Development of a generation dispatch algorithm for power plants to achieve economic efficiency in power system operation

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ABSTRACT

In power system operation, economic generation dispatch aims to maintain the balance between power generation and load demand while minimizing electricity production cost. Conventional approaches are usually based on complex mathematical optimization methods, which require iterative calculations with high computational effort and are difficult to implement in real-time dispatch centers. This paper proposes a generation dispatch algorithm based on a linear relationship between generator outputs and total system demand. The proposed method significantly reduces computation time while maintaining adequate accuracy and economic efficiency, making it suitable for real-time and automated power system operation.

Keywords: Economic dispatch; Power system operation; Generation allocation; Optimal control; Real-time computation.

1. INTRODUCTION

In power system operation, the problem of generation power allocation among generating units plays a central role in simultaneously satisfying two fundamental requirements: (i) balancing generation and load demand while maintaining the operational constraints of the power grid, and (ii) minimizing electricity production costs under time-varying load conditions. These issues have been systematically presented in textbooks on power system operation and stability analysis, and are closely associated with mathematical modeling and optimal control theory. Classical domestic and international references indicate that economic operation problems are strongly related to Economic Dispatch (ED), Optimal Power Flow (OPF), and constrained optimization techniques [1-4, 6-8].

However, as the scale of power systems increases and operational constraints become more complex, many conventional solution methods based on nonlinear optimization or mathematical programming require a large number of iterative computations. This leads to difficulties in automation and fast response for real-time operation, especially in situations where system states and load profiles must be continuously updated. Moreover, the growing integration of renewable energy sources and various forms of uncertainty—such as wind and solar power, flexible loads, and energy storage systems—necessitates that the generation power allocation problem consider uncertainty scenarios, robust optimization frameworks, and/or emission-aware optimization approaches. Recent studies have extended traditional ED models by incorporating emission considerations and environmental constraints [9], developing robust cost-optimization models under worst-case scenarios with aggregated generation–load components [10], as well as reviewing and evaluating OPF-based techniques for operational decision-making in distribution systems and power systems with high penetration of renewable energy sources [11].

Motivated by the above requirements, this paper focuses on developing a generation power allocation algorithm aimed at reducing computational burden while ensuring solution correctness under operational constraints. The proposed approach is presented through clear illustrations of cost function models and power allocation relationships, facilitating simulation implementation and comparative analysis using commonly adopted computational tools in power system research [1, 3, 7, 12].

2. PROBLEM FORMULATION AND DEVELOPMENT OF A POWER ALLOCATION ALGORITHM FOR POWER PLANTS

Assume that the power system consists of N generating units producing electrical power. In this study, transmission line losses are considered negligible and therefore are neglected. According to the reference [1], the operating cost characteristics of each generating unit can generally be represented by the following equation:

$$Z_i = a_i P_i^2 + b_i P_i + c_i \tag{1}$$

where Z_i is the operating cost of the i-th generating unit, P_i denotes the output power of the i-th generating unit, a_i, b_i, c_i are the quadratic, linear, and constant cost coefficients of the i-th generating unit, respectively.

Based on the above equation, it can be observed that the relationship between the cost function and the generated power is commonly approximated by a parabolic function.

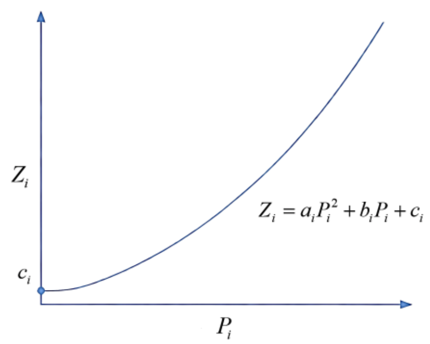

Figure 1. *Cost–power characteristic curve.*

At a given time instant, the total load demand can be considered constant, i.e.,

$$P_1 + P_2 + \cdots + P_N = P_\Sigma \tag{2}$$

The problem considered here is to determine $P_1 + P_2 + \cdots + P_N$ such that the total operating cost is minimized. According to reference [1], this can be expressed as follows:

$$Z_\Sigma = Z_1 + Z_2 + Z_3 + \cdots + Z_N \tag{3}$$

In the existing literature, this problem is typically solved using a Lagrange-based optimization approach. Although this method is mathematically rigorous and accurate, it does not explicitly show the direct relationship between the total cost and the system load and requires solving an additional unknown, namely the Lagrange multiplier. To address these limitations, this paper proposes a variable substitution–based approach that enables a direct solution of the cost equations.

