# A sensitivity approach for calculating power losses and bus voltages in radial distribution grids

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*Received 23 Jan. 2026; Revised 24 Mar. 2026; Accepted 10 Apr. 2026; Published 25 Apr. 2026.*

**DOI:** https://doi.org/10.54939/1859-1043.j.mst.110.2026.63-73

## ABSTRACT

*This paper presents a sensitivity-based method for the calculation of power losses and nodal voltages in power distribution systems. Sensitivity factors of power losses and voltage magnitudes with respect to nodal power injections based on the exploitation of the radial structure of distribution grids are rigorously represented. The power losses and voltages attained from this sensitivity approach are validated using a six-bus distribution system and compared with the Alternating Current Power Flow (ACPF) method that solves non-linear power flow equations iteratively and provides accurate solutions. The calculated findings show that sensitivity method errors are minor and highly acceptable in comparison with the ACPF method when power injections vary slightly. For voltages, the computed results from both techniques are nearly the same.*

**Keywords:** Radial distribution grids; Power flow; Sensitivity analysis; Power loss sensitivity factors; Voltage sensitivity factors.

## 1. INTRODUCTION

The approach based on sensitivity factors, such as loss factors and voltage sensitivity factors, plays an essential role in power system analysis [1]. The first reason is that the ever-increasing penetration of distributed generations (DGs) [2], which makes distribution systems active and smarter, requires their impacts to be accurately assessed and fast enough. In terms of computational speed, the sensitivity methods are better than Alternating Current Power Flow (ACPF), such as the Newton-Raphson (NR) method and the Power Summation method (PSM). However, the accuracy of sensitivity approaches in comparison with ACPF-based methods needs to be further analyzed [3].

Furthermore, the distribution-level electricity markets are considered to be an effective means to manage and exploit a large amount of DGs. The locational marginal price (LMP) of the wholesale markets in transmission systems has been extended to be deployed in the distribution-level retail markets [4]. The decomposition of distribution-level LMP into components, including marginal prices for real power, reactive power, congestion, losses, and voltage, enhances the effectiveness of market operation. Nevertheless, to decompose the distribution-level LMP, it is necessary to derive the sensitivity factors, namely the loss factors and voltage sensitivity factors [5].

In transmission systems, sensitivity factors, for instance, Power Transfer Distribution Factors (PTDF), are usually determined using the Direct Current Power Flow (DCPF) [6]. Nonetheless, the DCPF-based methodologies cannot be applied to distribution systems due to the high R/X ratio. For distribution systems, the authors in [7] developed a method for network reconfiguration utilizing the sensitivity of active power loss with respect to branch impedance, which requires the construction of a Jacobian matrix. Similarly, the sensitivity factors proposed in [3] and [8] are calculated with the need of constructing the Jacobian matrix, which is computationally complicated. In terms of the radial topology of power distribution systems, authors in [9] and [10] exploited this feature to identify network topology and efficiently control distributed voltage, respectively.

This paper presents a sensitivity-based method for computing power losses and nodal voltages in radial power distribution systems. This sensitivity-based approach is a highly efficient tool for evaluating network performance, enabling engineers to instantly predict how changes in load affect system efficiency and voltage stability without complex power flow iterations. The main contributions of this research are:

- To propose uncomplicated and effective mathematical expressions for recursively determining power loss and voltage sensitivity factors in radial distribution systems, without the need to establish the Jacobian matrix.
- To rigorously describe a step-by-step procedure to calculate sensitivity factors for a six-bus radial distribution system;
- To compare the errors of power losses and voltage magnitudes from the ACPF and the sensitivity approach.

The paper is structured into four sections. Section 2 presents mathematical expressions for determining sensitivity factors of power losses and nodal voltages with respect to nodal power injections in radial distribution power networks, followed by formulas for calculating power losses and voltage magnitudes using these sensitivity factors. Numerical results and discussions using the six-bus radial distribution system are given in Section 3, and the conclusions are inferred in Section 4.

## 2. PROBLEM

### 2.1. Theoretical foundations

The topological structure of power distribution grids is usually radial, often called a tree structure, with the distribution substation as the root bus. The following principles are deployed to number the tree topology of distribution systems:

- The index of the root bus is 1, and all buses of the distribution systems are numbered from 1 to *N*, where *N* is the number of buses.
- The index of a receiving bus has to be greater than that of its sending node.

Consider a distribution line as shown in Figure 1. The notations in this figure encompass:

- $P_{ik}$ and $Q_{ik}$ are the power flow at the sending bus *i*;
- $P'_{ik}$ and $Q'_{ik}$ are active power and reactive power flow at the receiving end *k*, respectively;
- $U_i$ and $U_k$ are the voltage magnitudes at nodes *i* and *k*, respectively;
- $R_{ik}$ and $X_{ik}$ are the resistance and reactance of branch *ik*, respectively.

