Research

Dynamic torque analysis of a coaxial-actuated 3-DOF spherical parallel mechanism under varying working orientation

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ABSTRACT

The coaxial-actuated three-degree-of-freedom (3-DOF) spherical parallel mechanism is a novel parallel robotic architecture that offers several advantages over conventional pan–tilt mechanisms and hexapods, including high structural stiffness, compact architecture, large payload capacity, low error accumulation, and high positioning accuracy. Furthermore, the coaxial arrangement of the mechanism limbs significantly reduces the installation space, improves structural symmetry, and facilitates integration into aerial platforms. However, the input driving torque varies considerably with the working orientation of the mechanism, directly affecting actuator selection, operational performance, and effective workspace utilization. This paper investigates the input driving torque characteristics of a coaxial-actuated 3-DOF spherical parallel mechanism under different working orientations. A virtual prototype of the mechanism is developed using Simscape Multibody and Simulink in MATLAB to evaluate the variation of the input driving torque. The working orientations are examined over an inclination angle ranging from 0° to 180° with an interval of 15°. Based on the simulation results, the distribution of the driving torque, the variation trend of the input driving torque with respect to the working orientation, and the orientations corresponding to the maximum torque demand are identified and analyzed. The results provide a comprehensive understanding of the relationship between working orientation and the input driving torque characteristics of the mechanism. The findings also establish a scientific basis for actuator selection, mechanical structure optimization, and control strategy development for spherical parallel mechanisms employed in applications requiring high positioning accuracy, high structural rigidity, and reliable operational stability.

Keywords: 3-DOF spherical parallel mechanism; Simulation; Driving torque; Working orientation; Aerial platforms.

1. INTRODUCTION

Spherical parallel mechanisms (SPMs) have been extensively studied due to their capability of generating pure rotational motion while maintaining high structural rigidity, high positioning accuracy, and compact architecture. Compared with conventional serial manipulators, SPMs exhibit lower moving inertia, reduced error accumulation, and higher load-carrying capability, making them suitable for robotic wrists, camera stabilization systems, medical robots, precision positioning equipment, and aerospace applications [1-3].

In recent years, considerable research has focused on the kinematic modeling, workspace analysis, singularity analysis, stiffness evaluation, and dynamic characteristics of 3-degree-of-freedom spherical parallel mechanisms. In particular, inverse and forward kinematic models have been established to improve motion accuracy and computational efficiency [4], while workspace and singularity analyses have been conducted to optimize mechanism geometry and avoid performance degradation near singular configurations [5]. Dynamic models based on the Newton–Euler method, the Lagrange formulation, and the principle of virtual work have also been proposed for evaluating actuator torque and motion performance [6-9].

Although these studies have provided important theoretical foundations for the design and

control of spherical parallel mechanisms, several limitations remain. Most existing investigations evaluate the input driving torque under a fixed installation orientation or along predefined motion trajectories. The influence of the working orientation of the entire mechanism relative to gravity has received comparatively little attention. In practical applications, especially in unmanned aerial vehicles and airborne electro-optical platforms, the mechanism continuously operates under varying attitudes. Consequently, the gravitational loading acting on the moving links changes significantly, resulting in considerable variations in the required actuator torque. Ignoring this effect may lead to inappropriate actuator selection, increased energy consumption, degraded control performance, and inefficient utilization of the mechanism workspace.

Furthermore, the recently developed coaxial-actuated 3-DOF spherical parallel mechanism introduces additional advantages over conventional SPMs by placing all actuators along a common axis. This configuration reduces installation space, enhances structural symmetry, simplifies mechanical integration, and is particularly suitable for compact aerial platforms. However, due to its unique structural arrangement, the variation of input driving torque under different working orientations has not yet been systematically investigated. Existing dynamic analyses mainly concentrate on motion characteristics under a single installation posture and therefore cannot provide sufficient guidance for actuator selection and system optimization under multi-orientation operating conditions.

