Determining the temperature-dependent thermal conductivity of phenolic resin/glass fiber composite materials using the inverse method

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Received 15 Jun. 2026; Revised 18 Aug. 2026; Accepted 04 Sep. 2026; Published 16 Sep. 2026.

DOI: https://doi.org/10.54939/1859-1043.j.mst.FMC.2026.28-36

ABSTRACT

This study presents a method for estimating the temperature-dependent thermal conductivity of phenolic/glass fiber composite materials using the Inverse Heat Transfer Problem (IHTP). The thermal conductivity, assumed to be constant and a linear function of temperature, is determined through fitting simulation results with experimental data. Temperature data obtained from the heating experiment are used to construct the objective function according to the least squares criterion between the calculated and measured temperatures. The forward heat transfer problem is described by an unsteady one-dimensional heat conduction equation with a temperature-dependent thermal conductivity. After discretization using the Finite Volume Method (FVM), the heat partial differential equation (PDE) was transformed into a system of ordinary differential equations (ODE). After that, the numerical solution of the ODE and the parameter optimization were carried out in the MATLAB environment, with the optimization performed using the "fminsearch" built-in function. The results show that the temperature-dependent thermal conductivity model describes the heat transfer process more accurately than the constant thermal conductivity model. The results confirm that the IHTP method is an effective tool for determining the thermal conductivity of composite materials under experimental conditions.

Keywords: The temperature-dependent thermal conductivity; Inverse Heat Transfer Problem (IHTP); Phenolic/glass-fiber composite.

1. INTRODUCTION

Glass fiber-reinforced phenolic composite materials (GPC), a charring ablator, are widely used in thermal protection systems of rocket engines, high-temperature combustion chambers, aerospace structures, and parts subjected to extreme heat loads due to their excellent heat resistance, low density, and effective thermal insulation properties [1, 4, 6]. During exposure to high-temperature gas streams, this material will form different regions (Figure 1): The virgin material, the pyrolysis zone and the porous char layer [1, 10], leading to thermophysical properties that also change with temperature and in different zones.

Thermal conductivity is one of the important thermophysical parameters that determines directly the heat transfer rate and temperature distribution of materials. In the design and thickness calculation of thermal protection systems, accurately characterizing the temperature dependence of thermal conductivity is essential for improving the predictive accuracy and reliability of heat transfer models. Although glass fiber-reinforced phenolic composites have been extensively investigated with respect to their fabrication processes, mechanical performance, and thermal resistance [7, 8], studies focusing on the experimental determination and mathematical characterization of temperature-dependent thermal conductivity remain limited, especially in the pre-pyrolysis (virgin zone) temperature range.

According to traditional experimental methods, determining the thermal conductivity coefficient requires specialized equipment and high costs and is sometimes difficult to apply to composite materials with heterogeneous structures or only provides average values within a certain

temperature range. Therefore, with the increasingly strong development of computers in recent decades, numerical processing of Inverse Heat Transfer Problems (IHTP) has been widely studied and applied to determine thermophysical parameters from measured temperature data [2, 5].

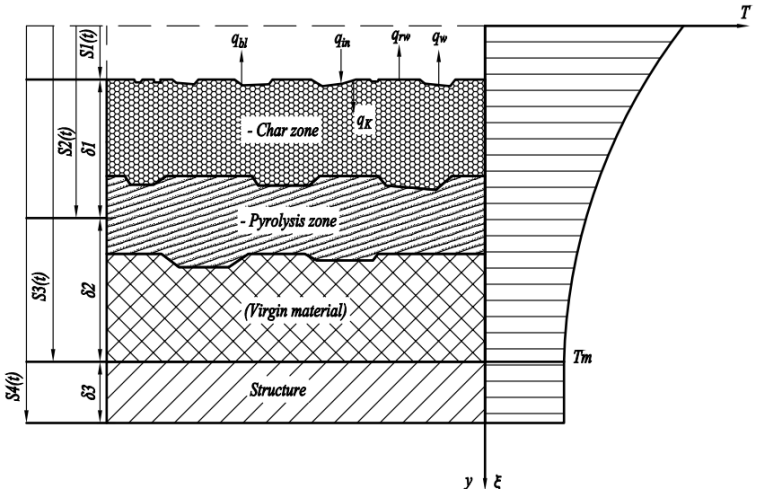


Figure 1. *Physical model of the charring ablator thermal protection layer undergoing exposure to a high-temperature gas stream [1].*