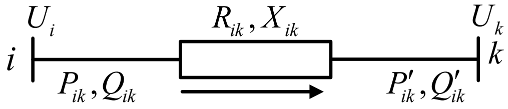


***Figure 1.*** *A two-bus distribution system.*

#### *2.1.1. Assumption*

To derive the mathematical expressions for the power loss and voltage sensitivity factors efficiently, the following assumption is made: the voltage magnitude at bus k remains constant when the demand changes slightly. This assumption simplifies the calculation of partial derivatives, enabling the determination of sensitivity factors without constructing a Jacobian matrix.

#### *2.1.2. Formulas*

##### *2.1.2.1. Power loss sensitivity factors for distribution*

A $(N \times N)$ matrix **T** is defined, in which element $T(k, j)$ is equal to 1 if bus *j* belongs to a sub-tree whose root bus is *k*; otherwise, element $T(k, j)$ equals 0.

Based on matrix **T**, the branch power flow of line *ik* at the receiving bus is given as follows:

$$\begin{aligned} P'_{ik} &= \sum_{j=k}^{N} T(k,j).\left(P_{\mathrm{D}j} - P_{\mathrm{G}j}\right) + \sum_{j=k+1}^{N} T(k,j).\Delta P_{mj} \\ Q'_{ik} &= \sum_{j=k}^{N} T(k,j).\left(Q_{\mathrm{D}j} - Q_{\mathrm{G}j}\right) + \sum_{j=k+1}^{N} T(k,j).\Delta Q_{mj} \end{aligned} \tag{1}$$

The power flow of branch *ik* at the sending bus is calculated using the following equations:

$$P_{ik} = P'_{ik} + \Delta P_{ik}; \quad Q_{ik} = Q'_{ik} + \Delta Q_{ik} \tag{2}$$

where:

- $m$ is the parent bus of the child node $j$;
- $\Delta P_{ik}$ and $\Delta Q_{ik}$ are active and reactive power losses of branch *ik*.

The real and reactive power losses of each line *ik* are determined below:

$$\Delta P_{ik} = \frac{\left(P'_{ik}\right)^2 + \left(Q'_{ik}\right)^2}{U_k^2}.R_{ik}; \quad \Delta Q_{ik} = \frac{\left(P'_{ik}\right)^2 + \left(Q'_{ik}\right)^2}{U_k^2}.X_{ik} \tag{3}$$

It is assumed that the voltage magnitude at bus $k$ remains constant when demand changes slightly. As a result, the sensitivity of power losses of line *ik* with respect to load at bus $j$ is written as follows:

$$\frac{\partial \Delta P_{ik}}{\partial P_{\mathrm{D}j}} = \left(2P'_{ik}\frac{\partial P'_{ik}}{\partial P_{\mathrm{D}j}} + 2Q'_{ik}\frac{\partial Q'_{ik}}{\partial P_{\mathrm{D}j}}\right).\frac{R_{ik}}{U_k^2}; \quad \frac{\partial \Delta Q_{ik}}{\partial P_{\mathrm{D}j}} = \left(2P'_{ik}\frac{\partial P'_{ik}}{\partial P_{\mathrm{D}j}} + 2Q'_{ik}\frac{\partial Q'_{ik}}{\partial P_{\mathrm{D}j}}\right).\frac{X_{ik}}{U_k^2} \tag{4}$$

$$\frac{\partial \Delta P_{ik}}{\partial Q_{\mathrm{D}j}} = \left(2P'_{ik}\frac{\partial P'_{ik}}{\partial Q_{\mathrm{D}j}} + 2Q'_{ik}\frac{\partial Q'_{ik}}{\partial Q_{\mathrm{D}j}}\right).\frac{R_{ik}}{U_k^2}; \quad \frac{\partial \Delta Q_{ik}}{\partial Q_{\mathrm{D}j}} = \left(2P'_{ik}\frac{\partial P'_{ik}}{\partial Q_{\mathrm{D}j}} + 2Q'_{ik}\frac{\partial Q'_{ik}}{\partial Q_{\mathrm{D}j}}\right).\frac{X_{ik}}{U_k^2} \tag{5}$$

The sensitivity of the total power losses is the sum of the sensitivity of power losses of all branches in distribution systems:

$$\frac{\partial \Delta P}{\partial P_{\mathrm{D}j}} = \sum_{\forall \mathrm{line}\, ik}\frac{\partial \Delta P_{ik}}{\partial P_{\mathrm{D}j}}; \quad \frac{\partial \Delta Q}{\partial P_{\mathrm{D}j}} = \sum_{\forall \mathrm{line}\, ik}\frac{\partial \Delta Q_{ik}}{\partial P_{\mathrm{D}j}}; \quad \frac{\partial \Delta P}{\partial Q_{\mathrm{D}j}} = \sum_{\forall \mathrm{line}\, ik}\frac{\partial \Delta P_{ik}}{\partial Q_{\mathrm{D}j}}; \quad \frac{\partial \Delta Q}{\partial Q_{\mathrm{D}j}} = \sum_{\forall \mathrm{line}\, ik}\frac{\partial \Delta Q_{ik}}{\partial Q_{\mathrm{D}j}} \tag{6}$$

