# Using a MILP-based model for solving the optimal power flow problem in distribution networks integrated with step-voltage regulators

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## ABSTRACT

*This paper proposes a mixed-integer linear programming (MILP) model for the optimal power flow (OPF) problem in distribution grids with step-voltage regulators (SVRs). The OPF problem aims to minimize the total cost of the distribution grid, including the cost of purchasing effective and reactive power from the transmission grid, the cost of generating effective power, and the reactive power of distributed power sources. The constraints considered include the power flow equations, the node voltage constraints, the branch transmission power limits, the power factor limits at the connection point, and the SVR constraint. The MILP model of the OPF problem in the distribution grid was developed from the mixed-integer nonlinear programming (MINLP) model by linearizing the power flow equations and building an accurate linear SVR model. The proposed MILP model is evaluated on an IEEE33-node grid with different load scenarios using the GAMS programming language and the CPLEX commercial solver. The calculation results show that the MILP model is computationally efficient, and the optimal pressure distribution of SVR reduces the operating costs of the distribution grid.*

**Keywords:** Optimal Power Flow (OPF); Mixed-integer linear programming; Step-voltage regulators (SVR); Distribution network; Local Marginal Price (LMP).

## 1. INTRODUCTION

Nowadays, global power systems, including that of Vietnam, are undergoing a structural transition from a monopolistic framework toward a competitive market environment. The evolution of electricity markets typically progresses through three distinct stages: the generation competitive market, the wholesale electricity market, and the retail electricity market. Among these, the retail market model is increasingly being adopted worldwide due to its ability to ensure equity and price transparency. In a retail electricity market, the Market Operator (MO) is responsible for aggregating sell-offers from generation companies, buy-bids from power purchasers, and data regarding ancillary service/reserve markets. Upon consolidating these bidding profiles, the MO performs market clearing by solving the Optimal Power Flow (OPF) problem [1]. Based on the market-clearing results, MO determines the locational marginal prices, which serve as the foundation for financial settlements in the day-ahead market. In the day-ahead market, performing Optimal Power Flow (OPF) calculations is critical to ensuring equitable and transparent market settlements. Consequently, the mathematical model must satisfy stringent requirements, including computational reliability, execution speed, and practical feasibility. Some researchers have proposed the Lambda iteration method to address the Economic Dispatch problem for power generation units [2]. However, using iterative algorithms to find the global optimum can suffer from slow convergence or fail to achieve the global optimal solution. Furthermore, these models typically account for only simplified constraints. To incorporate more complex operational constraints and improve the accuracy of the results, researchers have proposed second-order cone programming relaxation methods to solve the OPF problem [3]. This model offers flexibility and can handle complex operational constraints in power systems.

Nevertheless, such complexity can lead to computational challenges when applied to large-scale power systems. Alternatively, another approach employs an iterative algorithm to solve the OPF problem while incorporating demand response [4], however, the integration of a price-responsive load model significantly degrades solution accuracy and computational efficiency. To address this, researchers have proposed an alternative OPF formulation that incorporates system security constraints. This model employs robust optimization techniques to identify and account for various contingency scenarios, thereby ensuring secure system operation under all operating conditions [5]. The limitations of this approach include difficulty in standardizing contingency scenarios, a high computational burden, and failure to account for the increasing penetration of renewable energy sources. Furthermore, a significant challenge in solving the OPF problem lies in the nonlinearity of the branch flow limit constraints. To overcome this nonlinearity, researchers have proposed a regular-polygon linearization method [6]. This approach guarantees that the obtained solution is the global optimum. In addition, researchers have proposed a hybrid method combining the Lagrange multiplier method with deep reinforcement learning to solve the real-time OPF problem. This method can handle large-scale power systems and account for the impacts of system contingencies. Furthermore, by not requiring an exact mathematical model of the power system, it significantly reduces the computational burden [7].