Based on the above considerations, this study proposes an inverse heat transfer approach to determine the temperature-dependent thermal conductivity of glass fiber-reinforced phenolic composites in the pre-pyrolysis region (virgin material). The thermal conductivity is assumed to vary linearly with temperature according to:$\lambda(T) = a + bT$ *(with T [K])*, where the coefficients *a* and *b* are identified by minimizing the discrepancy between the calculated and experimentally measured temperatures. To evaluate the necessity and effectiveness of considering the temperature dependence of thermal conductivity, two thermal conductivity models are investigated:

Case 1: a linear temperature-dependent model, $\lambda(T)=a+bT$ *(with T [K])*,

Case 2: λ=const.

The thermal conductivity parameters obtained from each inverse analysis are subsequently employed in a one-dimensional transient forward heat transfer model to predict the temperature history, and the numerical results are compared with the experimental measurements. This comparison enables assessment of the predictive capability of each model and identification of the most appropriate thermal conductivity representation for the investigated material. The proposed methodology provides an effective approach for determining the thermal conductivity of virgin phenolic composites and supplies reliable thermophysical properties for developing more accurate thermal response models of thermal protection systems operating under high-temperature environments.

2. THEORETICAL BASIS

2.1. Forward heat transfer problem

Consider a sample of a glass fiber-reinforced phenolic composite material with thickness (L). To calculate the thickness of the heat-treated layer, several assumptions are used in this paper as follows:

- The heat transfer process in the sample is assumed to be one-dimensional (1D), perpendicular to the heating surface.

- The density remains constant with temperature throughout the experiment.

- The experiment was conducted only at temperatures lower than the temperature before the degradation reaction occurred.

Based on the energy balance equation and Fourier's law of heat conduction, the differential equation of heat conduction, boundary conditions, and initial conditions of the problem are expressed as follows:

$$\frac{\partial \rho CT}{\partial t} + \nabla.\left(-\lambda \frac{\partial T}{\partial x}\right) = 0 \qquad \text{, with } 0 \le x \le L \tag{1}$$

$$T\big|_{t=0} = T_0 = const \tag{2}$$

$$\rho C(T)\frac{\Delta x}{2}\frac{\partial T}{\partial t}\bigg|_{x=0} = \lambda(T)\frac{(T_2 - T_1)}{\Delta x} - h\left(T(L,t) - T_e\right) \text{ and } T\big|_{x=L} = T_w \tag{3}$$

Where:

- T – Temperature *[K];*

- T_0 – Initial temperature of the test sample *[K];*

- $\lambda(T) = \lambda_0(1 + \beta T)$ the temperature-dependent thermal conductivity (W/m·K); β is called the temperature coefficient of thermal conductivity [3]. Therefore, in this study, the thermal conductivity is regarded as $\lambda(T) = a + bT$, T is temperature [K] and a and b are constant ($b \neq 0$)

- ρ - Density of the composite [kg/m^3];

- Cp – Heat capacity of the composite [$J/kg{\cdot}K$].

- h – Convective heat transfer coefficient between the composite surface and the environmental air, [W/m^2K]

- At the right-hand position, i.e., $x = L$, the problem follows boundary conditions of type I, with a surface temperature $Tw = 443$ *[K]*.

- At left-hand position, $x = 0$, the problem follows boundary conditions of type III; heat released at this position is exchanged by convection with the air, with a convective heat transfer coefficient $h = 25$ [$W/m^2.K$].