According to equation (1), the sensitivity factors of the branch power flows with respect to the real power of demand are expressed below.

$$\frac{\partial P'_{ik}}{\partial P_{\mathrm{D}j}} = T(k,j) + \sum_{j=k+1}^{N} T(k,j).\frac{\partial \Delta P_{mj}}{\partial P_{\mathrm{D}j}}; \quad \frac{\partial Q'_{ik}}{\partial P_{\mathrm{D}j}} = \sum_{j=k+1}^{N} T(k,j).\frac{\partial \Delta Q_{mj}}{\partial P_{\mathrm{D}j}} \tag{7}$$

$$\frac{\partial P'_{ik}}{\partial Q_{\mathrm{D}j}} = \sum_{j=k+1}^{N} T(k,j).\frac{\partial \Delta P_{mj}}{\partial Q_{\mathrm{D}j}}; \quad \frac{\partial Q'_{ik}}{\partial Q_{\mathrm{D}j}} = T(k,j) + \sum_{j=k+1}^{N} T(k,j).\frac{\partial \Delta Q_{mj}}{\partial Q_{\mathrm{D}j}} \tag{8}$$

Then, equations (7)-(8) are substituted into mathematical expressions (4)-(5). The power loss factors for the distribution are recursively defined.

*2.1.2.2. Voltage sensitivity factors*

The voltage drop along the distribution line *ik* can be derived as follows:

$$\Delta U_{ik} = \frac{P_{ik}R_{ik} + Q_{ik}X_{ik}}{U_i} \tag{9}$$

The voltage magnitude at bus $k$ can be determined using the following formula:

$$U_k = U_i - \Delta U_{ik} = U_i - \frac{P_{ik}R_{ik} + Q_{ik}X_{ik}}{U_i} \tag{10}$$

The sensitivity factors of voltage magnitude with respect to nodal power injections are represented as follows:

$$\begin{aligned}\frac{\partial U_k}{\partial P_{\mathrm{D}j}} &= \frac{\partial U_i}{\partial P_{\mathrm{D}j}} - \frac{1}{U_i^2}\left[\left(R_{ik}\frac{\partial P_{ik}}{\partial P_{\mathrm{D}j}} + X_{ik}\frac{\partial Q_{ik}}{\partial P_{\mathrm{D}j}}\right)U_i - \frac{\partial U_i}{\partial P_{\mathrm{D}j}}\left(P_{ik}R_{ik} + Q_{ik}X_{ik}\right)\right] \\ &= \frac{\partial U_i}{\partial P_{\mathrm{D}j}}\left(1 + \frac{\Delta U_{ik}}{U_i}\right) - \frac{1}{U_i}\left(R_{ik}\frac{\partial P_{ik}}{\partial P_{\mathrm{D}j}} + X_{ik}\frac{\partial Q_{ik}}{\partial P_{\mathrm{D}j}}\right)\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\partial U_k}{\partial Q_{\mathrm{D}j}} &= \frac{\partial U_i}{\partial Q_{\mathrm{D}j}} - \frac{1}{U_i^2}\left[\left(R_{ik}\frac{\partial P_{ik}}{\partial Q_{\mathrm{D}j}} + X_{ik}\frac{\partial Q_{ik}}{\partial Q_{\mathrm{D}j}}\right)U_i - \frac{\partial U_i}{\partial Q_{\mathrm{D}j}}\left(P_{ik}R_{ik} + Q_{ik}X_{ik}\right)\right] \\ &= \frac{\partial U_i}{\partial Q_{\mathrm{D}j}}\left(1 + \frac{\Delta U_{ik}}{U_i}\right) - \frac{1}{U_i}\left(R_{ik}\frac{\partial P_{ik}}{\partial Q_{\mathrm{D}j}} + X_{ik}\frac{\partial Q_{ik}}{\partial Q_{\mathrm{D}j}}\right)\end{aligned} \tag{12}$$

If the voltage magnitudes are assumed to be constant when the power injections change, then the voltage sensitivity factors are calculated below:

$$\frac{\partial U_k}{\partial P_{\mathrm{D}j}} = -\frac{1}{U_i}\left(R_{ik}\frac{\partial P_{ik}}{\partial P_{\mathrm{D}j}} + X_{ik}\frac{\partial Q_{ik}}{\partial P_{\mathrm{D}j}}\right); \quad \frac{\partial U_k}{\partial Q_{\mathrm{D}j}} = -\frac{1}{U_i}\left(R_{ik}\frac{\partial P_{ik}}{\partial Q_{\mathrm{D}j}} + X_{ik}\frac{\partial Q_{ik}}{\partial Q_{\mathrm{D}j}}\right) \tag{13}$$

The sensitivity of sending-end power flows can be estimated based on the following mathematical statements.