To overcome these limitations, this paper presents a comprehensive investigation of the driving torque characteristics of a coaxial-actuated 3-DOF spherical parallel mechanism under different working orientations. A virtual prototype of the mechanism is established in MATLAB Simscape Multibody and Simulink to perform dynamic simulations [10]. The mechanism is evaluated over an inclination range from 0° to 180° with an interval of 15° (Figure 2). The obtained results clarify the influence of the working orientation on the input driving torque, identify the orientations corresponding to the maximum torque demand, and provide practical references for actuator sizing, structural optimization, and control strategy development.

2. MATERIALS AND METHODS

2.1. Mechanism architecture

The *mechanical system* under investigation is a coaxial-actuated 3-degree-of-freedom (3-DOF) spherical parallel mechanism (SPM) with a 3-RRR configuration, as illustrated in Figure 1. It consists of a fixed base (1) and three identical kinematic limbs equally distributed around the central axis. Each limb comprises a rotary plate (2), a proximal link (3), and a distal link (4), which ultimately connect to a common moving platform (5). The actuators installed beneath the fixed base (1) provide the driving torque, which is transmitted to the rotary plate (2) through Joint I. Since the rotary plate (2) is rigidly connected to the proximal link (3), the input torque is subsequently conveyed to the distal link (4) via Joint II.

The distal link (4) then transmits the torque to the moving platform (5) through Joint III, enabling the mechanism to achieve the desired rotational motion in different directions. The orientation of the moving platform relative to the fixed platform is described by the roll–pitch–yaw (RPY) angles, denoted by α, β and γ corresponding to rotations about the $\boldsymbol{x}$-, $\boldsymbol{y}$-, and $\boldsymbol{z}$-axes, respectively. The generalized joint coordinates of the mechanism are defined as: $\mathbf{q} = [q_1, q_2, q_3]^T$ where q_i $(i = 1, 2, 3)$ represent the angular displacements of the three coaxially actuated input joints.

The working orientation of the proposed mechanism is defined by the installation inclination angle θ_i, which is the angle between the normal axis of the fixed platform and the global vertical direction, as shown in Figure 2. As the inclination angle varies, the direction of the gravity vector relative to the mechanism changes, leading to different gravitational load distributions on the moving links. Consequently, the actuator torque required to achieve the same motion varies with

the installation orientation. Therefore, the inclination angle θ_i is selected as the primary variable for the dynamic analysis in this study. To systematically evaluate its influence, simulations are performed over a range of installation inclinations while maintaining the same motion trajectory and operating conditions. This approach enables a quantitative comparison of the actuator torque characteristics under different working orientations.

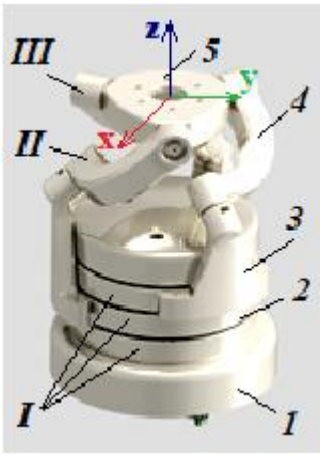


Figure1. *a) The structure of a 3-DOF spherical parallel manipulator with a 3-RRR and coaxial actuated input joint.*

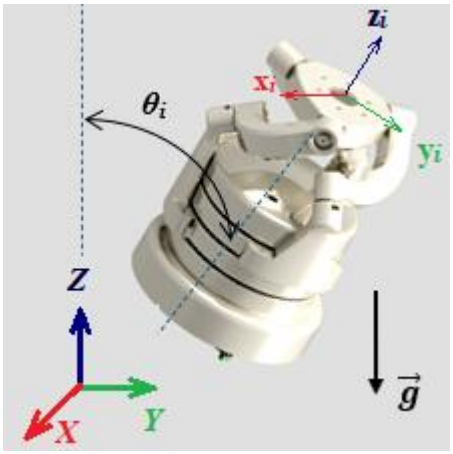


Figure 2. *The orientations of the mechanism investigated in this study vary as the angle θ_i ranges from 0° to 180° with an increment of 15°.*

