{v}_{c0} \tag{26}$$

Substituting (22) and (16) into (26) yields: $\begin{bmatrix} \dot{V}_{11} \\ \dot{V}_{12} \end{bmatrix} = \rho \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix}$ (27)

and then substituting (14) and (27) into (25) gives: $\dot{V}_1 = -\rho V_1$ (28)

From the Lyapunov function above, we conclude that V_1 converges to zero. If $|v_{c0i}| < v_{u2i}, i = 1,2,$ then $v_{c1} = v_{c0}$ and $V_1 = \frac{1}{2} z^T z$. Hence, z converges to zero.

3.1.2. Simulation of the control law based on the model for the wheeled mobile robot (WMR).

Kinematic parameters of the WMR: $r = 100mm, a = 250mm, m = 45kg, I = 7kgm^2$;

The parameters of the proposed controller for each agent include the control gains and the auxiliary velocity parameters: $\mu = 0.01, k_1 = 10, k_2 = 5, k_3 = 4, \rho = [15 \quad 57.6]^T$; (29)

For a simulation duration of $t = 80s$ the desired trajectory is defined as $x_d = 2\sin(0.1t)$; $y_d = 2 - 2\cos(0.1t)$; $\theta_d = 0.1t - \pi/2$;

At the initial point: $q(0) = [-0.5 \quad -0.5 \quad -0.5]^T$; $\dot{q}(0) = [0 \quad 0 \quad 0]^T$ (30)

We obtain the following simulation results:

✓ In the absence of disturbances and model uncertainties

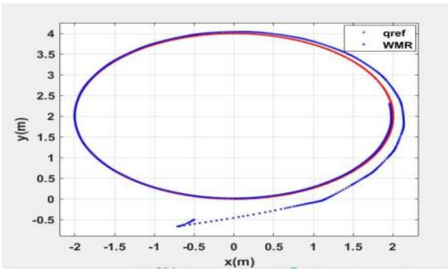

Figure 3. *Trajectory response using the model-based controller.*

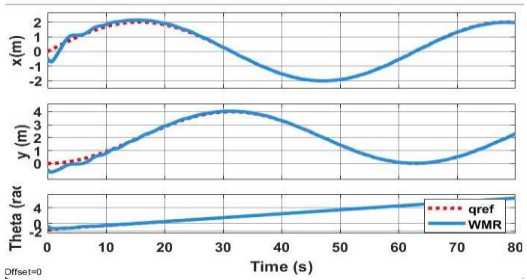

Figure 4. *Deviation between q_{ref} and q.*

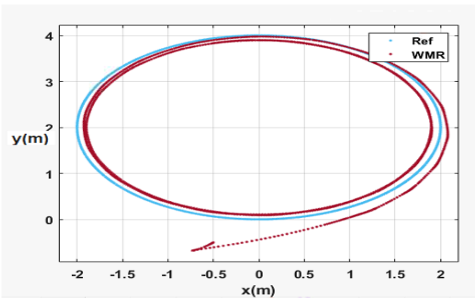

Figure 5. *Trajectory response using the model-based controller.*

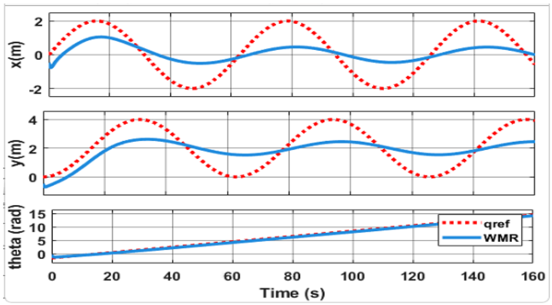

Figure 6. *Deviation between q_{ref} and q.*

When the model parameters are accurately identified and the robot operates under ideal conditions with no external disturbances, the model-based controller enables the robot to perfectly track the desired trajectory.

✓ In the presence of disturbances and model uncertainties

It is clearly observed that when external disturbances are present and the model parameters are accurately estimated, the model-based controller can no longer enable the robot to track the desired trajectory. The error at this time is very large, nearly 1.5m (Figure 5 and Figure 6).