$$\begin{aligned}&\frac{\partial P_{ik}}{\partial P_{\mathrm{D}j}} = \frac{\partial P'_{ik}}{\partial P_{\mathrm{D}j}} + \frac{\partial \Delta P_{ik}}{\partial P_{\mathrm{D}j}}; \quad \frac{\partial Q_{ik}}{\partial P_{\mathrm{D}j}} = \frac{\partial Q'_{ik}}{\partial P_{\mathrm{D}j}} + \frac{\partial \Delta Q_{ik}}{\partial P_{\mathrm{D}j}} \\ &\frac{\partial P_{ik}}{\partial Q_{\mathrm{D}j}} = \frac{\partial P'_{ik}}{\partial Q_{\mathrm{D}j}} + \frac{\partial \Delta P_{ik}}{\partial Q_{\mathrm{D}j}}; \quad \frac{\partial Q_{ik}}{\partial Q_{\mathrm{D}j}} = \frac{\partial Q'_{ik}}{\partial Q_{\mathrm{D}j}} + \frac{\partial \Delta Q_{ik}}{\partial Q_{\mathrm{D}j}}\end{aligned} \tag{14}$$

*2.1.2.3. Calculation of power losses and nodal voltages using sensitivity factors*

It is assumed that the sensitivity factors of power losses and voltage magnitudes have been calculated from the initial power flow. When the active and reactive powers of the load at bus *j* vary by small amounts of $\Delta P_{\mathrm{D}j}$ and $\Delta Q_{\mathrm{D}j}$, the variation of total power losses and nodal voltages can be given as follows:

$$\mathrm{D}P = \frac{\partial \Delta P}{\partial P_{\mathrm{D}j}}.\Delta P_{\mathrm{D}j} + \frac{\partial \Delta P}{\partial Q_{\mathrm{D}j}}.\Delta Q_{\mathrm{D}j} \tag{15}$$

$$\mathrm{D}U_k = \frac{\partial U_k}{\partial P_{\mathrm{D}j}}.\Delta P_{\mathrm{D}j} + \frac{\partial U_k}{\partial Q_{\mathrm{D}j}}.\Delta Q_{\mathrm{D}j} \tag{16}$$

The equations for calculating the total power losses and voltage magnitudes after the change of load can be represented below:

$$\Delta P^{\mathrm{new}} = \Delta P^{\mathrm{old}} + \mathrm{D}P; \quad U_k^{\mathrm{new}} = U_k^{\mathrm{old}} + \mathrm{D}U_k \tag{17}$$

## 2.2. Experiment preparation

*2.2.1. Instrumentation*

The proposed sensitivity-based method is implemented and validated using the MATPOWER software package [11]. This software is used to solve the initial power flow and provides the benchmark results (using the Newton-Raphson method) to compare with the results obtained from the sensitivity analysis.

*2.2.2. Experimental materials*

The numerical analysis is conducted on a six-bus radial distribution system as shown in Figure 2. The parameters of this test system are configured as follows:

- System voltage: The rated voltage is 10 kV (U base). The voltage magnitude at the root point (Bus 1) is set as 1.05 per unit.
- Line parameters: The resistance and reactance of the distribution lines are identical, with $R_0$ = 0.33 ohm/km and $X_0$ = 0.395 ohm/km, respectively.
- Base power: The calculations are based on a reference power of $S_{base}$ = 1000 kVA.

## 3. RESULTS AND DISCUSSION

### 3.1. Input data

In this section, the sensitivity factors, total power losses, and voltage magnitudes are calculated for the six-bus distribution system shown in Figure 2. The rated voltage of this system is equal to 10 kV. The active and reactive outputs of each distributed generation (DG) are given below ($S_{base}$ = 1000 kVA):

$$\dot{S}_{DG1} = 0.5765 + j1.0000 \text{ p.u.}; \quad \dot{S}_{DG2} = 1.0000 + j0.3287 \text{ p.u.}; \quad \dot{S}_{DG3} = 1.0000 + j0.3287 \text{ p.u.}$$

The line resistance and reactance are identical and equal to 0.33 Ω/km and 0.395 Ω/km, respectively. Furthermore, the voltage magnitude of the root point (bus 1) is set as 1.05 p.u. ($U_{cb}$ = 10 kV). The real and reactive power of demand (D) at each bus in this system is given as below:

$$\dot{S}_{D2} = 1.4 + j0.7; \dot{S}_{D3} = 1.2 + j0.45; \dot{S}_{D4} = 0.8 + j0.5; \dot{S}_{D5} = 1 + j0.6; \dot{S}_{D6} = 2.5 + j1.2$$

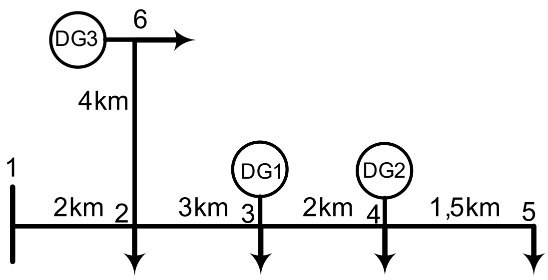


***Figure 2.*** *Six-bus distribution system.*

### 3.2. Methods and simulation tools

The sensitivity-based approach presented in section 2 is applied to calculate total power losses and nodal voltage magnitudes in radial distribution networks. This method utilizes the sensitivity factors of power losses and bus voltages with respect to nodal active and reactive power injections, which are derived from the initial operating point of the system. To evaluate the accuracy of the proposed approach, the Newton–Raphson–based alternating current power flow (ACPF) method is employed as a reference for comparison. The ACPF method solves the nonlinear power flow equations iteratively and is widely regarded as providing accurate steady-state solutions for power systems. All simulations are carried out using the MATPOWER software package. An initial power flow is first performed to obtain the base values of branch power flows and bus voltage magnitudes. Based on this initial solution, the sensitivity factors are calculated and subsequently used to estimate the variations in power losses and nodal voltages under load changes.

