Adaptive output feedback control of induction motors with unknown parameters

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ABSTRACT

This paper considers an induction motor model controlled by stator currents in the presence of unknown resistance, inductance, and load torque. A parameter-linearized regression model is derived from the control-signal dynamics, enabling the application of the Dynamic Regressor Extension and Mixing (DREM) technique. Using measurements of the rotor angular velocity, an estimation algorithm for the unknown parameters and a rotor-flux observer are developed. The proposed dynamic control law stabilizes the motor at a constant reference speed and guarantees global stability of the closed-loop system.

Keywords: Dynamic parameterization; Output control; Adaptive control; DREM; Induction motor.

1. INTRODUCTION

Induction motors are extensively utilized in industrial and transportation systems due to their robustness, low cost, and high reliability. Nevertheless, the design of high-performance control strategies for induction motors remains challenging in the presence of uncertainties in key electrical and mechanical parameters, such as stator resistance, inductance, and load torque, which may be unknown or time-varying. These uncertainties can significantly deteriorate control performance, particularly in sensor-limited scenarios where only partial state measurements are available [1].

In [2], a class of electromechanical systems relevant to alternating current (AC) electric motors is studied, and an output-feedback passive control law is proposed. This work provides an important contribution to the theory of output-feedback passive control for AC machines under realistic assumptions on system structure and parameter availability. The main result identifies a class of machines for which the output torque-tracking problem can be addressed using passive output-feedback control. The invertibility condition established in [2] is analogous to the full-rank condition of the decoupling matrix in geometric control theory [3].

Several approaches have been developed for the estimation of unknown constant parameters, including least-squares methods [4], gradient- and integral-based adaptive schemes [5, 6], and optimization-based techniques [7]. In this work, the Dynamic Regressor Extension and Mixing (DREM) method [8] is adopted due to its ability to combine the advantages of least-squares and gradient-based adaptive estimation. By applying linear filtering to regression models with time-varying coefficients and exploiting the swapping lemma, DREM converts a vector regression into a set of scalar regressions, thereby enabling efficient estimation of unknown parameters using measurable or computable signals.

This paper proposes an output-feedback control strategy for an induction motor with unknown stator resistance, inductance, and load torque. The proposed approach integrates a DREM-based parameter and rotor-flux estimation scheme with a nonlinear backstepping control design [9, 10]. A novel control law is developed to ensure online estimation of the unknown parameters while guaranteeing closed-loop stability and asymptotic convergence of both torque- and flux-tracking errors.

The main contributions of this paper are summarized as follows:

1. A linear regression model is derived to enable DREM-based estimation of unknown motor parameters and rotor flux.
2. An output-feedback control law is developed by integrating parameter and flux estimation with nonlinear backstepping control, while ensuring closed-loop stability.
3. A rigorous stability analysis is provided to establish global asymptotic convergence of both estimation and tracking errors.
4. Numerical simulation results are presented to demonstrate the effectiveness and robustness of the proposed method under parameter uncertainties.

The remainder of this paper is organized as follows. Section 2 presents the induction motor model and problem formulation, followed by the development of the DREM-based parameter estimation algorithm, the proposed control design, and the corresponding stability analysis. Section 3 presents the simulation results. Finally, Section 4 concludes the paper.

2. METHODOLOGY

2.1. Problem statements

We consider the dynamic model of an induction motor [13].

$$\dot{\boldsymbol{\lambda}} = -\frac{R}{L}\boldsymbol{\lambda} + \frac{R}{L}\boldsymbol{u} \tag{1}$$

$$\tau = \frac{n_p}{L}\boldsymbol{u}^T \boldsymbol{J}\,\boldsymbol{\lambda} \tag{2}$$

$$D\dot{\omega} = \tau - \tau_L \tag{3}$$

where $\boldsymbol{\lambda}(t) \in \mathbb{R}^2$ is the rotor flux vector, $\boldsymbol{u}(t) \in \mathbb{R}^2$ represents the stator currents serving as the control input, $\omega(t) \in \mathbb{R}$ is the rotor angular velocity, assumed to be measured without error, $\tau \in \mathbb{R}$ denotes the electromagnetic torque, and $\tau_L \in \mathbb{R}$ is the load torque, which is assumed to be constant. The parameters $R > 0$, $L > 0$, and $D > 0$ correspond to the resistance, inductance, and rotor inertia, respectively; $n_p \in \mathbb{Z}_+$ is the number of pole pairs, and

$$\boldsymbol{J} \coloneqq \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