2.2. Dynamic analysis

The dynamic model of the proposed coaxial-actuated 3-DOF spherical parallel mechanism is formulated using the generalized rigid-body equation of motion in the joint space:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q},\theta) = \tau \tag{1}$$

where $\mathbf{q}$ is the vector of generalized joint coordinates, $\mathbf{M(q)}$ is the generalized inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ represents the Coriolis and centrifugal effects, $\mathbf{G}(\mathbf{q},\theta)$ is the gravity torque vector, and $\boldsymbol{\tau}$ denotes the actuator torque vector. Since the objective of this study is to evaluate the influence of the installation inclination, the gravity term is explicitly expressed as a function of the working orientation angle θ.

The generalized inertia matrix is obtained from the kinetic energy of the moving platform and the three identical limbs by mapping their inertial properties into the joint space through the corresponding Jacobian matrices:

$$\mathbf{M}(\mathbf{q}) = \mathbf{M}_p + \sum_{i=1}^{3} \mathbf{M}_i$$

where $\mathbf{M}_p$ and $\mathbf{M}_i$ denote the inertia matrices of the moving platform and the i-th limb, respectively. Because all joint axes of the spherical parallel mechanism intersect at a common center of rotation, each moving link performs pure rotational motion about this point. Accordingly, the translational kinetic energy is incorporated into the equivalent rotational inertia using the parallel-axis (Steiner) theorem, resulting in a symmetric positive-definite inertia matrix.

The Coriolis and centrifugal matrix is determined from the velocity-dependent terms of the dynamic model:

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \tag{2}$$

To evaluate the effect of the installation orientation, the gravity vector in the base coordinate system is expressed as a function of the inclination angle θ. Assuming that the mechanism is

inclined about the Y-axis, the transformed gravity vector is

$$g(\theta) \tag{3}$$

where g = 9.81 m/s^2.

The generalized gravity torque is obtained by projecting the gravitational forces of the moving platform and the three limbs into the joint space through their corresponding position Jacobians:

$$\mathbf{G}(\mathbf{q},\theta) \tag{4}$$

For an ideal spherical parallel mechanism, where the center of mass of the moving platform coincides with the common center of rotation, the gravity torque generated by the moving platform becomes zero. Equation (4) is therefore simplified to

$$\mathbf{G}(\mathbf{q},\theta) = \sum_{i=1}^{3} \mathbf{J}_{vi}^{T} m_i \mathbf{g}(\theta) \tag{5}$$

which is used throughout this study to investigate the influence of working orientation on the actuator torque.

2.3. Dynamic modeling and simulation

To analyze the torque characteristics under varying spatial configurations, a digital twin was constructed in MATLAB Simscape Multibody. Originating from a SolidWorks geometric model, the multibody system was fully established via the Simscape Multibody Link. Within this environment, the components are modeled as absolute rigid bodies with constant masses and inertia tensors.