To assess the performance of the sensitivity-based method, a series of load variation scenarios is considered. Specifically, the active and reactive power demands at bus 5 are increased by 2.5%, 5.0%, 7.5%, 10.0%, 12.5%, and 15.0%, while maintaining a constant power factor. The results obtained using the sensitivity-based approach are then compared with those obtained from the Newton–Raphson ACPF method.

### 3.3. Simulation results and comments

Using MATPOWER software, the solution of the initial power flow is revealed in table 1.

***Table 1.*** *Results of initial power flow.*

| Line (*i-k*) | $S'_{ik}$ (p.u.) | $\Delta U_{ik}$ (p.u.) | $U_k$ (p.u.) |
|---|---|---|---|
| **1-2** | $4.4031 + j1.8879$ | 0.0439 | 1.0061 |
| **2-3** | $1.4394 + j0.2403$ | 0.0174 | 0.9886 |
| **3-4** | $0.8072 + j0.7799$ | 0.0118 | 0.9769 |
| **4-5** | $1.0000 + j0.6000$ | 0.0088 | 0.9681 |
| **2-6** | $1.5000 + j0.8713$ | 0.0346 | 0.9714 |

*3.3.1. Results for power loss sensitivity factors*

To deploy the equation (1), the power flows at the receiving bus of lines for the six-bus distribution system are represented as follows:

$$P'_{45} = P_{D5};\ P'_{34} = P_{D4} - P_{DG2} + P_{D5} + \Delta P_{45};\ P'_{23} = P_{D3} - P_{DG1} + P_{D4} - P_{DG2} + P_{D5} + \Delta P_{34} + \Delta P_{45}$$

$$P'_{26} = P_{D6} - P_{DG3};\ P'_{12} = P_{D2} + P_{D3} - P_{DG1} + P_{D4} - P_{DG2} + P_{D5} + P_{D6} - P_{DG3} + \Delta P_{23} + \Delta P_{34} + \Delta P_{45} + \Delta P_{26}$$

$$Q'_{45} = Q_{D5};\ Q'_{34} = Q_{D4} - Q_{DG2} + Q_{D5} + \Delta Q_{45};\ Q'_{23} = Q_{D3} - Q_{DG1} + Q_{D4} - Q_{DG2} + Q_{D5} + \Delta Q_{34} + \Delta Q_{45}$$

$$Q'_{26} = Q_{D6} - Q_{DG3}$$

$$Q'_{12} = Q_{D2} + Q_{D3} - Q_{DG1} + Q_{D4} - Q_{DG2} + Q_{D5} + Q_{D6} - Q_{DG3} + \Delta Q_{23} + \Delta Q_{34} + \Delta Q_{45} + \Delta Q_{26}$$

The sensitivity factors of power losses with respect to the real power of the load at bus 5 are rigorously described as follows.

- Sensitivity factors of power loss for the branch connecting node 4 and node 5:

$$\frac{\partial P'_{45}}{\partial P_{D5}} = 1;\ \frac{\partial Q'_{45}}{\partial P_{D5}} = 0$$

$$\frac{\partial \Delta P_{45}}{\partial P_{D5}} = \left(2P'_{45}\frac{\partial P'_{45}}{\partial P_{D5}} + 2Q'_{45}\frac{\partial Q'_{45}}{\partial P_{D5}}\right)\times\frac{R_{45}}{U_5^2} = \left(2\times1.0000\times1 + 2\times0.6000\times0\right)\times\frac{1.5\times0.0033}{0.9681^2} = 0.01056$$

$$\frac{\partial \Delta Q_{45}}{\partial P_{D5}} = \left(2P'_{45}\frac{\partial P'_{45}}{\partial P_{D5}} + 2Q'_{45}\frac{\partial Q'_{45}}{\partial P_{D5}}\right)\times\frac{X_{45}}{U_5^2} = \left(2\times1.0000\times1 + 2\times0.6000\times0\right)\times\frac{1.5\times0.00395}{0.9681^2} = 0.01264$$