The control objective is to design a control law that asymptotically stabilizes the rotor speed at the prescribed reference value, under the assumption that the resistance, inductance, and load torque are unknown and satisfy

$$\lim_{t\to\infty}(\omega(t) - \omega^*) = 0 \tag{4}$$

2.2. Controller design

2.2.1. Estimation of unknown parameters

Proposition 1: [11] Let the control signal $\boldsymbol{u}(t) \in \mathbb{R}^2$ be generated by a dynamic system described by

$$\boldsymbol{u}(t) = \boldsymbol{h}_u^T\boldsymbol{\chi}(t), \dot{\boldsymbol{\chi}}(t) = \boldsymbol{\Gamma}_0\boldsymbol{\chi}(t) + \boldsymbol{b_0}\boldsymbol{v}(t) \tag{5}$$

with the initial condition $\boldsymbol{\chi}(0) \in \mathbb{R}^2$ and a new control input $\boldsymbol{v}(t) \in \mathbb{R}^2$. The vectors are defined as $\boldsymbol{h_u} = \mathrm{col}(\boldsymbol{I}, 0, \dots, 0), \boldsymbol{b_0} = \mathrm{col}(0, \dots, 0, \boldsymbol{I}), \boldsymbol{I}$ - the identity matrix with appropriate dimensions.

Then, for the system described by (1) – (3), the following relation can be derived:

$$z(t) = \boldsymbol{\gamma}^T\boldsymbol{q}(t) \tag{6}$$

where $z(t) \in \mathbb{R}$ and $\boldsymbol{q}(t) = \mathrm{col}(q_1, \dots, q_6) \in \mathbb{R}^6$ are computable functions, and $\boldsymbol{\gamma} =$

$\gamma(R, L, \tau_L) = \text{col}(\gamma_1, \dots, \gamma_6) \in \mathbb{R}^6$ is a vector of constant unknown parameters.

Proof:

Define $a = \frac{R}{L}$ and $b = \frac{n_p}{L}$; then equation (1) – (3) can be rewritten as

$$\dot{\boldsymbol{\lambda}} = -a\boldsymbol{\lambda} + a\boldsymbol{\chi_1} \tag{7}$$

$$D\dot{\omega} + \tau_L = b\boldsymbol{\chi_1^T J\lambda} \tag{8}$$

Differentiating (8) twice yields

$$D\ddot{\omega} = \left(\boldsymbol{\chi_2^T} - a\boldsymbol{\chi_1^T}\right)b\boldsymbol{J\lambda} \tag{9}$$

$$D\dddot{\omega} - \boldsymbol{\chi_2^T}ab\boldsymbol{J\chi_1} = \left(\boldsymbol{\chi_3^T} - 2a\boldsymbol{\chi_2^T} + a^2\boldsymbol{\chi_1^T}\right)b\boldsymbol{J\lambda} \tag{10}$$

Equations (8) and (9) are expressed in matrix form as

$$\begin{bmatrix} D\dot{\omega} + \tau_L \\ D\ddot{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\chi_1^T} \\ \boldsymbol{\chi_2^T} - a\boldsymbol{\chi_1^T} \end{bmatrix} b\boldsymbol{J\lambda}, \tag{11}$$

where we define

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{\chi_1^T} \\ \boldsymbol{\chi_2^T} - a\boldsymbol{\chi_1^T} \end{bmatrix}$$

Substituting the expression for $b\boldsymbol{J\lambda}$ obtained from (11) into (10) gives

$$\begin{aligned} D\dddot{\omega} - \boldsymbol{\chi_2^T}ab\boldsymbol{J\chi_1} &= \left(\boldsymbol{\chi_3^T} - 2a\boldsymbol{\chi_2^T} + a^2\boldsymbol{\chi_1^T}\right)\mathbf{W}^{-1}\begin{bmatrix} D\dot{\omega} + \tau_L \\ D\ddot{\omega} \end{bmatrix} \\ &= \left(\boldsymbol{\chi_3^T} - 2a\boldsymbol{\chi_2^T} + a^2\boldsymbol{\chi_1^T}\right)\frac{1}{\det(\mathbf{W})}\text{adj}(\mathbf{W})\begin{bmatrix} D\dot{\omega} + \tau_L \\ D\ddot{\omega} \end{bmatrix} \end{aligned} \tag{12}$$