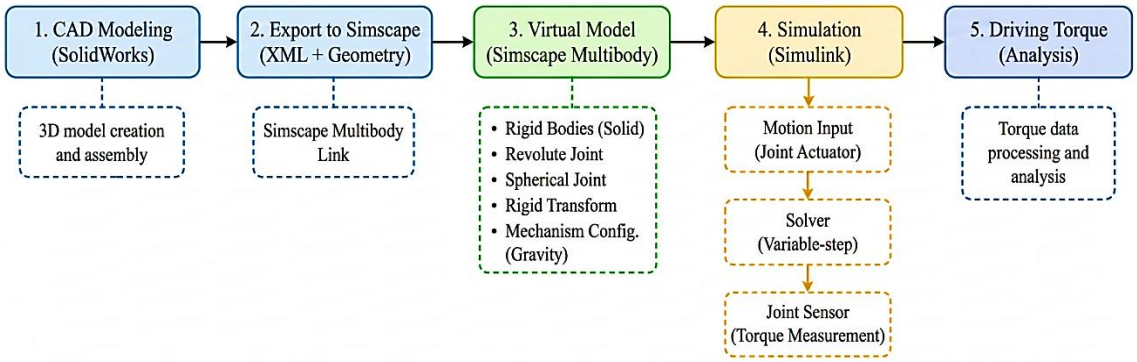
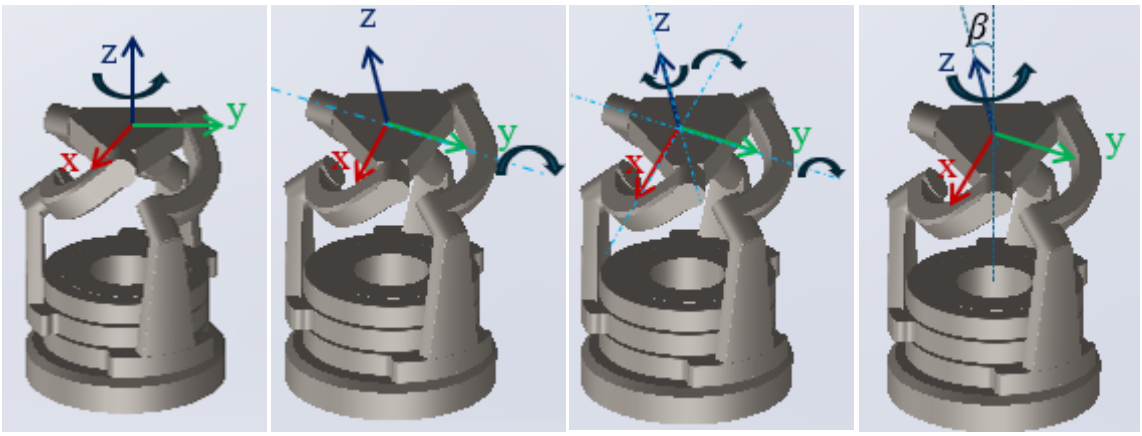


Figure 3. *Workflow of the virtual dynamic model and driving torque evaluation.*

The kinematic topology is rigorously interconnected via *Revolute* and *Spherical Joints*, as well as *Rigid Transforms*. Subjected to the gravitational field (*Mechanism Configuration*), the mechanism executes the prescribed motion profiles dictated by *Joint Actuators*. The resulting feedback torques are captured by *Joint Sensors* and routed to Simulink, where a variable-step numerical solver is applied to resolve the nonlinear differential equations. Throughout the simulation scenarios, all motion trajectories and external loads are strictly kept constant. The sole variable altered is the installation inclination angle—sweeping from 0° to 180°—to accurately evaluate the impact of gravity on the torque distribution of the mechanism.

Four representative operating scenarios are considered to evaluate the influence of working orientation on the actuator torque characteristics, as shown in Figure 4: (a) continuous rotation about the vertical Z-axis, (b) oscillatory motion about a horizontal axis, (c) combined roll–pitch–yaw motion, and (d) inclined installation with an inclination angle β. These scenarios represent typical operating conditions of spherical parallel mechanisms. To ensure a fair comparison, all simulation parameters are kept identical, while only the motion pattern or the installation inclination is varied.