- Sensitivity factors of power loss in branch connecting between node 3 and node 4:

$$\frac{\partial P'_{34}}{\partial P_{D5}} = 1 + \frac{\partial \Delta P_{45}}{\partial P_{D5}} = 1 + 0.01056 = 1.01056;\ \frac{\partial Q'_{34}}{\partial P_{D5}} = \frac{\partial \Delta Q_{45}}{\partial P_{D5}} = 0.01264$$

$$\frac{\partial \Delta P_{34}}{\partial P_{D5}} = \left(2P'_{34}\frac{\partial P'_{34}}{\partial P_{D5}} + 2Q'_{34}\frac{\partial Q'_{34}}{\partial P_{D5}}\right)\times\frac{R_{34}}{U_4^2}$$

$$= 2\times\left(0.8072\times1.01056 + 0.7799\times0.01264\right)\times\frac{2\times0.0033}{0.9769^2} = 0.01142$$

$$\frac{\partial \Delta Q_{34}}{\partial P_{D5}} = \frac{\partial \Delta P_{34}}{\partial P_{D5}}\times\frac{X_{34}}{R_{34}} = 0.01367$$

- Sensitivity factors of power loss for the branch connecting between node 2 and node 3:

$$\frac{\partial P'_{23}}{\partial P_{D5}} = 1 + \frac{\partial \Delta P_{34}}{\partial P_{D5}} + \frac{\partial \Delta P_{45}}{\partial P_{D5}} = 1 + 0.01142 + 0.01056 = 1.02198;$$

$$\frac{\partial Q'_{23}}{\partial P_{D5}} = \frac{\partial \Delta Q_{34}}{\partial P_{D5}} + \frac{\partial \Delta Q_{45}}{\partial P_{D5}} = 0.01367 + 0.01264 = 0.02631$$

$$\frac{\partial \Delta P_{23}}{\partial P_{D5}} = \left(2P'_{23}\frac{\partial P'_{23}}{\partial P_{D5}} + 2Q'_{23}\frac{\partial Q'_{23}}{\partial P_{D5}}\right) \times \frac{R_{23}}{U_3^2}$$

$$= 2\times\left(1.4394\times1.02198 + 0.2403\times0.02631\right)\times\frac{3\times0.0033}{0.9886^2} = 0.02993$$

$$\frac{\partial \Delta Q_{23}}{\partial P_{D5}} = \frac{\partial \Delta P_{23}}{\partial P_{D5}} \times \frac{X_{23}}{R_{23}} = 0.03583$$

The calculated results of power loss sensitivity factors for the six-bus distribution system are depicted in table 2.

***Table 2.*** *Power loss sensitivity factors for six-bus system.*

| **Load (*j*)** | $\partial\Delta P / \partial P_{Dj}$ | $\partial\Delta P / \partial Q_{Dj}$ | $\partial\Delta Q / \partial P_{Dj}$ | $\partial\Delta Q / \partial Q_{Dj}$ |
|---|---|---|---|---|
| **2** | 0.05742 | 0.02462 | 0.06873 | 0.02947 |
| **3** | 0.08912 | 0.02991 | 0.10667 | 0.03580 |
| **4** | 0.10168 | 0.04205 | 0.12170 | 0.05033 |
| **5** | 0.11384 | 0.04935 | 0.13627 | 0.05907 |
| **6** | 0.10303 | 0.05111 | 0.12333 | 0.06118 |

*3.3.2. Results for voltage sensitivity factors*

The sensitivity factors of voltage magnitude at all buses with respect to active power injection at node 2 are computed using the following expressions. Similarly, the voltage sensitivity factors for the six-bus system are sketched in table 3.

$$\frac{\partial U_2}{\partial P_{D2}} = -\frac{1}{U_1}\left(R_{12}\frac{\partial P_{12}}{\partial P_{D2}} + X_{12}\frac{\partial Q_{12}}{\partial P_{D2}}\right) = -\frac{1}{1.05}\times\left(0.0066\times1.05742 + 0.0079\times0.06873\right) = -0.00716$$

$$\frac{\partial U_3}{\partial P_{D2}} = \frac{\partial U_2}{\partial P_{D2}}\times\left(1 + \frac{\Delta U_{23}}{U_2}\right) - \frac{1}{U_2}\left(R_{23}\frac{\partial P_{23}}{\partial P_{D2}} + X_{23}\frac{\partial Q_{23}}{\partial P_{D2}}\right)$$

$$= -0.00716\times\left(1 + \frac{0.0174}{1.0061}\right) - \frac{1}{1.0061}\times\left(0.0099\times0 + 0.01185\times0\right) = -0.00728$$

$$\frac{\partial U_4}{\partial P_{D2}} = \frac{\partial U_3}{\partial P_{D2}}\times\left(1 + \frac{\Delta U_{34}}{U_3}\right) - \frac{1}{U_3}\left(R_{34}\frac{\partial P_{34}}{\partial P_{D2}} + X_{34}\frac{\partial Q_{34}}{\partial P_{D2}}\right)$$

$$= -0.00728\times\left(1 + \frac{0.0118}{0.9886}\right) - \frac{1}{0.9886}\times\left(0.0066\times0 + 0.0079\times0\right) = -0.00737$$