3.2. Design of an adaptive neural network controller considering velocity constraints

3.2.1. Adaptive neural network control algorithm considering velocity constraints

For the neural network-based controller, according to [20], the following adaptive neural network control law is proposed:

$$\tau_{dk} = -\rho_1 B_1^{-1}\left(\begin{bmatrix}A_1'\\A_2'\end{bmatrix}\otimes\begin{bmatrix}V_{11}\\V_{12}\end{bmatrix}\right) + B_1^{-1}\left\{\begin{bmatrix}A_1'\\A_2'\end{bmatrix}\otimes(v-v_{c1})\otimes\left(\hat{W}^T S(Z)\right)\right\} \tag{31}$$

where ρ_1 is the control gain; $\hat{W}^T S(Z)$ represents the neural network; $S(Z)=\left[S_1(Z),S_2(Z),\ldots,S_n(Z)\right]^T$, $n=64$ denotes the basis function of the RBF neural network with input; $Z=\left[\dot{v}_{c0}^T,\dfrac{A_1}{(v_1-v_{c11})},\dfrac{A_2}{(v_2-v_{c12})},v^T\right]^T$, $S(Z)$ is the Gaussian function; $\hat{W}=\left[\hat{W}_1,\hat{W}_2\right]$, $\hat{W}_i=\left[\hat{W}_{i1},\hat{W}_{i2},\ldots,\hat{W}_{in}\right]^T$ is the weight matrix of the neural network, updated according to the following adaptive law:

$$\dot{\hat{W}}_i = -\Gamma_i\left[S(Z_i)(v_i-v_{c1i})+\sigma_i\hat{W}_i\right], i=1,2 \tag{32}$$

In which $\Gamma=\left[\Gamma_1,\Gamma_2\right]^T$ is a positive definite matrix, $\sigma=\left[\sigma_1,\sigma_2\right]^T$ is a small constant matrix. The neural network term $\hat{W}^T S(Z)$ is an approximation of $\hat{W}^{*T} S(Z)$ which is defined as follows:

$$W^{*T}S(Z) = M_1\dot{v}_{c0} + \begin{bmatrix}\dfrac{A_1}{(v_1-v_{c11})}\\[2mm] \dfrac{A_2}{(v_2-v_{c12})}\end{bmatrix}\otimes F_1 + \varepsilon_2 \tag{33}$$

Where $\varepsilon_2=\left[\varepsilon_{21},\varepsilon_{22}\right]^T$ denotes the approximation error. In this work it is assumed that $\varepsilon_2 \approx 0$. Figure 7 shows the block diagram of the controller.

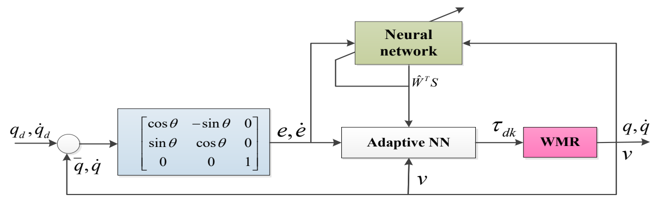

***Figure 7.** Block diagram of the proposed controller.*

✓ ***Stability analysis of the neural-network-based control law***

Theorem 2: According to [20], for the system described by (5), with the control law (32) and the adaptive law (33), if the initial states satisfy the velocity constraints $|v_i|<v_{u1i}, i=1,2$ then the

system states will not violate the imposed constraints. Moreover, when $|v_{c0i}| < v_{u2i}, i = 1,2,$ the closed-loop system is semi-globally ultimately bounded; in this case the error signals z and e converge to the compact sets Ω_2 and Ω_3 respectively, which are defined as follows:

$$\Omega_2 := \left\{ z \in R^2 \, \|z\| \le \sqrt{D} \right\} \tag{34}$$

$$\Omega_3 := \left\{ e \in R^3 \Big| \|e\| < \left(\sqrt{\frac{1}{k_1}\varepsilon_4} + \frac{\varepsilon_3}{2k_1} \right) + 2\left(\sqrt{\frac{k_2}{k_3}\varepsilon_4} + \frac{\varepsilon_3}{2k_3} \right) + l\left\{ \left(\sqrt{\frac{1}{k_1}\varepsilon_4} + \frac{\varepsilon_3}{2k_1} \right) + 2\left(\sqrt{\frac{k_2}{k_3}\varepsilon_4} + \frac{\varepsilon_3}{2k_3} \right) \right\} \right\} \tag{35}$$