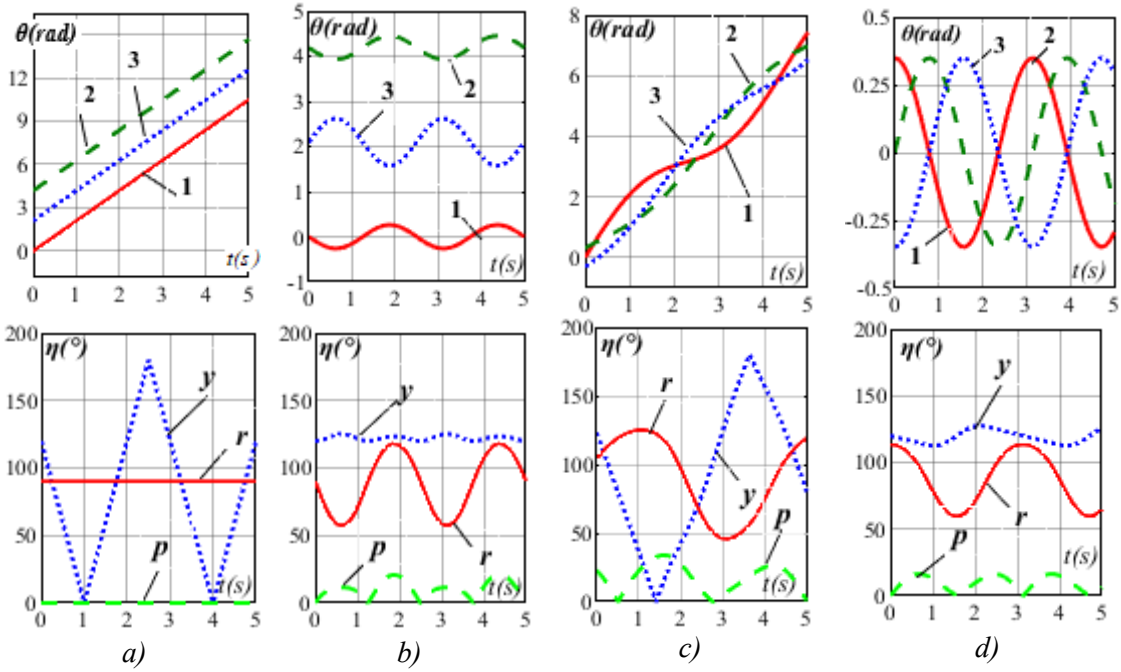
a) *b)* *c)* *d)*

Figure 4. *Representative simulation scenarios: (a) continuous rotation about the vertical Z-axis; (b) oscillatory motion about a horizontal axis; (c) combined roll–pitch–yaw motion; (d) inclined installation with inclination angle β.*

3. RESULTS AND DISCUSSION

3.1. Torque curves

Figure 5 shows the trajectories of the actuating links and the corresponding end-effector orientations for four representative operating scenarios, which are used to evaluate the actuator torque characteristics under different working orientations.



a) *b)* *c)* *d)*

Figure 5. *Trajectories of actuating links 1, 2, and 3 (top row) and the corresponding end-effector trajectories along the roll, pitch, and yaw axes (bottom row) under four simulation scenarios: (a) continuous rotation about the vertical axis, (b) oscillatory motion about a horizontal axis, (c) combined roll–pitch–yaw motion, and (d) inclined installation with an inclination angle of β.*

Figure 6 illustrates the actuator torque profiles of the proposed coaxial-actuated spherical parallel mechanism under four representative operating cases at the characteristic vertical working

orientation ($\theta = 0°$). In all cases, the three actuators exhibit similar trends, indicating that the coaxial arrangement provides a relatively balanced torque distribution among the parallel branches.

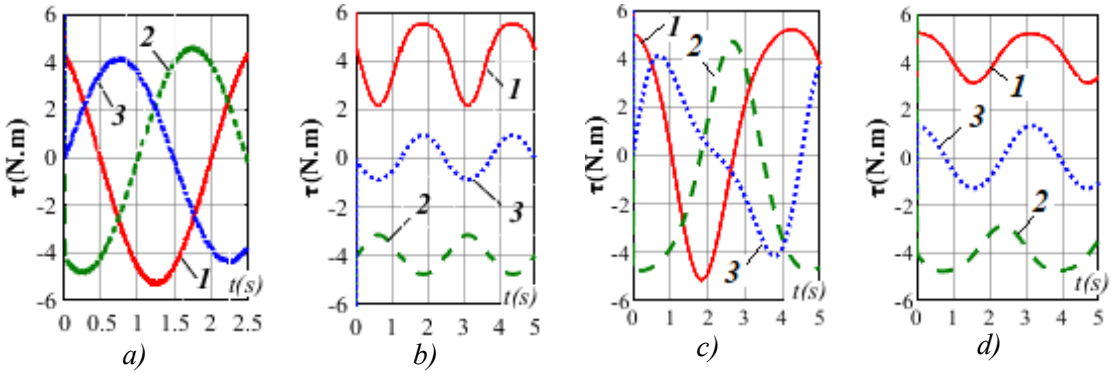


Figure 6. *Input torque curves of the three actuators at the working orientation θ = 0°.*