$$\frac{\partial U_5}{\partial P_{D2}} = \frac{\partial U_4}{\partial P_{D2}}\times\left(1 + \frac{\Delta U_{45}}{U_4}\right) - \frac{1}{U_4}\left(R_{45}\frac{\partial P_{45}}{\partial P_{D2}} + X_{45}\frac{\partial Q_{45}}{\partial P_{D2}}\right)$$

$$= -0.00737\times\left(1 + \frac{0.0088}{0.9769}\right) - \frac{1}{0.9769}\times\left(0.00495\times0 + 0.005925\times0\right) = -0.00744$$

$$\frac{\partial U_6}{\partial P_{D2}} = \frac{\partial U_2}{\partial P_{D2}} \times \left(1 + \frac{\Delta U_{26}}{U_2}\right) - \frac{1}{U_2}\left(R_{26}\frac{\partial P_{26}}{\partial P_{D2}} + X_{26}\frac{\partial Q_{26}}{\partial P_{D2}}\right)$$
$$= -0.00716 \times \left(1 + \frac{0.0346}{1.0061}\right) - \frac{1}{1.0061} \times \left(0.0132 \times 0 + 0.0158 \times 0\right) = -0.00741$$

***Table 3.*** *Voltage sensitivity factors for the six-bus system.*

| **Load (*j*)** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|
| $\partial U_2 / \partial P_{Dj}$ | –0.00716 | –0.00765 | –0.00784 | –0.00803 | –0.00786 |
| $\partial U_2 / \partial Q_{Dj}$ | –0.00790 | –0.00798 | –0.00817 | –0.00828 | –0.00831 |
| $\partial U_3 / \partial P_{Dj}$ | –0.00729 | –0.01832 | –0.01879 | –0.01925 | –0.00800 |
| $\partial U_3 / \partial Q_{Dj}$ | –0.00804 | –0.02001 | –0.02047 | –0.02075 | –0.00845 |
| $\partial U_4 / \partial P_{Dj}$ | –0.00737 | –0.01854 | –0.02587 | –0.02651 | –0.00809 |
| $\partial U_4 / \partial Q_{Dj}$ | –0.00813 | –0.02025 | –0.02888 | –0.02926 | –0.00855 |
| $\partial U_5 / \partial P_{Dj}$ | –0.00744 | –0.01870 | –0.02610 | –0.03195 | –0.00817 |
| $\partial U_5 / \partial Q_{Dj}$ | –0.00821 | –0.02043 | –0.02914 | –0.03567 | –0.00863 |
| $\partial U_6 / \partial P_{Dj}$ | –0.00741 | –0.00791 | –0.00811 | –0.00830 | –0.02259 |
| $\partial U_6 / \partial Q_{Dj}$ | –0.00817 | –0.00826 | –0.00845 | –0.00856 | –0.02507 |

*3.3.3. Results for power losses and nodal voltages using sensitivity factors*

It is assumed that there are respective increases of 2.5%, 5.0%, 7.5%, 10%, 12.5%, and 15.0% in the power consumed by the load at bus 5, with the power factor remaining constant. The power flow results using the sensitivity approach are compared with those of the Newton-Raphson (NR) method, as shown in tables 4 and 5.

***Table 4.*** *Comparison of power losses between two methods.*

| **Increased level of load 5** | **Active power losses (p.u.)** | | | **Reactive power losses (p.u.)** | | |
|---|---|---|---|---|---|---|
| | **Sensitivity method** | **NR method** | **Error (%)** | **Sensitivity method** | **NR method** | **Error (%)** |
| 2.5% | 0.23281 | 0.23305 | 0.104 | 0.27867 | 0.27895 | 0.102 |
| 5.0% | 0.23639 | 0.23695 | 0.236 | 0.28296 | 0.28362 | 0.233 |
| 7.5% | 0.23998 | 0.24092 | 0.391 | 0.28725 | 0.28838 | 0.389 |
| 10.0% | 0.24357 | 0.24496 | 0.571 | 0.29155 | 0.29321 | 0.568 |
| 12.5% | 0.24715 | 0.24908 | 0.773 | 0.29584 | 0.29814 | 0.770 |
| 15.0% | 0.25074 | 0.25326 | 0.996 | 0.30013 | 0.30315 | 0.994 |

From both table 4 and table 5, it can be seen that when the load power at bus 5 varies by 2.5% to 15.0%, the errors in total real and reactive power losses climb to just under 1%. At the same time, the errors of voltage magnitudes between the two techniques are insignificant and can be ignored. Therefore, the sensitivity-based approach is reasonable for determining power losses and node voltages as load power changes in power distribution systems.