Where $D = 4\dfrac{C_3}{\rho_2}, \varepsilon_3 = \sqrt{D}, \varepsilon_4 = \dfrac{\varepsilon_3^2}{4k_1} + \dfrac{\varepsilon_3^2}{4k_2k_3}$, and l is the smallest value satisfying the given condition $l > 5,\ \dfrac{10}{lk_2v_{1d\min}} < 0.2, \dfrac{k_3}{2k_2l^2} < 0.5, \dfrac{k_3}{k_2v_{1d\min}l} < 0.1,$

$$1.1v_{2\max}\frac{10}{lk_2v_{1d\min}} + v_{1d\max}\left(\frac{1}{l} + \frac{11v_{1d\max}}{lv_{1d\min}} + \frac{k_3}{5l}\right)\frac{10}{lk_2v_{1d\min}} < 0.2,$$

and $\dfrac{k_3}{lk_2v_{1d\min}}\left(1 + 1.1k_2v_{1d\max}\dfrac{10}{v_{1d\min}} + \dfrac{k_3}{5}\right) < 0.2.$ with C_3, ρ_2, k_2, k_3 is a positive constant.

<u>Proof:</u> Consider the following Lyapunov function $V_2 = V_1 + \dfrac{1}{2}\sum_{i=1}^{2} M_{1ii}^{-1}\Gamma_i^{-1}\tilde{W}_i^T\tilde{W}_i$ (36)

With $\tilde{W}_i = \hat{W}_i - W_i^*$ is the weighted error.

Derivative (36) with respect to time we get: $\dot{V}_2 = \dot{V}_1 + \sum_{i=1}^{2} M_{1ii}^{-1}\Gamma_i^{-1}\tilde{W}_i^T\dot{\tilde{W}}_i$ (37)

V_2 can also be divided into two parts corresponding to v_1 and v_2, respectively

$$V_2 = V_{21} + V_{22} \Rightarrow \dot{V}_2 = \dot{V}_{21} + \dot{V}_{22} \tag{38}$$

$$\Rightarrow \begin{bmatrix} \dot{V}_{21} \\ \dot{V}_{22} \end{bmatrix} = \begin{bmatrix} \dot{V}_{11} \\ \dot{V}_{12} \end{bmatrix} + \begin{bmatrix} M_{111}^{-1}\Gamma_1^{-1}\tilde{W}_1^T\dot{\tilde{W}}_1 \\ M_{122}^{-1}\Gamma_2^{-1}\tilde{W}_2^T\dot{\tilde{W}}_2 \end{bmatrix} = A \otimes \dot{v} - (v - v_{c1}) \otimes \dot{v}_{c0} + M_1^{-1}\begin{bmatrix} \Gamma_1^{-1}\tilde{W}_1^T\dot{\tilde{W}}_1 \\ \Gamma_2^{-1}\tilde{W}_2^T\dot{\tilde{W}}_2 \end{bmatrix} \tag{39}$$

By substituting (5) and $\tilde{W}_i = \hat{W}_i - W_i^*$ into (39), we obtain:

$$\begin{bmatrix} \dot{V}_{21} \\ \dot{V}_{22} \end{bmatrix} = A \otimes (M_1^{-1}(B_1\tau - F_1)) - (v - v_{c1}) \otimes \dot{v}_{c0} + M_1^{-1}\begin{bmatrix} \Gamma_1^{-1}\tilde{W}_1^T\dot{\tilde{W}}_1 \\ \Gamma_2^{-1}\tilde{W}_2^T\dot{\tilde{W}}_2 \end{bmatrix} \tag{40}$$

Using the control law (32) and $\tilde{W}_i = \hat{W}_i - W_i^*$, we have:

$$\begin{bmatrix} \dot{V}_{21} \\ \dot{V}_{22} \end{bmatrix} = -\rho_1 M_1^{-1}\begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} + M_1^{-1}\begin{bmatrix} \Gamma_1^{-1}\tilde{W}_1^T\dot{\tilde{W}}_1 \\ \Gamma_2^{-1}\tilde{W}_2^T\dot{\tilde{W}}_2 \end{bmatrix} + M_1^{-1}\left\{ (v - v_{c1}) \otimes (W^* + \tilde{W})^T S(Z) \right\} - A \otimes (M_1^{-1}F_1) - (v - v_{c1}) \otimes \dot{v}_{c0} \tag{41}$$