3.2. Maximum torque

Figures 7 and 8 present the maximum actuator torque under different working orientations ranging from 0° to 180° with an interval of 15°. Figure 7 illustrates the torque distribution using a heatmap, whereas Figure 8 depicts the variation of the maximum actuator torque with respect to the working orientation.

The color scale in Figure 7 is used to represent the torque magnitude, where blue indicates lower values and red denotes higher values, providing an intuitive visualization of the torque distribution under different operating cases and working orientations. The results indicate that the maximum actuator torque varies significantly with the working orientation of the mechanism. Among the four investigated operating cases, Case 2 generally requires the highest driving torque, whereas Case 3 exhibits a more balanced load distribution and lower torque demand. Furthermore, the maximum torque tends to reach higher values at the extreme working orientations (θ = 0° and θ = 180°) and decreases at intermediate orientations, particularly around θ = 90°.

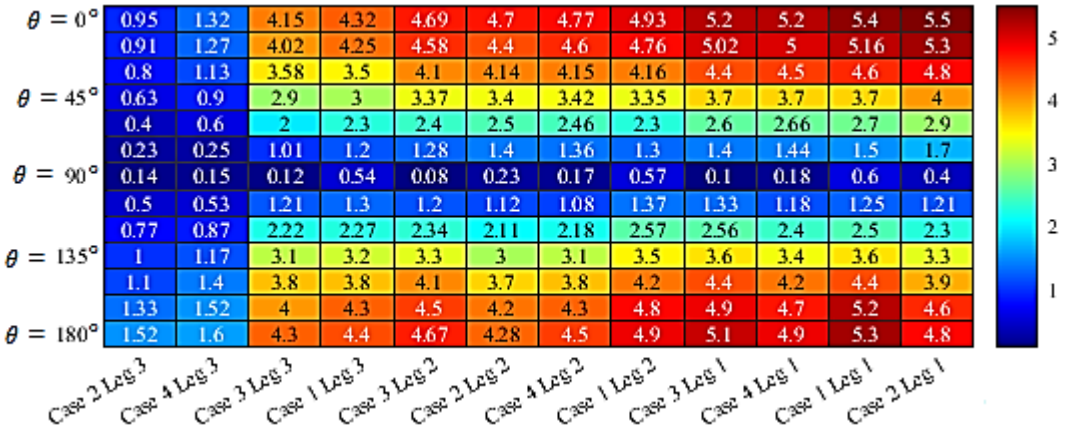


Figure 7. *Actuator torque values at different working orientations for four representative operating cases.*

As illustrated in Figure 8, the maximum actuator torque exhibits an approximately U-shaped variation, reaching its highest values near the upright (0°) and inverted (180°) configurations and its minimum around 90°. This behavior confirms that working orientation has a much greater influence on actuator torque than the load difference among individual actuators, indicating that gravitational loading is the dominant factor affecting the driving torque.

The actuator torques remain well balanced throughout the investigated workspace, and no

actuator experiences persistent overload under any operating condition. Minor load differences occur only at orientations associated with larger gravitational moments, demonstrating the effectiveness of the coaxial architecture in distributing the driving loads.