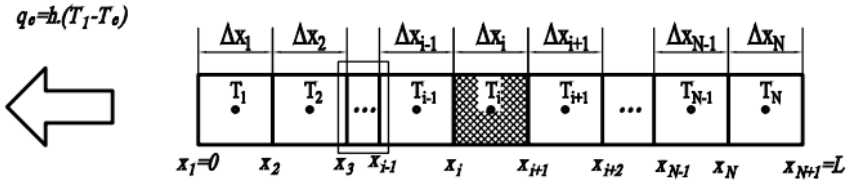


Figure 2. *Discrete mesh generation using the finite volume method.*

In this method, the partial differential equation is only discretized in space; the result of this discretization is a system of ordinary differential equations at nodal points in space. Then this system of ordinary differential equations will be solved using tools for solving ordinary differential equations. The advantage of this method is that the computation time is very fast, the solution has high accuracy, and the programming time is also much faster than discretizing both spatial and time dimensions in the classical way [5]. On that basis, equation (1) is expressed for element i as follows:

$$\int_{(i-1)\Delta x}^{i\Delta x} \frac{\partial\left(c_p \rho T\right)}{\partial t} dx - \int_{(i-1)\Delta x}^{i\Delta x} \nabla.\left(\lambda \frac{\partial T}{\partial x}\right) dx = 0 \tag{4}$$

Using discrete approximation, the thermal energy accumulation within the element is approximated as follows:

$$\int_{(i-1)\Delta x}^{i\Delta x} \frac{\partial\left(c_p \rho T\right)}{\partial t} dx = \Delta x\left(c_p \rho\right)\Big|_{T_i} \frac{dT_i}{dt} \tag{5}$$

The heat flux component due to conduction is integrated and approximated as:

$$\int_{(i-1)\Delta x}^{i\Delta x} \nabla.\left(\lambda \frac{\partial T}{\partial x}\right) dx = \left(\lambda_{i+1,i} \frac{T_{i+1} - T_i}{\Delta x} - \lambda_{i,i-1} \frac{T_i - T_{i-1}}{\Delta x}\right) \quad (6)$$

Combining equations (5) and (6), we obtain the ordinary differential equation representing the temperature change process of element i over time.

$$\frac{dT_i}{dt} = \frac{1}{\Delta x \left(c_p \rho\right)\Big|_{T_i}} \left(\lambda_{i+1,i} \frac{T_{i+1} - T_i}{\Delta x} - \lambda_{i,i-1} \frac{T_i - T_{i-1}}{\Delta x}\right) \quad (7)$$

At the x = 0 surface, the sample is in contact with air. Using the approximation method, we have:

$$\rho C(T) \frac{\Delta x}{2} \frac{\partial T}{\partial t}\Big|_{x=0} = \lambda(T) \frac{(T_2 - T_1)}{\Delta x} - h(T_1 - T_e) \quad (8)$$

At the surface x = L with $T_{N+1} = T_w$, represented under discrete boundary conditions:

$$\frac{dT_N}{dt} = \frac{1}{\Delta x \left(c_p \rho\right)\Big|_{T_N}} \left(\lambda_{N+1,N} \frac{T_{N+1} - T_N}{\Delta x} - \lambda_{N,N-1} \frac{T_N - T_{N-1}}{\Delta x}\right) \quad (9)$$

Grid convergence was verified by incrementally increasing the number of elements from N = 30 to 101. The results indicated that the calculated parameters and the objective function show negligible changes with further mesh refinement, with relative differences between refined grids of less than 1% (Table 1). Therefore, N = 101 was selected as a suitable compromise between numerical accuracy and computational time and was subsequently used in the IHTP calculations.

Table 1. *Grid convergence analysis with different numbers of control volumes.*

Number of control volumes, N	a	b	Calculation time (s)	Relative difference
30	*0.3716*	*0.0005*	*44.734*	-
60	*0.3903*	*0.0005*	*111.190*	*4.79%*
80	*0.3945*	*0.0005*	*185.936*	*1.06%*
91	*0.3961*	*0.0005*	*257.908*	*0.40%*
101	*0.3971*	*0.0005*	*362.914*	*0.25%*