To solve the OPF problem, the Alternating Direction Method of Multipliers (ADMM) approach offers rapid convergence for large-scale power systems [8]. While Sequential Linear Programming (SLP) guarantees a global optimal solution, its computational complexity scales significantly with system size [9]. Alternatively, Stochastic Dual Dynamic Programming optimally allocates power generation while accounting for transmission limits [10]. Recently, Second-Order Cone Programming (SOCP) has been increasingly adopted for distribution networks due to its convexity, which guarantees a global optimum with negligible approximation error [11].

In practice, incorporating voltage-regulating devices mitigates voltage deviations and power losses, but accurate mathematical modeling substantially increases computational complexity [12]. Although exact linearization of On-Load Tap Changer models struggles with complex nonlinear constraints, combining a Mixed-Integer Linear Programming (MILP) formulation with SOCP facilitates accurate modeling of these devices and topology constraints [13]. Additionally, an integrated optimization model provides an efficient solution for voltage regulation and service restoration following grid faults [14]. Another study proposed a MILP optimization approach integrated with network reconfiguration.

From the aforementioned methods, it can be seen that there are numerous approaches to solving the OPF problem. However, Linear Programming (LP) models remain widely adopted in the industry due to their computational efficiency and, crucially, their ability to guarantee a global optimal solution. This paper proposes an optimal power flow framework that integrates voltage regulating devices. Initially, non-linear formulations are employed to describe the network topology constraints and the operational limits of the voltage-regulating equipment. Subsequently, the paper introduces linearization techniques to handle these non-linearities, transforming the original formulation into a comprehensive linear model. The employment of this LP framework ensures that the resulting solution is the global optimum.

The paper is structured into four sections. Section 2 develops the MILP model for the OPF problem by formulating the initial mixed-integer nonlinear programming (MINLP) model with voltage regulation devices and applying linearization techniques to its nonlinear constraints, ensuring a global optimal solution. Numerical results and discussions using the IEEE 33-bus distribution system are given in Section 3, and the conclusions are inferred in Section 4.

## 2. PROBLEM

### 2.1. Theoretical foundations

*2.1.1. Assumption*

To effectively linearize the power flow equations, the following assumptions are made: the system operates in a steady state, branch power losses are ignored because they are insignificant relative to the power flows, and nodal voltages are approximated at 1 per unit.

*2.1.2. Non-linear programming formulation for optimal power flow considering voltage regulating devices*

The objective function of the proposed model aims to minimize the total system cost, formulated as follows:

$$\min \sum_{i \in \Omega_{\mathrm{DG}}} \left( C_i^{\mathrm{PG}} P_i^{\mathrm{G}} + C_i^{\mathrm{QG}} \hat{Q}_i^{\mathrm{G}} \right) \tag{1}$$

where, $C_i^{\mathrm{PG}}$ and $C_i^{\mathrm{QG}}$ represent the active and reactive power bid prices of the *i*-th generating unit; $P_i^{\mathrm{G}}$ and $\hat{Q}_i^{\mathrm{G}}$ denote the active and reactive power outputs of the power source at bus *i*, respectively; $\Omega_{\mathrm{DG}}$ is the set of buses connected to distributed generators.

Branch flow limit constraints:

$$P_{ij}^2 + Q_{ij}^2 \le \left( S_{ij}^{\max} \right)^2; \; ij \in \Omega_{\mathrm{L}} \tag{2}$$

where, $P_{ij}$ and $Q_{ij}$ are the active and reactive power flows on branch *ij*; $S_{ij}^{\max}$ represents the apparent power flow limit of branch *ij*; $\Omega_{\mathrm{L}}$ denotes the set of all branches in the power grid.

Generation capacity constraints:

$$P_i^{\mathrm{G,min}} \le P_i^{\mathrm{G}} \le P_i^{\mathrm{G,max}}; \quad i \in \Omega_{\mathrm{DG}} \tag{3}$$

$$Q_i^{\mathrm{G,min}} \le Q_i^{\mathrm{G}} \le Q_i^{\mathrm{G,max}}; \quad i \in \Omega_{\mathrm{DG}} \tag{4}$$

where, $P_i^{\mathrm{G,min}}$ and $P_i^{\mathrm{G,max}}$ are the minimum and maximum active power limits of the generating unit at bus *i*, respectively; $Q_i^{\mathrm{G,min}}$ and $Q_i^{\mathrm{G,max}}$ denote the minimum and maximum reactive power limits of the generating unit at bus *i*, respectively.