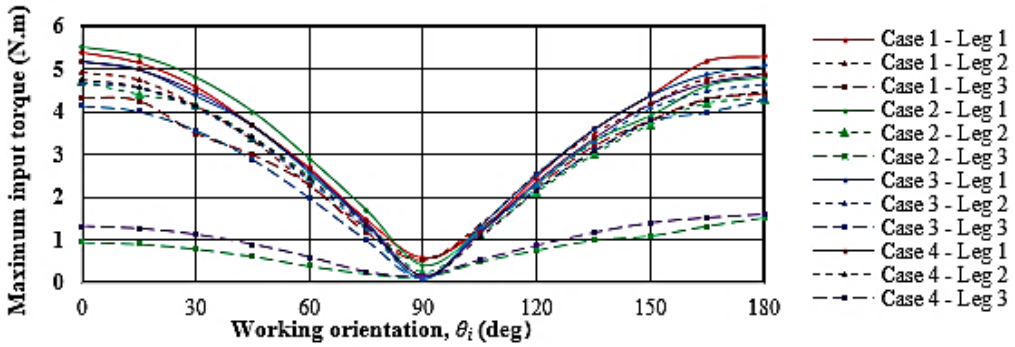


Figure 8. *Variation of maximum input torque with working orientation.*

As the mechanism rotates from the vertical to the horizontal configuration, the gravitational moment arm decreases, resulting in lower actuator torque. Beyond 90°, the gravitational effect increases again, causing the torque demand to rise. Therefore, operating the mechanism near intermediate orientations can effectively reduce peak torque and improve energy efficiency.

Overall, changing the installation orientation from 0° to 90° reduces the peak actuator torque by nearly one order of magnitude, whereas changing the operating scenario alters the peak torque by less than 10%. This finding confirms that working orientation, rather than motion pattern, is the governing factor in actuator torque demand

4. CONCLUSIONS

This paper investigated the influence of working orientation on the input torque characteristics of a coaxial-actuated 3-DOF spherical parallel mechanism through dynamic modeling based on the Newton–Euler formulation and simulations in MATLAB Simscape Multibody. The mechanism was evaluated at installation inclinations ranging from 0° to 180° under representative operating scenarios.

The results show that the maximum input torque follows a nearly symmetric U-shaped distribution with respect to the working orientation, reaching its highest values near 0° and 180° and its minimum around 90°. In addition, the coaxial architecture provides balanced load sharing among the three actuators throughout the investigated workspace. These findings quantitatively demonstrate the influence of installation orientation on actuator loading and provide a useful basis for actuator sizing, structural design, and installation planning of spherical parallel mechanisms.

Future work will focus on experimental validation using a physical prototype and on extending the dynamic model to include payload variation, joint friction, structural compliance, and torque-aware trajectory optimization for practical robotic applications.

REFERENCES

[1]. S. Bai and J. Angeles, "*A review of spherical motion generation using either spherical parallel manipulators or spherical motors*", Mechanism and Machine Theory, Vol. 140, pp. 377–404, (2019). DOI: 10.1016/j.mechmachtheory.2019.06.012.

[2]. C. M. Gosselin, "*Simulation and computer-aided kinematic design of three-degree-of-freedom spherical parallel manipulators*", Journal of Robotic Systems, Vol. 12, No. 12, pp. 857–871, (1995). DOI: 10.1002/rob.4620121209.

[3]. C. M. Gosselin and J. Angeles, "*On the direct kinematics of spherical three-degree-of-freedom parallel manipulators of general architecture*", Journal of Mechanical Design, Vol. 116, No. 2, pp. 594–598, (1994). DOI: 10.1115/1.2919419.

[4]. I. Tursynbek, A. Niyetkaliye and A. Shintemirov, "*Computation of unique kinematic solutions of a spherical parallel manipulator with coaxial input shafts*", Proceedings of the IEEE 15th International Conference on Automation Science and Engineering (CASE), Vancouver, BC, Canada, pp. 1640–1645, (2019). DOI: 10.1109/COASE.2019.8843090.

[5]. M. Karouia and J. M. Hervé, "*Asymmetrical 3-DOF spherical parallel mechanisms*", European Journal of Mechanics - A/Solids, Vol. 24, No. 1, pp. 47–57, (2005). DOI: 10.1016/j.euromechsol.2004.10.001.

[6]. A. Hassani, A. Bataleblu, S. A. Khalilpour, H. D. Taghirad and P. Cardou, "*Dynamic models of spherical parallel robots for model-based control schemes*", arXiv preprint arXiv:2110.00491, (2021).