2.2. Inverse heat transfer problem

The basis of this method is usually expressed in the following form: $f(X) \rightarrow Min$. With f being the objective function of the problem, X is called the input variable, $X = [a, b]$. Accordingly, for each value of the input variable X (a, b), the returned objective function is the sum of the squared differences between the simulated and experimental values. The smaller the result of the objective function, the smaller the difference between the simulated and experimental results. The problem of determining the parameters of the equation $\lambda(T)=a+bT$ *(T, [K])* becomes the problem of determining the minimum point of the function *f(X)*. The goodness of fit between the model and the experiment is evaluated through the objective function:

$$\mathrm{ER}R(a,b) = \sum_{i=1,n} \left(T_{sim,i} - T_{\exp,i}\right)^2 \quad (10)$$

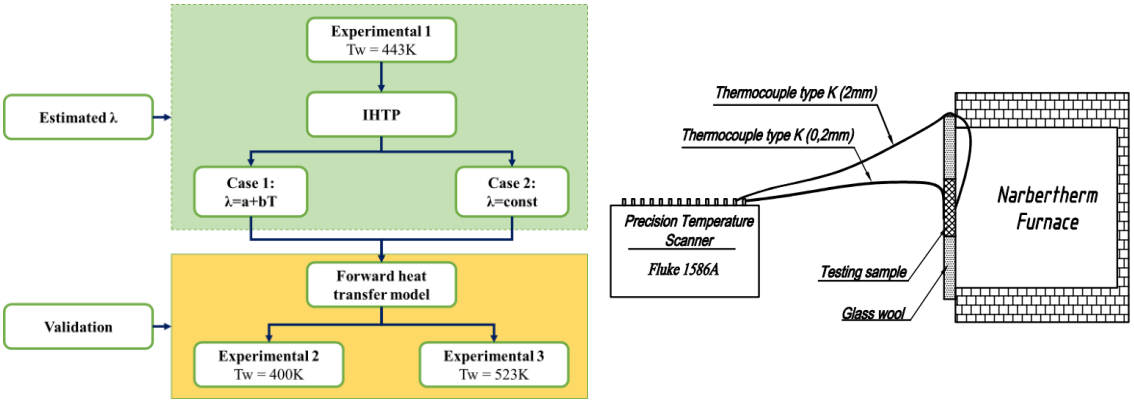
Where: + $T_{sim,i}$ is the theoretically calculated temperature;

+ $T_{exp,i}$ is the experimental temperature;

In this paper, the problem of finding the minimum value is determined using the “fminsearch” function built into MATLAB software.

3. EXPERIMENTAL

The research procedure is shown in Figure 3. Temperature data obtained from Experiment 1 were used as input data for the inverse problem to determine the thermal conductivity of the material in Case 1 and Case 2. The determined thermal conductivity was then fed into a forward heat transfer model to simulate Experiments 2 and 3. The simulation results were compared with experimental data to evaluate the accuracy of the determined thermal conductivity.



(a) *(b)*

Figure 3. *Methodology of the proposed approach: (a) inverse–forward heat transfer analysis and (b) experimental setup.*

The composite specimens were cured at 160 °C under a pressure of 25 MPa for 30 minutes using a direct thermocompression machine (symbol: Starmark) with a pressure of up to 30 tons and a temperature of up to 230 °C at the Institute of Missile, Academy of Military Science and Technology (dimensions: 70×70×20 mm (length × width × height)).

Table 2. *Input parameters for calculation.*

Property	Symbol	Value	Source
Density	ρ	*1785 [kg/m^3]*	measured
Heat capacity	c	*58.0123+3.1188.T [J/kg.K]*	measured
Surface temperature	T_w	*443 [K]*	measured
Thickness of sample	L	*20x10^{-3} [m]*	measured
Initial temperature of sample.	T_0	*304 [K]*	measured
Convective heat transfer coefficient	h	*25 [W/m^2.K]*	[3]
Environmental Temperature	T_e	*304 [K]*	measured
Throughout this paper, temperature is expressed in kelvin (K) and converted from the measured sample temperature, T_{sample}(°C), according to: $T = T_{sample}$+273 (K).			