Reactive power remuneration constraints for DGs:

$$\hat{Q}_i^{\mathrm{G}} \ge Q_i^{\mathrm{G}}; \; i \in \Omega_{\mathrm{DG}} \tag{5}$$

$$\hat{Q}_i^{\mathrm{G}} \ge -Q_i^{\mathrm{G}}; \; i \in \Omega_{\mathrm{DG}} \tag{6}$$

Bus voltage limit constraints:

$$U_i^{\min} \le U_i \le U_i^{\max}; \; i = 1,...,N \tag{7}$$

where, $U_i$ is the voltage at bus *i*; $U_i^{\min}$ and $U_i^{\max}$ are the minimum and maximum operating voltage limits, respectively;

System-wide power balance constraints:

$$\sum_{i \in \Omega_{\mathrm{DG}}} P_i^{\mathrm{G}} = \sum_{i=2}^{N} P_i^{\mathrm{D}} + \Delta P \tag{8}$$

$$\sum_{i \in \Omega_{\mathrm{DG}}} Q_i^{\mathrm{G}} = \sum_{i=2}^{N} Q_i^{\mathrm{D}} + \Delta Q \tag{9}$$

where, $P_i^{\mathrm{D}}$ and $Q_i^{\mathrm{D}}$ are the active and reactive power demands at bus *i*, respectively; $\Delta P$ and $\Delta Q$ denote the total active and reactive power losses of the network.

Voltage regulation device constraints

$$U_i = \left(1 + \frac{n.e_0}{100}\right).U_j \tag{10}$$

where, $U_i$ and $U_j$ are voltages at the primary and secondary sides of the voltage regulating device; *n* denotes the tap position of the voltage regulator; $e_0$ represents the voltage step size per tap.

*2.1.3. Linear formulation of the power flow problem*

When the power system operates in a steady state, branch power losses are negligible compared to the branch power flows. Concurrently, the nodal voltages are approximately equal to 1 per unit. Therefore, the linearized power flow model is formulated as follows:

$$\sum_{k\in\Omega_{\mathrm{TP}}(i)} P_{ki} - \sum_{j\in\Omega_{\mathrm{GP}}(i)} P_{ij} = P_i^{\mathrm{G}} - P_i^{\mathrm{D}}; i = 2,...,N \tag{11}$$

$$\sum_{k\in\Omega_{\mathrm{TP}}(i)} Q_{ki} - \sum_{j\in\Omega_{\mathrm{GP}}(i)} Q_{ij} = Q_i^{\mathrm{G}} - Q_i^{\mathrm{D}}; i = 2,...,N \tag{12}$$

$$U_i = U_j - \left(r_{ij}P_{ij} + x_{ij}Q_{ij}\right); i = 2,...,N; j \in \Omega_{\mathrm{TP}}\left(i\right) \tag{13}$$

where, $r_{ij}$ and $x_{ij}$ are the resistance and reactance of branch *ij*; $\Omega_{\mathrm{TP}}(i)$ and $\Omega_{\mathrm{GP}}(i)$ denote the set of buses transmitting power to bus *i* and the set of buses receiving power from bus *i*.

*2.1.4. Linearization of transmission capacity constraints*

The original transmission capacity limits are inherently non-linear, which can adversely affect the solver's ability to find a solution. This section presents a linearization method for these constraints to ensure that a global optimal solution is achieved. The regular polygon linearization technique [6] is employed, as illustrated in Figure 1.