[7]. V. T. Le, "*Kinematic and dynamic analysis of a serial manipulator leg for a spherical parallel robot with three coaxial revolute joints*", TNU Journal of Science and Technology, Vol. 231, No. 02/S, pp. 162–167, (2026). DOI: 10.34238/tnu-jst.14329.

[8]. T. N. Nguyen, H. K. Le and T. D. Duong, "*Modeling and control of a coaxial parallel spherical mechanism*", WSEAS Transactions on Systems and Control, Vol. 19, pp. 373–384, (2024). DOI: 10.37394/23203.2024.19.39.

[9]. A. Le, T. N. Nguyen, H. K. Le et al., "*Inertial line-of-sight stabilization using a 3-DOF spherical parallel manipulator with coaxial input shafts*", arXiv preprint arXiv:2312.02641, (2023).

[10]. Trần Trung Kiên, Vũ Đức Cường, Nguyễn Danh Huy, và Nguyễn Trung Kiên, *"Điều khiển chế độ trượt cho robot Stewart Platform: Tiếp cận dựa trên mô hình bán vật lý"*, Tạp chí Nghiên cứu Khoa học và Công nghệ quân sự, số Đặc san FEE 2024, tr. 20-26, (2024) (in Vietnamese).

TÓM TẮT

Đặc tính mô-men dẫn động đầu vào của cơ cấu chấp hành song song cầu ba bậc tự do với các khâu đầu vào đồng trục tại các tư thế làm việc khác nhau

Cơ cấu chấp hành song song cầu ba bậc tự do (3-DOF) với các khâu đầu vào đồng trục là một dạng cơ cấu song song mới, mang lại nhiều ưu điểm so với các cơ cấu nối tiếp truyền thống, bao gồm độ cứng kết cấu cao, kiến trúc nhỏ gọn, khả năng mang tải lớn, sai số tích lũy thấp và độ chính xác định vị vượt trội. Bên cạnh đó, việc bố trí các cơ cấu chấp hành đầu vào theo dạng đồng trục giúp giảm không gian lắp đặt, tăng tính đối xứng của hệ thống và tạo thuận lợi cho việc tích hợp trên các nền tảng bay. Tuy nhiên, mô-men dẫn động đầu vào thay đổi đáng kể theo tư thế làm việc của cơ cấu, ảnh hưởng trực tiếp đến việc lựa chọn cơ cấu chấp hành, hiệu suất vận hành và khả năng khai thác hiệu quả không gian làm việc. Bài báo này nghiên cứu đặc tính mô-men dẫn động của cơ cấu chấp hành song song cầu ba bậc tự do với các khâu đầu vào đồng trục tại các tư thế làm việc khác nhau. Một mô hình ảo của cơ cấu được xây dựng trong môi trường Simscape Multibody và Simulink trên nền MATLAB nhằm phân tích sự biến thiên của mô-men dẫn động đầu vào. Các tư thế làm việc được khảo sát trong dải góc nghiêng từ 0° đến 180° với bước thay đổi 15°. Dựa trên kết quả mô phỏng, phân bố mô-men dẫn động, quy luật biến thiên mô-men theo tư thế làm việc, cũng như các tư thế tương ứng với giá trị mô-men cực đại được xác định và phân tích. Kết quả thu được cung cấp cái nhìn toàn diện về mối quan hệ giữa tư thế làm việc và đặc tính mô-men dẫn động của cơ cấu. Đồng thời, nghiên cứu cũng cung cấp cơ sở khoa học cho việc lựa chọn cơ cấu chấp hành, tối ưu hóa thiết kế cơ khí và phát triển chiến lược điều khiển đối với các cơ cấu chấp hành song song cầu trong các ứng dụng yêu cầu độ chính xác cao, độ cứng kết cấu lớn và độ ổn định vận hành.

Từ khoá: Cơ cấu chấp hành song song cầu 3 bậc tự do; Mô phỏng; Mô-men dẫn động; Tư thế làm việc; Nền tảng trên thiết bị bay.