Indeed, from equation (2), the following relationship can be derived:

$$P_N = P_\Sigma - \left(P_1 + P_2 + \cdots + P_{N-1}\right) \tag{4}$$

Substituting P_N from (4) into the cost function in (1) yields:

$$Z_N = a_N \left[P_\Sigma - \left(P_1 + P_2 + \cdots + P_{N-1}\right)\right]^2 + b_N \left[P_\Sigma - \left(P_1 + P_2 + \cdots + P_{N-1}\right)\right] + c_N \tag{5}$$

By expanding equation (5), we obtain:

$$
\begin{aligned}
Z_N &= a_N\left[P_\Sigma{}^2 - 2P_\Sigma\left(P_1+P_2+\cdots+P_{N-1}\right) + \left(P_1+P_2+\cdots+P_{N-1}\right)^2 \right] \\
&+ b_N\left[P_\Sigma - \left(P_1+P_2+\cdots+P_{N-1}\right)\right] + c_N = a_N\left(P_1+P_2+\cdots+P_{N-1}\right)^2 \\
&-\left(2P_\Sigma a_N + b_N\right)\left(P_1+P_2+\cdots+P_{N-1}\right) + \left(a_N P_\Sigma{}^2 + b_N P_\Sigma + c_N\right)
\end{aligned} \tag{6}
$$

The total cost is given by:

$$
\begin{aligned}
Z_\Sigma &= Z_1+Z_2+\cdots+Z_{N-1}+Z_N = \sum_{i=1}^{N-1} a_i P_i^2 + \sum_{i=1}^{N-1} b_i P_i + \sum_{i=1}^{N-1} c_i + a_N\left(P_1+P_2+\cdots+P_{N-1}\right)^2 \\
&-\left(2P_\Sigma a_N + b_N\right)\left(P_1+P_2+\cdots+P_{N-1}\right) + \left(a_N P_\Sigma{}^2 + b_N P_\Sigma + c_N\right)
\end{aligned} \tag{7}
$$

Equation (7) indicates that the total cost depends only on $N-1$ variables; hence, only these $N-1$ variables need to be determined, with the remaining one obtained from (4).

To minimize the total cost, we take the partial derivative of Z_Σ with respect to P_i, yielding:

$$
\frac{\partial Z_\Sigma}{\partial P_i} = 0, i = 1,2,...,N-1
$$

$$
\frac{\partial Z_\Sigma}{\partial P_i} = 2a_i P_i + b_i + 2a_N\left(P_1+P_2+\cdots+P_{N-1}\right) - \left(2P_\Sigma a_N + b_N\right) = 0 \ \left(for\ i\ =\ 1,2,\ldots,N-1\right) \tag{8}
$$

Dividing both sides of equation (8) by $2a_N$ and rearranging terms, we obtain:

$$
P_1+P_2+\cdots+P_{N-1} + \frac{a_i}{a_N}P_i = \frac{2P_\Sigma a_N + b_N - b_i}{2a_N} = P_\Sigma + \frac{b_N - b_i}{2a_N} \ \left(for\ i\ =\ 1,2,\ldots,N-1\right) \tag{9}
$$

From expression (9), the following $N-1$ linear equations are obtained:

$$
\begin{cases}
\left(\dfrac{a_1}{a_N}+1\right)P_1 + P_2 + \cdots + P_{N-1} = P_\Sigma + \dfrac{b_N - b_1}{2a_N} \\
P_1 + \left(\dfrac{a_2}{a_N}+1\right)P_2 + \cdots + P_{N-1} = P_\Sigma + \dfrac{b_N - b_2}{2a_N} \\
\vdots \\
P_1 + P_2 + \cdots + \left(\dfrac{a_{N-1}}{a_N}+1\right)P_{N-1} = P_\Sigma + \dfrac{b_N - b_{N-1}}{2a_N}
\end{cases} \tag{10}
$$