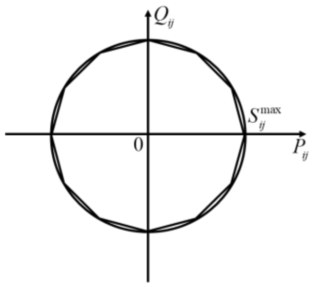


***Figure 1.*** *Regular polygon linearization model.*

Quadratic constraints, such as the branch transmission capacity limits, are restricted by points within a circular region. By dividing this circle into 12 circular arcs, a 12-sided regular polygon is formed using 12 chords to linearize these quadratic constraints. The resulting system of linear equations is formulated as follows:

$$\alpha_{c,0}P_{ij} + \alpha_{c,1}Q_{ij} + \alpha_{c,2}S_{ij}^{\max} \le 0; \quad c = 1,...,12 \tag{14}$$

where $\alpha_{c,0}$, $\alpha_{c,1}$, and $\alpha_{c,2}$ are the coefficients of the linearized constraints

*2.1.5. Linearized model of the voltage regulation device*

Assume that a voltage regulation device is installed on an arbitrary branch *ij*. The equivalent circuit of the regulator is represented by branch *ii'*, which is connected in series with an ideal transformer having a tap ratio $\left(t_{ij} = 1 + n.e_0/100\right)$, as illustrated in Figure 2.

By defining: $n = \sum_{m\in\Omega(\mathrm{SVR})} m.b_m; \ \sum_{m\in\Omega(\mathrm{SVR})} b_m = 1$ (15)

where, $b_m$ is a binary variable that takes the value of 0 when the transformer tap position is zero, and 1 otherwise, *m* ($m \neq 0$); Ω(SVR) denotes the set of possible tap positions for the voltage regulation device.

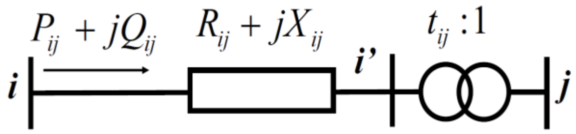


***Figure 2.*** *Equivalent circuit of the voltage regulator.*

Expression (10) is linearized as follows:

$$U_{i'} = \left(1 + \frac{n.e_0}{100}\right).U_j = U_j + \left(\sum_{m\in\Omega(\mathrm{SVR})} m.b_m.U_j.e_0\right) \Big/ 100 \qquad (16)$$

The non-linear term $U_j.b_m$ is further linearized by introducing an auxiliary variable:

$$x_{j,m} = U_j.b_m$$

Consequently, expression (16) is transformed into:

$$U_{i'} = U_j + \left(\sum_{m\in\Omega(\mathrm{SVR})} m.x_{j,m}.e_0\right) \Big/ 100 \qquad (17)$$

The linearization constraints are formulated as follows:

$$-M.b_m \le x_{j,m} \le M.b_m;\ m \in \Omega(\mathrm{SVR}) \qquad (18)$$

$$U_j - M\left(1 - b_m\right) \le x_{j,m} \le U_j + M\left(1 - b_m\right); m \in \Omega(\mathrm{SVR}) \qquad (19)$$

where *M* represents a sufficiently large constant

*2.1.6. Mixed-integer linear programming model for optimal power flow considering voltage regulation devices.*

The proposed MILP model for the optimal power flow problem considering voltage regulation consists of the objective function (1) and constraints (3)-(4), (5)-(6), (7), (11)-(13), (14),(15), (17)-(19).

**2.2. Experiment preparation**

*2.2.1. Instrumentation*

The OPF model is implemented using the GAMS modeling language and solved with the CPLEX solver. All computations are conducted on a personal computer equipped with an AMD Ryzen 5 5600G processor at 3.9 GHz and 32GB of RAM. The computational time is 0.5s, with a relative gap tolerance of zero.

*2.2.2. Experimental materials*

The numerical analysis is conducted on the IEEE 33-bus distribution network, as illustrated in Figure 3. The parameters of this test system are configured as follows:

- Base system: All parameters are calculated using a per-unit system with a base power of $S_{base}$ = 5 MVA and a base voltage of $U_{base}$ = 12.66 kV.
- Operational limits: The required voltage limits are set between 0.95 and 1.05 p.u. Additionally, all branches share a common transmission capacity limit of 5 MVA.
- Voltage regulators configuration: Step-voltage regulators (SVR) are installed on branches 7–8, 2–19, and 6–26. Each regulator features 17 tap positions with a step voltage variation of 0.625% per tap.