Since M_1^{-1} is a positive definite diagonal matrix, we have:

$$\begin{bmatrix} \dot{V}_{21} \\ \dot{V}_{22} \end{bmatrix} = -\rho_1 M_1^{-1}\begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} + M_1^{-1}\begin{bmatrix} \Gamma_1^{-1}\tilde{W}_1^T\dot{\tilde{W}}_1 \\ \Gamma_2^{-1}\tilde{W}_2^T\dot{\tilde{W}}_2 \end{bmatrix} + M_1^{-1}\left\{ (v - v_{c1}) \otimes (\tilde{W}^T S(Z) + \varepsilon_2 \right\} \tag{42}$$

Substituting (32) into (42) and using $\tilde{W}_i = \hat{W}_i - W_i^*$, we obtain:

$$\begin{bmatrix} \dot{V}_{21} \\ \dot{V}_{22} \end{bmatrix} = -\rho_1 M_1^{-1} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} - M_1^{-1} \begin{bmatrix} \sigma_1 \tilde{W}_1^T \hat{W}_1 \\ \sigma_2 \tilde{W}_2^T \hat{W}_2 \end{bmatrix} + M_1^{-1} \{(v - v_{c1}) \otimes \varepsilon_2\} \tag{43}$$

$$\le -\rho_1 M_1^{-1} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} + \frac{1}{2} M_1^{-1} \begin{bmatrix} \|v_1 - v_{c11}\|^2 \\ \|v_2 - v_{c12}\|^2 \end{bmatrix} - \frac{1}{2} M_1^{-1} \begin{bmatrix} \sigma_1 \|\tilde{W}_1\|^2 \\ \sigma_2 \|\tilde{W}_2\|^2 \end{bmatrix} + \frac{1}{2} M_1^{-1} \begin{bmatrix} \sigma_1 \|W_1^*\|^2 \\ \sigma_2 \|W_2^*\|^2 \end{bmatrix} + \frac{1}{2} M_1^{-1} \begin{bmatrix} \|\bar{\varepsilon}_{21}\|^2 \\ \|\bar{\varepsilon}_{22}\|^2 \end{bmatrix}. \tag{44}$$

To compare V_{1i} and $\frac{1}{2}(v_i - v_{c1i})^2, i = 1,2,$ by partially differentiate V_{1i} with respect to v_i:

$$\frac{\partial V_{1i}}{\partial v_i} = A_i \Leftrightarrow \frac{\partial^2 V_{1i}}{\partial v_i^2} = 1 + \frac{h_i(v_i)}{\left(v_{u11} - |v_1|\right)^2} \ge 1 \tag{45}$$

the partial derivative of $\frac{1}{2}(v_i - v_{c1i})^2$ with respect to v_i, we have: $V_{1i} \ge \frac{1}{2}\left(v_i - v_{c1i}\right)^2$ (46)

From equations (40) and (46), it follows that

$$\begin{bmatrix} \dot{V}_{21} \\ \dot{V}_{22} \end{bmatrix} \le -(\rho_1 - 1) M_1^{-1} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} + \frac{1}{2} M_1^{-1} \begin{bmatrix} \|\bar{\varepsilon}_{21}\|^2 \\ \|\bar{\varepsilon}_{22}\|^2 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} M_{111}^{-1} \sigma_1 \|\tilde{W}_1\|^2 \\ M_{122}^{-1} \sigma_2 \|\tilde{W}_2\|^2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} M_{111}^{-1} \sigma_1 \|W_1^*\|^2 \\ M_{122}^{-1} \sigma_2 \|W_2^*\|^2 \end{bmatrix} \le -\rho_2 \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \tag{47}$$

$$\Rightarrow \dot{V}_2 \le -\rho_2 V_2 + C_3 \text{ Where: } \rho_2 = \min\left(\left(\rho_1 - 1\right) \lambda_{\min}\left(M_1^{-1}\right), \min_{i=1,2}\left(\sigma_i \Gamma_i\right)\right);\ C_3 = C_1 + C_2 \tag{48}$$

According to the Lyapunov function described above, it can be concluded that V_2 can converge to an arbitrarily small value by increasing ρ_2. If the auxiliary velocity v_{c0} is large, or even exceeds the constraint bounds $|v_{c01}| > v_{u11}$ or $|v_{c02}| > v_{u12}$ the proposed method can effectively limit the velocity within the admissible constraints to ensure safety, which is even more critical than maintaining tracking stability. If $|v_{c0i}| < v_{u2i}, i = 1,2,$ we have $v_{c1} = v_{c0}$ and $\frac{1}{2}(v - v_{c0})^T (v - v_{c0}) < V_3$.