By expressing the above equations in matrix form, we obtain:

$$
\begin{pmatrix}
\left(\dfrac{a_1}{a_N}+1\right) & 1 & \cdots & 1 \\
1 & \left(\dfrac{a_2}{a_N}+1\right) & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 1 & \cdots & \left(\dfrac{a_{N-1}}{a_N}+1\right)
\end{pmatrix}
\begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{N-1} \end{pmatrix}
=
\begin{pmatrix}
P_\Sigma + \dfrac{b_N - b_1}{2a_N} \\
P_\Sigma + \dfrac{b_N - b_2}{2a_N} \\
\vdots \\
P_\Sigma + \dfrac{b_N - b_{N-1}}{2a_N}
\end{pmatrix} \tag{11}
$$

To simplify the solution of the equation, we define:

$$A=\begin{pmatrix} \left(\frac{a_1}{a_N}+1\right) & 1 & \cdots & 1 \\ 1 & \left(\frac{a_2}{a_N}+1\right) & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & \left(\frac{a_{N-1}}{a_N}+1\right) \end{pmatrix} P=\begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{N-1} \end{pmatrix} \quad (12)$$

$$B=\begin{pmatrix} P_\Sigma+\frac{b_N-b_1}{2a_N} \\ P_\Sigma+\frac{b_N-b_2}{2a_N} \\ \vdots \\ P_\Sigma+\frac{b_N-b_{N-1}}{2a_N} \end{pmatrix} \quad (13)$$

Accordingly, equation (11) can be rewritten as:

$$A.P=B \quad (14)$$

Under this formulation, the solution for P is obtained as follows:

$$P=A^{-1}.B \quad (15)$$

3. SIMULATION STUDY

In the considered case study, three generating units are assumed, and their cost functions are defined as follows:

$$Z_1=2P_1^2+3P_1+3 \quad (16)$$

$$Z_2=3P_2^2+P_2+2 \quad (17)$$

$$Z_3=5P_3^2+2P_3+2 \quad (18)$$

Accordingly, the required total load demand P_Σ is satisfied at minimum cost as follows:

$$P_\Sigma \geq c_1+c_2+c_3=3+2+2=7 \quad (19)$$

From (12), the following results are obtained:

$$A=\begin{pmatrix} \left(\frac{2}{5}+1\right) & 1 \\ 1 & \left(\frac{3}{5}+1\right) \end{pmatrix}=\begin{pmatrix} \frac{7}{5} & 1 \\ 1 & \frac{8}{5} \end{pmatrix} \quad (20)$$

According to linear algebra [5], for a second-order (2×2) matrix, if:

$$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (21)$$

The inverse matrix can be obtained as:

$$A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{22}$$

where $\det(A)$ is given by:

$$\det(A) = ad - cb \tag{23}$$

Using the matrix result in (20) for A and the expression for $\det(A)$ in (23), we obtain:

$$\det(A) = \left(\frac{2}{5}+1\right)\left(\frac{3}{5}+1\right) - 1 = \left(\frac{7}{5}\times\frac{8}{5}\right) - 1 = \frac{56-25}{25} = \frac{31}{25} \tag{24}$$

Hence, the inverse matrix A^{-1} given in (22) is obtained as:

$$A^{-1} = \frac{25}{31}\begin{pmatrix} \frac{8}{5} & -1 \\ -1 & \frac{7}{5} \end{pmatrix} = \begin{pmatrix} \frac{40}{31} & -\frac{25}{31} \\ -\frac{25}{31} & \frac{35}{31} \end{pmatrix} \tag{25}$$

Similarly, the value of B can be computed using (13), yielding:

$$B = \begin{pmatrix} P_\Sigma + \frac{2-3}{10} \\ P_\Sigma + \frac{2-1}{10} \end{pmatrix} = \begin{pmatrix} P_\Sigma - \frac{1}{10} \\ P_\Sigma + \frac{1}{10} \end{pmatrix} \tag{26}$$

Finally, the solution P is obtained using (15):

$$P = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} \frac{40}{31} & -\frac{25}{31} \\ -\frac{25}{31} & \frac{35}{31} \end{pmatrix}\begin{pmatrix} P_\Sigma - \frac{1}{10} \\ P_\Sigma + \frac{1}{10} \end{pmatrix} \tag{27}$$