- Market and load data: The active and reactive power prices at the point of common coupling with the transmission grid (LMP) are 16 $/MWh and 6 $/MVArh, respectively. The load is assumed to be price-inelastic (fixed load).

## 3. RESULTS AND DISCUSSION

### 3.1. Input data

The diagram and parameters of the IEEE 33-node distribution network are illustrated in Figure 3 and Table 1, respectively. The load is assumed to be price-inelastic (fixed load). The model is evaluated under two primary scenarios: the first considers the network without SVR, while the second incorporates SVR. Each scenario is simulated at two distinct loading levels: 100% and 150% of the base load.

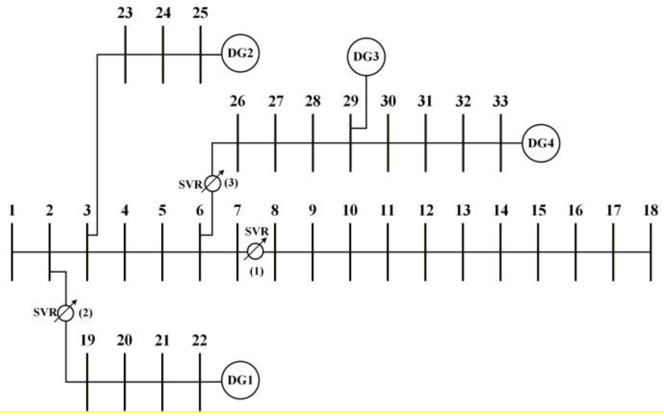


***Figure 3.*** *IEEE 33-bus distribution network.*

### 3.2. Numerical results and discussions

The bus voltages corresponding to each loading level are detailed in Table 2. For various loading conditions, the integration of voltage regulators operating at optimal tap settings ensures that the voltage profile remains within permissible limits and avoids operating at boundary points. Specifically, in the 100% base load scenario, the voltage at bus 22 is reduced from 1.05 to 1.0077 per unit. Furthermore, in the 150% load case, the voltage at bus 18 improves significantly, increasing from 0.95 to 0.9970 per unit.

The power output of the generating sources is presented in Table 3. Under the base load condition, the integration of voltage regulation devices into the network results in a significant reduction in the reactive power drawn from the transmission grid. At the 150% load level, the voltage at bus 18 reaches its lower limit in the scenario without SVR. Consequently, the system is forced to dispatch power from DG4 to maintain stability. In contrast, with the integration of SVR, no buses operate at their lower voltage boundaries, eliminating the need for power dispatch from DG4. Furthermore, there is a notable reduction in the reactive power drawn from the transmission grid. The optimal tap positions of the voltage regulation devices are presented in Table 4.

The electricity procurement costs are detailed in Table 5. It is observed that the integration of SVR leads to a reduction in both the cost of reactive power purchased from the transmission grid and the payments made to DGs. In both simulated scenarios (100% and 150% of the base load levels), the total operational expenditure of the network is significantly minimized when voltage regulation devices are incorporated into the system.

***Table 1.*** *Distributed generation data.*

| Bus | $P_G^{min}$ (kW) | $P_G^{max}$ (kW) | $\cos\varphi^{min}$ | Active power offer price ($/MWh) | Reactive power offer price ($/MVArh) |
|---|---|---|---|---|---|
| 22 | 0 | 600 | 0.95 | 12 | 2 |
| 25 | 0 | 600 | 0.95 | 15 | 3 |
| 29 | 0 | 1000 | 0.95 | 13 | 4 |
| 33 | 0 | 600 | 0.95 | 17 | 5 |