In this experiment, the sample was heated from one side using a Nabertherm furnace (SN 217837), while the opposite side exchanged heat with the air via convection. The side surfaces of the sample were insulated with glass wool to minimize heat loss and lateral heat flow. Consequently, heat flow occurred primarily perpendicular to the heated surface; that is, through the thickness of the sample, allowing the problem to be treated as one-dimensional heat transfer. Temperatures at the heated surface and the opposite surface were continuously measured using K-type thermocouples (2.0 mm, 0–1200 °C range; 0.2 mm, 0–300 °C range, respectively) and recorded at 25-second intervals using a Fluke 1586A data logger (Figure 3b). Since the actual temperature at the heated surface (Tw) was measured directly, this value was used as a boundary condition (Type I) for the

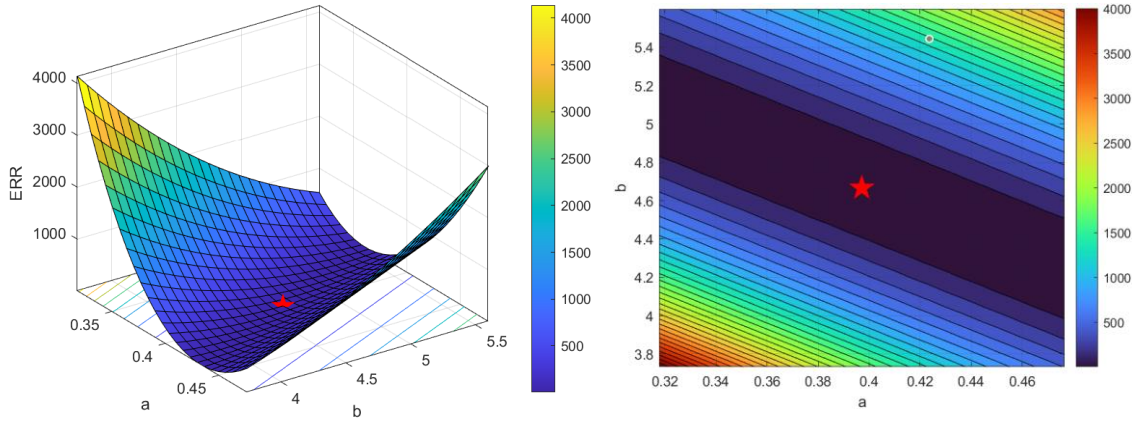
mathematical model. To validate the inverse method, the sample's thermal conductivity at room temperature (304 K) was independently measured using C-Therm equipment in accordance with ASTM D7984 and compared with the results obtained from the inverse method. All experiments were conducted at the Institute of Technology, General Department of Defense Industry.

The input parameters for the computational simulation are given in Table 2.

4. RESULTS AND DISCUSSION

4.1. Estimation λ

Figure 4 and Figure 5, respectively, present the response surface and contour plot of the ERR objective function for two cases of selecting the starting point of the optimization process. **Case 1:** Starting point $x_0 = [a_0, b_0] = [0.3, 0.0001]$, corresponding to simultaneously optimizing two parameters a and b. **Case 2:** Starting point $x_0 = a_0 = 0.3$, corresponding to the case where only parameter a is optimized when the thermal conductivity coefficient is assumed to be temperature-independent (λ=const).



(a) (b)

Figure 4. *Optimization results for determining the linear thermal conductivity function: (a) response surface and (b) contour plot.*

The results show that in both cases (Table 3), the objective function only exhibits one global minimum within the domain under investigation, demonstrating that the inverse problem has a definite solution and the fminsearch built-in function converges stably to the optimal solution. The red point on the graph represents the position of the minimum of the objective function, corresponding to the optimal set of parameters obtained after the optimization process (Figure 4a and Figure 5).

Table 3. *Identified thermal conductivity values.*

Case	Thermal conductivity [W/m.K]	ERR
Case 1	$\lambda(T) = 0.3971 + 0.0005T^*$	5.2277
Case 2	$\lambda = 0.5784$	16.276
Experimental value (ASTM D7984-16)	$\lambda = 0.570$	-
Reference value (ГОСТ 20437-89)	$\lambda = 0.6$	-
*Throughout this paper, temperature is expressed in kelvin (K) and converted from the measured sample temperature, T_{sample}(°C), according to: $T = T_{sample}+273$ (K).		