Hence, it follows that:

$$P_1 = \frac{40}{31}\left(P_\Sigma - \frac{1}{10}\right) - \frac{25}{31}\left(P_\Sigma + \frac{1}{10}\right) = \frac{40}{31}P_\Sigma - \frac{25}{31}P_\Sigma - \frac{40}{31\times 10} - \frac{25}{31\times 10} = \frac{15}{31}P_\Sigma - \frac{13}{62} \tag{28}$$

$$P_2 = -\frac{25}{31}\left(P_\Sigma - \frac{1}{10}\right) + \frac{35}{31}\left(P_\Sigma + \frac{1}{10}\right) = -\frac{25}{31}P_\Sigma + \frac{35}{31}P_\Sigma + \frac{25}{31\times 10} + \frac{35}{31\times 10} = \frac{10}{31}P_\Sigma + \frac{6}{31} \tag{29}$$

For the conventional simulation case, (2) yields:

$$P_3 = P_\Sigma - P_1 - P_2 \tag{30}$$

Substituting (28) and (29) into (30) yields:

$$P_3 = P_\Sigma - \left(\frac{15}{31}P_\Sigma - \frac{13}{62}\right) - \left(\frac{10}{31}P_\Sigma + \frac{6}{31}\right) = \frac{6}{31}P_\Sigma + \frac{1}{62} \tag{31}$$

The analysis in Section 2 and the case study in Section 3 demonstrate that generator power allocation can be efficiently determined using linear expressions. The algorithm is illustrated by the flowchart in Figure 2.

Figure 2 illustrates the flowchart of the proposed algorithm. First, the cost function coefficient a_i, b_i, and c_i of the generating units are used to construct matrix A according to (12), followed by the computation of its inverse A^{-1}. This step represents the most computationally intensive operation of the algorithm and is performed only once for a given set of cost coefficients.

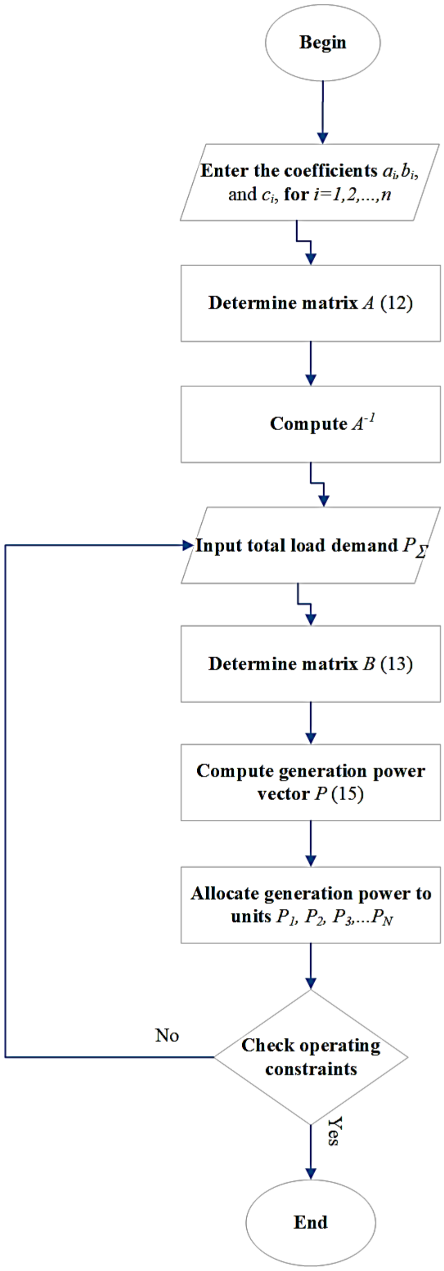

Figure 2. *Flowchart of the generation power allocation algorithm.*

When the total load demand P_Σ varies, the algorithm requires only the determination of matrix B using (13) and the direct computation of the generation power vector P using (15). These subsequent steps involve only simple linear operations and do not require any iterative procedure. As a result, the proposed method exhibits a clear computational advantage over classical Lagrange-based economic dispatch methods.

Based on (28), (29), and (31), the numerical results corresponding to the total load demand P_Σ are calculated and summarized in Table 1, where a comparison with the classical Lagrange-based method is presented. Both methods yield identical optimal power allocations and satisfy the total load demand, thereby confirming the correctness of the proposed approach.