***Table 2.*** *Bus voltage corresponding to each loading results.*

| ***U*** **(p.u.)** | **Loading level of 100%** | | **Loading level of 150%** | |
|---|---|---|---|---|
| | **Without SVR** | **With SVR** | **Without SVR** | **With SVR** |
| 1 | 1.0500 | 1.0500 | 1.0500 | 1.0500 |
| 2 | 1.0486 | 1.0487 | 1.0473 | 1.0473 |
| **2’** | **1.0485** | **1.0493** | **1.0471** | **1.0469** |
| 3 | 1.0410 | 1.0411 | 1.0335 | 1,0331 |
| 4 | 1.0367 | 1.0369 | 1.0260 | 1.0253 |
| 5 | 1.0327 | 1.0328 | 1.0188 | 1.0178 |
| 6 | 1.0222 | 1.0222 | 1.0004 | 0.9988 |
| **6’** | **1.0219** | **1.0144** | **0.9994** | **0.9842** |
| 7 | 1.0190 | 1.0191 | 0.9956 | 0.9940 |
| **7’** | **1.0145** | **1.0124** | **0.9889** | **0.9840** |
| **8** | **1.0145** | **0.9938** | **0.9889** | **1.0358** |
| 9 | 1.0087 | 0.9880 | 0.9882 | 1.0272 |
| 10 | 1.0034 | 0.9827 | 0.9722 | 1.0192 |
| 11 | 1.0026 | 0.9819 | 0.9710 | 1.0180 |
| 12 | 1.0012 | 0.9805 | 0.9689 | 1.0159 |
| 13 | 0.9956 | 0.9749 | 0.9605 | 1.0075 |
| 14 | 0.9935 | 0.9728 | 0.9574 | 1.0044 |
| 15 | 0.9922 | 0.9715 | 0.9555 | 1.0024 |
| 16 | 0.9910 | 0.9703 | 0.9536 | 1.0006 |
| 17 | 0.9891 | 0.9684 | 0.9508 | 0.9978 |
| 18 | 0.9886 | 0.9679 | **0.9500** | **0.9970** |
| **19** | **1.0485** | **0.9993** | **1.0471** | **0.9970** |
| 20 | 1.0489 | 1.0031 | 1.0476 | 0.9990 |
| 21 | 1.0491 | 1.0045 | 1.0482 | 1.0001 |
| 22 | **1.0500** | **1.0077** | **1.0500** | **1.0030** |
| 23 | 1.0396 | 1.0397 | 1.0303 | 1.0299 |
| 24 | 1.0374 | 1.0375 | 1.0248 | 1.0244 |
| 25 | 1.0384 | 1.0385 | 1.0242 | 1.0238 |
| **26** | **1.0219** | **0.9661** | **0.9994** | **1.0360** |
| 27 | 1.0216 | 0.9658 | 0.9981 | 1.0345 |
| 28 | 1.0196 | 0.9638 | 0.9917 | 1.0273 |
| 29 | 1.0185 | 0.9627 | 0.9875 | 1.0224 |
| 30 | 1.0152 | 0.9594 | 0.9830 | 1.0175 |
| 31 | 1.0114 | 0.9556 | 0.9781 | 1.0118 |
| 32 | 1.0106 | 0.9548 | 0.9771 | 1.0105 |
| 33 | 1.0103 | 0.9545 | 0.9770 | 1.0101 |

***Table 3.*** *Power output of DGs.*

| | Loading level of 100% | | | | Loading level of 150% | | | |
|---|---|---|---|---|---|---|---|---|
| | Without SVR | | With SVR | | Without SVR | | With SVR | |
| | $P_G$ (kW) | $Q_G$ (kVAr) | $P_G$ (kW) | $Q_G$ (kVAr) | $P_G$ (kW) | $Q_G$ (kVAr) | $P_G$ (kW) | $Q_G$ (kVAr) |
| External grid | 1515.0 | 1969.7 | 1515.0 | 1576.9 | 3270.5 | 2868.3 | 3372.5 | 2726.9 |
| DG1 | 600 | -195.6 | 600 | 197.2 | 600 | 22.23 | 600 | 197.2 |
| DG2 | 600 | 197.2 | 600 | 197.2 | 600 | 197.2 | 600 | 197.2 |
| DG3 | 1000 | 328.6 | 1000 | 328.6 | 1000 | 328.6 | 1000 | 328.6 |
| DG4 | 0 | 0 | 0 | 0 | 102.03 | 33.53 | 0 | 0 |