For Case 1, the contour lines exhibit elongated elliptical shapes rather than nearly circular

distributions. This feature indicates that the two unknown parameters, a and b, are correlated during the inverse identification process. In other words, the effect of a variation in one parameter can be partially compensated by an opposite variation in the other while maintaining a low value of the objective function (ERR=5.2277). Such parameter correlation is commonly encountered in inverse heat transfer problems involving the simultaneous estimation of multiple material properties from the same set of temperature measurements.

For Case 2, only one parameter is optimized, resulting in a simpler response surface and a faster, more stable convergence of the optimization process (Figure 5). However, the objective function value (ERR = 16.276) is higher than that obtained for Case 1, indicating that the assumption of a constant thermal conductivity is insufficient to accurately represent the heat transfer characteristics of the material. Although the identified thermal conductivity is λ=0.5784 W/(m·K), which differs by only approximately 3.6% from the reference value of 0.600 W/(m·K) reported in the literature according to ГОСТ 20437-89, the higher ERR suggests that neglecting the temperature dependence of thermal conductivity reduces the predictive accuracy of the heat transfer model.

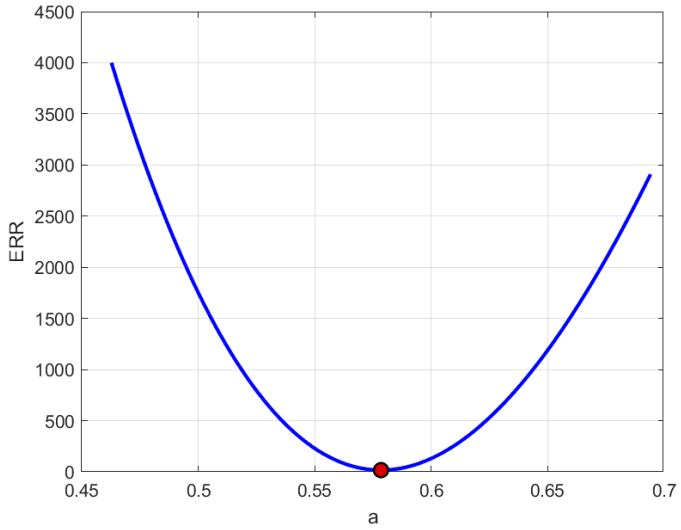


Figure 5. *Optimization results for determining the constant thermal conductivity case.*

4.2. Validation

For case 1, the thermal conductivity identified by the inverse heat transfer method at 304 K is 0.55 W/(m·K). This value agrees well with the authors' experimental measurement performed in accordance with ASTM D7984-16 and differs by only approximately 8.3% from the reference value of 0.600 W/(m·K) reported in the ГОСТ 20437-89 standard.

To assess the reliability of the results obtained, the paper conducted experiments using the thermal conductivity coefficients obtained from Table 1:

Experiment 2: Front surface temperature Tw = 400 K (Figure 6a).

Experiment 3: Front surface temperature Tw = 523 K (Figure 6b).

Figure 6 shows that the back surface temperature of the samples increased rapidly in the initial phase and gradually reached a steady state in both experiments. The model using the temperature-dependent thermal conductivity coefficient (λ = a + bT) fits the experimental data better, while the λ = Const model consistently predicted a higher value of approximately 4–7 K. This trend was consistent under both experimental conditions, suggesting that considering the dependence of the thermal conductivity coefficient on temperature more accurately describes the heat transfer process in the material.