***Table 5.*** *Comparison of nodal voltages between two approaches.*

| Node | Increased level of 5.0% | | | Increased level of 15.0% | | |
|---|---|---|---|---|---|---|
| | Sensitivity method | NR method | Error (%) | Sensitivity method | NR method | Error (%) |
| 2 | 1.0054 | 1.0054 | 0.000 | 1.0041 | 1.0041 | 0.002 |
| 3 | 0.9871 | 0.9871 | 0.000 | 0.9839 | 0.9838 | 0.005 |
| 4 | 0.9747 | 0.9747 | 0.001 | 0.9703 | 0.9702 | 0.007 |
| 5 | 0.9654 | 0.9654 | 0.001 | 0.9601 | 0.9600 | 0.008 |
| 6 | 0.9707 | 0.9707 | 0.000 | 0.9694 | 0.9694 | 0.003 |

***Table 6.*** *Comparison of power losses between two methods.*

| Decreased level of load 5 | Active power losses (p.u.) | | | Reactive power losses (p.u.) | | |
|---|---|---|---|---|---|---|
| | Sensitivity method | NR method | Error (%) | Sensitivity method | NR method | Error (%) |
| 2.5% | 0.22566 | 0.22548 | 0.080 | 0.27011 | 0.26990 | 0.078 |
| 5.0% | 0.22208 | 0.22179 | 0.131 | 0.26582 | 0.26547 | 0.132 |
| 7.5% | 0.21849 | 0.21816 | 0.151 | 0.26153 | 0.26113 | 0.153 |
| 10.0% | 0.21491 | 0.21460 | 0.147 | 0.25724 | 0.25687 | 0.144 |
| 12.5% | 0.21132 | 0.21110 | 0.104 | 0.25295 | 0.25268 | 0.107 |
| 15.0% | 0.20773 | 0.20767 | 0.003 | 0.24866 | 0.24858 | 0.032 |

***Table 7.*** *Comparison of nodal voltages between two approaches.*

| Node | Decreased level of 5.0% | | | Decreased level of 15.0% | | |
|---|---|---|---|---|---|---|
| | Sensitivity method | NR method | Error (%) | Sensitivity method | NR method | Error (%) |
| 2 | 1.0067 | 1.0067 | 0.000 | 1.0080 | 1.0080 | 0.000 |
| 3 | 0.9902 | 0.9902 | 0.000 | 0.9934 | 0.9934 | 0.000 |
| 4 | 0.9791 | 0.9791 | 0.000 | 0.9835 | 0.9834 | 0.010 |
| 5 | 0.9708 | 0.9708 | 0.001 | 0.9760 | 0.9760 | 0.000 |
| 6 | 0.9721 | 0.9721 | 0.001 | 0.9734 | 0.9734 | 0.000 |

It is hypothesized that the load at bus 5 undergoes sequential reductions of 2.5%, 5.0%, 7.5%, 10.0%, 12.5%, and 15.0% while maintaining a constant power factor. The power flow results obtained through the sensitivity-based approach are subsequently validated against the NR method, with the comparative data summarized in tables 6 and 7.

According to the data presented in tables 6 and 7, as the power demand at bus 5 scales down from 2.5% to 15.0%, the resulting errors in both active and reactive power losses remain remarkably small, peaking at only 0.153%. Simultaneously, the deviation in voltage magnitudes between the two computational techniques is negligible and can be safely disregarded. Consequently, the sensitivity-based framework demonstrates high precision in determining power losses and nodal voltages during load shedding or decrease scenarios in distribution networks.

## 4. CONCLUSIONS

This paper presents a method for constructing mathematical expressions for power-loss sensitivity factors and voltage sensitivity factors based on the radial structure of power distribution networks. These sensitivity factors obtained from the initial power flow computation are then used to calculate power losses and voltage magnitudes as nodal power injections change in a six-bus radial distribution system. The results reveal that the solutions obtained with the sensitivity approach are reasonably close to those of the ACPF method. These findings provide valuable information for the Distribution System Operator (DSO) to effectively operate and plan active distribution systems. While the proposed Jacobian-free sensitivity method demonstrates high efficiency for radial distribution grids, its application is currently limited to radial topologies. Future studies will explore adapting this framework to ring and meshed network configurations.

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## TÓM TẮT

**Áp dụng phương pháp độ nhạy để tính toán tổn thất công suất và điện áp nút trong lưới điện phân phối hình tia**

*Bài báo này trình bày phương pháp độ nhạy để tính toán tổn thất công suất và điện áp nút trong lưới điện phân phối. Các hệ số độ nhạy tổn thất công suất và điện áp theo công suất nút có xét đến cấu trúc hình tia của lưới điện phân phối được mô tả chi tiết. Lưới điện 6 nút được áp dụng để tính toán tổn thất công suất và điện áp nút theo phương pháp độ nhạy và các kết quả tính toán này được so sánh với phương pháp trào lưu công suất xoay chiều (ACPF). Phương pháp ACPF sử dụng kỹ thuật lặp để giải hệ phương trình phi tuyến mô tả hệ thống điện trong chế độ xác lập và các kết quả tính toán theo phương pháp này là chính xác. Các kết quả tính toán cho thấy sai số của phương pháp độ nhạy so với phương pháp ACPF là nhỏ và hoàn toàn có thể chấp nhận được. Với điện áp nút, kết quả tính toán của cả hai phương pháp gần như giống nhau.*

**Từ khóa:** Lưới điện phân phối hình tia; Trào lưu công suất; Phân tích độ nhạy; Hệ số độ nhạy tổn thất công suất; Hệ số độ nhạy điện áp.