***Table 4.*** *Tap position.*

| Tap position | Loading level (%) | |
|---|---|---|
| | 100% | 150% |
| $VR_{7–8}$ | 3 | –8 |
| $VR_{2–19}$ | 8 | 8 |
| $VR_{6–26}$ | 8 | –8 |

***Table 5.*** *Electricity procurement costs.*

| | Loading level (%) | | | |
|---|---|---|---|---|
| | 100% | | 150% | |
| Cost ($/h) | Without SVR | With SVR | Without SVR | With SVR |
| **Active power from the transmission grid** | 24.24 | 24.24 | 52.3274 | 53.96 |
| **Reactive power from the transmission grid** | 11.8185 | 9.4614 | 17.21 | 16.3614 |
| **Active power from DGs** | 29.2 | 29.2 | 30.9345 | 29.2 |
| **Reactive power from DGs** | 2.2972 | 2.3004 | 2.1181 | 2.3004 |
| **Total cost** | 67.5557 | 65.2018 | 102.5900 | 101.8218 |

## 4. CONCLUSIONS

This paper presents a mixed-integer linear programming (MILP) model for optimal power flow that considers voltage regulation devices. The model incorporated various operational constraints, including power flow equations and transmission capacity limits, which were linearized using the regular-polygon method. Additionally, constraints on distributed generation and voltage-control assets were incorporated. All formulated constraints were maintained in linear form to ensure the achievement of a globally optimal solution. The IEEE 33-bus radial distribution system was employed to evaluate the performance of the proposed model. Numerical results demonstrated the effectiveness of the approach in optimizing power flow distribution. Future research will focus on extending this model to calculate and decompose locational marginal price components within distribution networks.

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## TÓM TẮT

### Giải bài toán trào lưu công suất tối ưu sử dụng mô hình MILP cho lưới phân phối có thiết bị điều chỉnh điện áp bước

*Bài báo đề xuất mô hình quy hoạch tuyến tính nguyên thực hỗn hợp (MILP) để giải bài toán trào lưu công suất tối ưu (OPF) trong lưới điện phân phối có thiết bị điều chỉnh điện áp bước (SVR). Bài toán OPF có hàm mục tiêu là tối thiểu hóa tổng chi phí của lưới điện phân phối, bao gồm chi phí mua công suất tác dụng và công suất phản kháng từ lưới truyền tải, chi phí phát công suất tác dụng và công suất phản kháng của các nguồn điện phân tán (DG). Các ràng buộc được xem xét bao gồm hệ phương trình trào lưu công suất, ràng buộc điện áp nút, giới hạn công suất truyền tải trên các nhánh, giới hạn hệ số công suất tại điểm đấu nối và ràng buộc liên quan đến SVR. Mô hình MILP của bài toán OPF trong lưới phân phối được phát triển từ mô hình quy hoạch phi tuyến nguyên thực hỗn hợp (MINLP) bằng cách tuyến tính hóa hệ phương trình trào lưu công suất và xây dựng mô hình tuyến tính chính xác của SVR. Mô hình MILP đề xuất được đánh giá trên lưới điện 33 nút IEEE với các kịch bản tải khác nhau sử dụng ngôn ngữ lập trình GAMS và bộ giải thương mại CPLEX. Các kết quả tính toán cho thấy rằng mô hình MILP hiệu quả về mặt tính toán và nấc phân áp tối ưu của SVR giúp giảm chi phí vận hành của lưới phân phối.*

**Từ khóa:** Trào lưu công suất tối ưu (OPF); Mô hình tuyến tính nguyên thực hỗn hợp (MILP); Thiết bị điều chỉnh điện áp (SVR); Lưới điện phân phối; Giá biên nút (LMP).