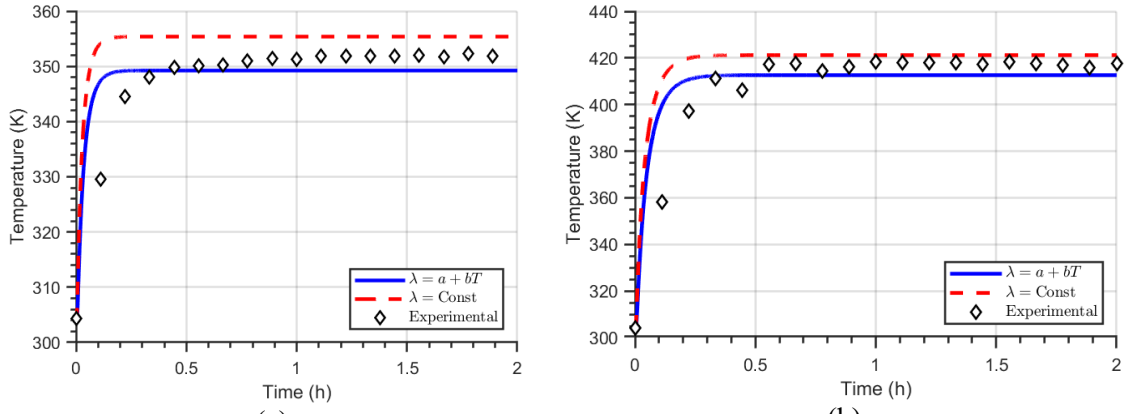


Figure 6. *Comparison of the predicted and experimental back-surface temperatures: (a) Experiment 2 (T_w=400 K) and (b) Experiment 3 (T_w=523 K).*

5. CONCLUSIONS

In this study, an inverse heat transfer method was employed to determine the thermal conductivity of glass fiber-reinforced phenolic composites from experimentally measured back-surface temperatures. The results show that the thermal conductivity is temperature dependent and can be expressed as:

$$\lambda(T) = 0.3971 + 0.0005T$$

Where: T [K], applied in the range $273 \leq T \leq 573$ K.

Compared with the constant thermal conductivity model, the proposed temperature-dependent model provides better agreement with the experimental temperature histories, confirming the importance of accounting for the temperature dependence of thermal conductivity. The proposed thermal conductivity function can be used in the design and thermal analysis of glass fiber-reinforced phenolic thermal protection systems. Moreover, the proposed inverse heat transfer approach can be extended to determine the thermophysical properties of other thermal protection materials.

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TÓM TẮT

Xác định hệ số dẫn nhiệt phụ thuộc nhiệt độ của vật liệu composite nhựa phenolic/sợi thủy tinh bằng phương pháp ngược

Nghiên cứu này trình bày phương pháp xác định hệ số dẫn nhiệt phụ thuộc nhiệt độ của vật liệu composite phenolic/sợi thủy tinh bằng phương pháp truyền nhiệt ngược (Inverse Heat Transfer Problem – IHTP). Hệ số dẫn nhiệt, được giả thiết là hằng số và hàm tuyến tính của nhiệt độ, được xác định thông qua quá trình khớp giữa kết quả mô phỏng và dữ liệu thực nghiệm. Dữ liệu nhiệt độ thu được từ thí nghiệm gia nhiệt được sử dụng để xây dựng hàm mục tiêu theo tiêu chuẩn bình phương tối thiểu giữa nhiệt độ tính toán và nhiệt độ đo được. Bài toán truyền nhiệt thuận được mô tả bằng phương trình dẫn nhiệt một chiều không ổn định với hệ số dẫn nhiệt phụ thuộc nhiệt độ. Hệ phương trình sau khi rời rạc hóa theo phương pháp phần tử thể tích được chuyển thành hệ phương trình vi phân thường. Quá trình giải phương trình vi phân và tối ưu tham số được thực hiện bằng phần mềm MATLAB. Các tham số của hàm hệ số dẫn nhiệt được xác định thông qua quá trình tối ưu hóa nhằm cực tiểu hóa hàm mục tiêu. Kết quả nghiên cứu cho thấy mô hình hệ số dẫn nhiệt phụ thuộc nhiệt độ mô tả quá trình truyền nhiệt chính xác hơn so với mô hình hệ số dẫn nhiệt không đổi. Kết quả khẳng định phương pháp IHTP là một công cụ hiệu quả để xác định hệ số dẫn nhiệt của vật liệu composite trong điều kiện thực nghiệm.

Keywords: Hệ số dẫn nhiệt phụ thuộc nhiệt độ; Phương pháp ngược; Composite phenolic/sợi thủy tinh.