Table 1. *ED results: Lagrange vs. proposed method.*

Method	**Proposed method**	**Lagrange**
Lambda	N/A	487/31
Number of iterations	N/A	2
P_Σ	7	7
P_1	197/62	197/62
P_2	76/31	76/31
P_3	85/62	85/62

As shown in Table 1, the Lagrange-based method requires the explicit computation of the Lagrange multiplier $\lambda = 487/31$ and converges within two iterations, whereas the proposed method directly determines the optimal generation outputs without computing the Lagrange multiplier. This demonstrates that the proposed method achieves the same optimal economic dispatch with reduced computational effort.

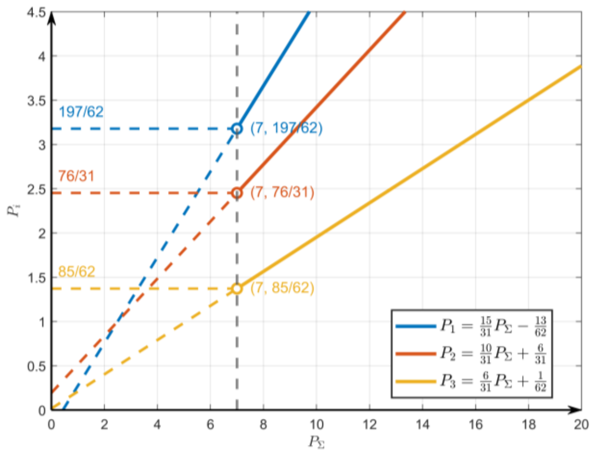

Figure 3. *Power allocation curves for generating units.*

Figure 3 depicts the power allocation curves of the generating units derived from the quadratic cost functions. The intersection point of the incremental cost curves occurs at $P_\Sigma = 7$, corresponding to the minimum-cost operating condition identified in Table 1.

At this operating point, all generating units share the same incremental cost, thereby satisfying the economic dispatch optimality condition. This graphical representation provides a clear visual confirmation of the numerical results summarized in Table 1.

4. CONCLUSIONS

The paper proposes a substitution-based approach that establishes an accurate and linear relationship between generator outputs and total load demand. The algorithm requires only simple linear computations within the iterative loop, while complex calculations are performed once outside the loop. This significantly reduces computational time and enables fast economic dispatch with real-time capability. The obtained results are equivalent to those of the Lagrange multiplier method for multi-unit systems, showing consistent simulation agreement.

Transmission loss modeling will be addressed in future work. The obtained results are consistent with those of the conventional Lagrange multiplier method for multi-unit generation systems, and the simulation outcomes further confirm this agreement.

As a result, the optimal generation power allocation can be determined efficiently without the explicit computation of the Lagrange multiplier. By transforming the economic dispatch problem into an unconstrained formulation, the proposed approach allows classical optimality conditions to be satisfied while reducing the overall computational effort. The method therefore provides a simple and effective analytical alternative to conventional Lagrange-based economic dispatch techniques.

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TÓM TẮT

Xây dựng thuật toán phân bổ công suất phát cho các nhà máy điện nhằm đạt hiệu quả kinh tế trong vận hành hệ thống điện

Trong vận hành hệ thống điện, việc phân bổ công suất phát kinh tế nhằm đảm bảo cân bằng công suất phát–tiêu thụ và tối thiểu hóa chi phí sản xuất điện. Các phương pháp giải truyền thống thường dựa trên tối ưu hóa toán học phức tạp, yêu cầu tính toán lặp với khối lượng lớn, dẫn đến thời gian xử lý kéo dài và khó đáp ứng yêu cầu thời gian thực tại trung tâm điều độ. Bài báo đề xuất một thuật toán phân bổ công suất phát dựa trên quan hệ tuyến tính giữa công suất phát của các nhà máy và tổng công suất phụ tải. Thuật toán giúp giảm thời gian tính toán so với các cách tiếp cận truyền thống, đồng thời vẫn đảm bảo độ chính xác và hiệu quả kinh tế, phù hợp cho điều độ tự động trong hệ thống điện hiện đại.

Từ khoá: Phân bổ công suất phát; Vận hành hệ thống điện; Điều độ kinh tế; Điều khiển tối ưu; Thời gian thực.