Substituting the values of the Cramer operators $\det(\boldsymbol{W}) = \boldsymbol{\chi_2^T J\chi_1}$ and $\text{adj}(\boldsymbol{W}) = [a\boldsymbol{J\chi_1} - \boldsymbol{J\chi_2} \quad \boldsymbol{J\chi_1}]$ into (12) yields the following relation

$$(D\dddot{\omega} - \boldsymbol{\chi_2^T}ab\boldsymbol{J\chi_1})\boldsymbol{\chi_2^T J\chi_1} = \left(\boldsymbol{\chi_3^T} - 2a\boldsymbol{\chi_2^T} + a^2\boldsymbol{\chi_1^T}\right)\begin{bmatrix} a\boldsymbol{J\chi_1} - \boldsymbol{J\chi_2} \\ \boldsymbol{J\chi_1} \end{bmatrix}^T \begin{bmatrix} D\dot{\omega} + \tau_L \\ D\ddot{\omega} \end{bmatrix} \tag{13}$$

which, after expansion, can be rewritten as

$$\begin{aligned} &D\dot{\omega}\boldsymbol{\chi_3^T J\chi_2} - D\ddot{\omega}\boldsymbol{\chi_3^T J\chi_1} + D\omega^{(3)}\boldsymbol{\chi_2^T J\chi_1} \\ &\quad = a(D\dot{\omega}\boldsymbol{\chi_3^T J\chi_1} - 2D\ddot{\omega}\boldsymbol{\chi_2^T J\chi_1}) - \tau_L\boldsymbol{\chi_3^T J\chi_2} + ab\boldsymbol{\chi_2^T J\chi_1\chi_2^T J\chi_1} \\ &\quad + a^2 D\,\dot{\omega}\boldsymbol{\chi_1^T J\chi_2} + a\tau_L\boldsymbol{\chi_3^T J\chi_1} + a^2\tau_L\boldsymbol{\chi_1^T J\chi_2} \end{aligned} \tag{14}$$

To eliminate the unavailable derivatives of the variable ω in (14), linear time-invariant filters of the form

$$\mathcal{H}_i[\cdot] = \frac{1}{(p+\alpha)^i}[\cdot]$$

are applied, where $p = \frac{d}{dt}$ denotes the differentiation operator, $\alpha > 0$ is a tuning parameter, and $i = 1,2,3$. These yields

$$\begin{aligned} &\mathcal{H}_3[D\dot{\omega}\boldsymbol{\chi_3^T J\chi_2} - D\ddot{\omega}\boldsymbol{\chi_3^T J\chi_1} + D\omega^{(3)}\boldsymbol{\chi_2^T J\chi_1}] \\ &\quad = a\mathcal{H}_3[D\dot{\omega}\boldsymbol{\chi_3^T J\chi_1} - 2D\ddot{\omega}\boldsymbol{\chi_2^T J\chi_1}] - \tau_L\mathcal{H}_3[\boldsymbol{\chi_3^T J\chi_2}] + ab\mathcal{H}_3[\boldsymbol{\chi_2^T J\chi_1\chi_2^T J\chi_1}] \\ &\quad + a^2\,\mathcal{H}_3[D\,\dot{\omega}\boldsymbol{\chi_1^T J\chi_2}] + a\tau_L\mathcal{H}_3[\boldsymbol{\chi_3^T J\chi_1}] + a^2\tau_L\mathcal{H}_3[\boldsymbol{\chi_1^T J\chi_2}] \end{aligned}$$

Using the Swapping Lemma [12], the unmeasurable derivatives of the speed ω are eliminated

through a sequence of transformations. In particular, for the first term on the left-hand side of (14), the Swapping Lemma yields

$$\begin{aligned}\mathcal{H}_3 D\dot{\omega}\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_2 &= D\frac{1}{(p+\alpha)^3}\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_2\dot{\omega}\\ &= D\frac{1}{(p+\alpha)^2}\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_2\frac{p}{p+\alpha}[\omega] - D\frac{1}{(p+\alpha)^3}(\boldsymbol{\chi}_4^T\boldsymbol{J}\boldsymbol{\chi}_2 + \boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_3)\frac{p}{p+\alpha}[\omega]\\ &= D\mathcal{H}_2\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_2\Omega_1 - D\mathcal{H}_3\boldsymbol{\chi}_4^T\boldsymbol{J}\boldsymbol{\chi}_2\Omega_1\end{aligned}$$

Applying the same procedure to the remaining terms in (14) and collecting the resulting terms, equation (14) can be expressed in the linear regression form (6), where the functions z and $\boldsymbol{q}$ are computed as

$$\begin{aligned}z = {} & D\boldsymbol{\chi}_2^T\boldsymbol{J}\boldsymbol{\chi}_1\Omega_3 - D\mathcal{H}_1\left[\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_1(\Omega_2+3\Omega_3)\right]\\ &+ D\mathcal{H}_2\left[\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_2(\Omega_1+2\Omega_2+3\Omega_3) + \boldsymbol{\chi}_4^T\boldsymbol{J}\boldsymbol{\chi}_1(2\Omega_2+3\Omega_3)\right]\\ &- D\mathcal{H}_3\left[\boldsymbol{\chi}_4^T\boldsymbol{J}\boldsymbol{\chi}_2(\Omega_1+2\Omega_2+3\Omega_3) + \boldsymbol{v}^T\boldsymbol{J}\boldsymbol{\chi}_1(\Omega_2+\Omega_3)\right]\end{aligned}$$