Since V_2 can converge to an arbitrarily small value by increasing ρ_2, it follows that z can also converge to any sufficiently small neighborhood. That is, for any $\varepsilon_3 > 0$ and the initial condition $v_i(0) < v_{u2i}, i = 1,2,$ let $\varepsilon_3 = \sqrt{\frac{4C_3}{\rho_2}}$, there exists a time $T > 0,$ such that for all $t > T$ and $|v_{c0i}| < v_{u2i}, i = 1,2,$ we have $\|v - v_{c0}\| < \varepsilon_3, z_1 < \varepsilon_3$ và $z_2 < \varepsilon_3$. According to the model-based control analysis presented in the previous section, it can be directly inferred that the tracking error e converges, and therefore

$$\Omega_3 = e \in R^3 \Big| \|e\| < \left(\sqrt{\frac{1}{k_1}\varepsilon_4 + \frac{\varepsilon_3}{2k_1}}\right) + 2\left(\sqrt{\frac{k_2}{k_3}\varepsilon_4 + \frac{\varepsilon_3}{2k_3}}\right) + l\left\{\left(\sqrt{\frac{1}{k_1}\varepsilon_4 + \frac{\varepsilon_3}{2k_1}}\right) + 2\left(\sqrt{\frac{k_2}{k_3}\varepsilon_4 + \frac{\varepsilon_3}{2k_3}}\right)\right\} \tag{49}$$

with Ω_3 being arbitrarily small by properly selecting k_1, k_2 và k_3. the overall closed-loop system is ultimately bounded in a semi-global sense, as stated in [21].

3.2.2. Algorithm simulation on Matlab

Recommended controller parameters: $\mu = 0.01; k_1 = 10; k_2 = 5; k_3 = 4; \rho = [1350, 21.6]^T$;

$\sigma = \begin{bmatrix} 0.0001 & 0.0001 \end{bmatrix}^T$; $\Gamma = \begin{bmatrix} 20000 & 800 \end{bmatrix}^T$; $v_{u1} = \begin{bmatrix} 1 & 1.5 \end{bmatrix}^T$; $v_{u2} = \begin{bmatrix} 0.8 & 1.3 \end{bmatrix}^T$. The centers of the 64 neural networks are selected in the region: $[-1,1]\times[-1,1]\times[-1,1]\times[-1,1]\times[-1,1]\times[-1,1]$. Simulation time, simulation trajectory, and starting point are the same as in section 3.1.2 above, in the case where the model parameters are not clearly defined and there are external disturbances. We obtain the following simulation results:

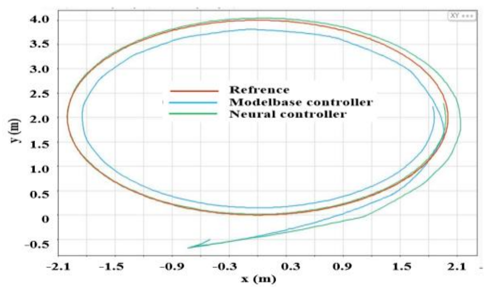

Figure 8. *Trajectory of the WMR.*

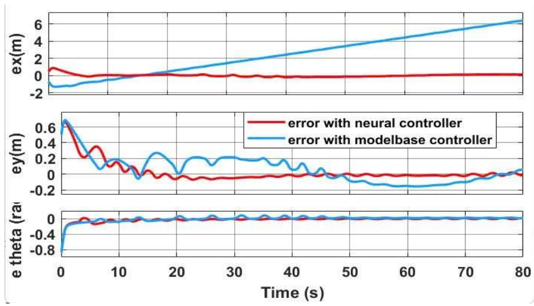

Figure 9. *Tracking error.*

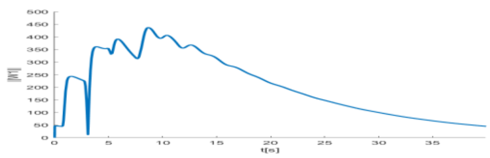

Figure 10. *Update of the first-layer weights.*

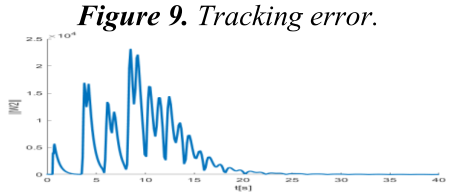

Figure 11. *Update of the second-layer weights.*

As shown by the simulation results, the WMR trajectory closely follows the desired path (figure 8), and the Lyapunov function rapidly converges to zero (Figures 10 and 11), verifying the effectiveness of the controller in (44). The linear and angular velocities remain within allowable limits, indicating satisfactory control performance, although the response speed is moderate. Comparative results (Figure 9) show that the model-based controller deviates from the desired trajectory as soon as external disturbances are introduced, whereas the ANN-based method maintains good adaptive tracking, with a steady-state error of approximately 0.0005 m. These results demonstrate that the ANN-based controller preserves trajectory tracking despite model uncertainties and disturbances through adaptive self-adjustment. The larger transient oscillations and slower convergence observed in ANN control are inherent characteristics of adaptive learning, as initially nonoptimal weights gradually converge, as guaranteed by Theorem 2. Furthermore, enforcing velocity and control constraints leads to a more conservative transient response. This reflects the typical trade-off between convergence speed and robustness in adaptive neural network control.

4. CONCLUSIONS

In this study, two trajectory-tracking control algorithms for wheeled mobile robots (WMRs) with velocity constraints have been proposed and analyzed. The first method is a model-based controller designed using the Backstepping algorithm, in which system stability is proven based on the Lyapunov criterion. This approach achieves high trajectory-tracking performance in simulations but requires precise knowledge of the robot's kinematic and dynamic parameters, which is difficult to ensure in practical applications. The second method is an adaptive controller based on an artificial neural network (ANN), capable of approximating nonlinear functions and compensating for parameter uncertainties. Although its trajectory tracking performance in simulations is slightly lower than that of model-based controllers, it demonstrates greater applicability in real-world environments due to its noise immunity and adaptability to model deviations. Simulation results in MATLAB/Simulink verify that both controllers ensure closed-loop system stability, maintain the end-effector motion closely along the desired trajectory, and keep both linear and angular velocities within their physical constraints. These outcomes confirm the correctness and effectiveness of the proposed control strategies. Future work will focus on experimental validation of the ANN-based adaptive controller on a real WMR platform, as well as optimization of the neural network structure and adaptation law to further improve control performance and reduce system response time.

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ABSTRACT

Bộ điều khiển thích nghi mạng nơron cho bài toán bám quỹ đạo robot di động bánh xe xét đến ràng buộc vận tốc

Bài báo này trình bày một phương pháp điều khiển thích nghi sử dụng mạng nơroncho bài toán bám quỹ đạo của robot di động bánh xe không toàn phương (WMR) dưới các ràng buộc về vận tốc. Mục tiêu điều khiển là đảm bảo chuyển động bám quỹ đạo ổn định ngay cả khi tồn tại bất định mô hình và nhiễu bên ngoài. Một mạng nơron (ANN) được sử dụng để xấp xỉ các phi tuyến chưa biết trong động học của robot, trong khi một luật điều khiển phụ trợ được thiết kế để bù các giới hạn vận tốc và đảm bảo vận hành an toàn trong miền cho phép. Thiết kế đề xuất kết hợp giữa phản hồi dựa trên mô hình và cơ chế thích nghi dựa trên dữ liệu nhằm nâng cao độ chính xác bám quỹ đạo và tính bền vững. Tính ổn định của toàn bộ hệ kín được chứng minh thông qua phân tích Lyapunov, đảm bảo sự hội tụ của sai số bám. Kết quả mô phỏng cho thấy bộ điều khiển dựa trên ANN đề xuất đạt được tốc độ hội tụ nhanh hơn, sai số xác lập nhỏ hơn và khả năng chống nhiễu tốt hơn so với các bộ điều khiển truyền thống dựa trên mô hình.

Từ khóa: Robot di động; Điều khiển thích nghi; Ràng buộc vận tốc; Mạng nơ-ron; Backstepping; Bám quỹ đạo.