$$\boldsymbol{q}_1 = -D\mathcal{H}_1\left[2\boldsymbol{\chi}_2^T\boldsymbol{J}\boldsymbol{\chi}_1\Omega_2\right] + D\mathcal{H}_2\left[\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_1(\Omega_1+4\Omega_2)\right] - D\mathcal{H}_3\left[(\boldsymbol{\chi}_4^T\boldsymbol{J}\boldsymbol{\chi}_1 + \boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_2)(\Omega_1+2\Omega_2)\right]$$

$$\boldsymbol{q}_2 = -\mathcal{H}_3\left[\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_2\right],\ \boldsymbol{q}_3 = \mathcal{H}_3\left[\left(\boldsymbol{\chi}_2^T\boldsymbol{J}\boldsymbol{\chi}_1\right)^2\right],\ \boldsymbol{q}_4 = D\mathcal{H}_2\left[\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_1\Omega_1\right] - D\mathcal{H}_3\left[\boldsymbol{\chi}_1^T\boldsymbol{J}\boldsymbol{\chi}_3\Omega_1\right]$$

$$\boldsymbol{q}_5 = \mathcal{H}_3\left[\boldsymbol{\chi}_3^T\boldsymbol{J}\boldsymbol{\chi}_1\right],\ \boldsymbol{q}_6 = \mathcal{H}_3\left[\boldsymbol{\chi}_1^T\boldsymbol{J}\boldsymbol{\chi}_2\right]$$

where $\Omega_i = p^i(p+\alpha)^{-i}[\omega]$, $i = 1,2,3$, and the parameter vector $\boldsymbol{\gamma}$ depends nonlinearly on the load torque τ_L and the parameters a and b, which are functions of the resistance R and the inductance L.

$$\boldsymbol{\gamma}^T = [a \quad \tau_L \quad ab\ a^2 \quad a\tau_L \quad a^2\tau_L]$$

This completes the proof.■

Following the DREM approach [8], six distinct positive constants α_l, $l = 1,\dots,6$, are selected, and the corresponding equations (6) are stacked to obtain

$$\boldsymbol{Z}(t) = \boldsymbol{Q}(t)\boldsymbol{\gamma}$$

with $\boldsymbol{Z}(t) = \mathrm{col}(z(t,\alpha_1),\dots,z(t,\alpha_6))$ and $\boldsymbol{Q}(t) = \mathrm{col}(q^T(t,\alpha_1),\dots,q^T(t,\alpha_6))$. The regressor $\boldsymbol{Q}(t)$ is then mixed using the Cramer operators.

$$\boldsymbol{\zeta}(t) = adj(\boldsymbol{Q}(t))\boldsymbol{Z}(t), \Delta(t) = det(\boldsymbol{Q}(t))$$

This procedure yields a set of regression equations of the form

$$\boldsymbol{\zeta}(t) = \Delta(t)\boldsymbol{\gamma} + \varepsilon(t)$$

with a common scalar regressor $\Delta(t) \in \mathbb{R}$ and a vector of computable functions $\boldsymbol{\zeta}(t) = \mathrm{col}(\zeta_1(t),\dots,\zeta_6(t)) \in \mathbb{R}^6$.

Next, based on the relations

$$R = n_p\frac{a}{b} = n_p\frac{\gamma_1^2}{\gamma_3}, L = n_p\frac{1}{b} = n_p\frac{\gamma_1}{\gamma_3}, \tau_L = \gamma_2$$

An estimation algorithm for the unknown parameters is constructed using the constants $\mu > 0$ and $\delta_0 > 0$.

$$\begin{aligned}&\hat{R} = n_p\frac{\hat{\gamma}_1^2}{\hat{\gamma}_3}, \hat{L} = n_p\frac{\hat{\gamma}_1}{\hat{\gamma}_3}, \hat{\tau}_L = \hat{\gamma}_2 \qquad (15)\\ &\dot{\hat{\gamma}}_i = \mu\Delta(\zeta_i - \Delta\hat{\gamma}_i), \quad i = 1,2,3\end{aligned}$$

2.2.2. Controller synthesis

The well-known field-oriented control (FOC) method [13] provides a framework for achieving

convergence of the torque-tracking error to zero. Motivated by this approach, a novel backstepping-based control law for $\boldsymbol{v}(t)$ in (5) is proposed below.

Proposition 2: The control law

$$\boldsymbol{u}(t) = (\boldsymbol{I} + V(t)\boldsymbol{J})\boldsymbol{\lambda_d}(t) \tag{16}$$

$$\dot{\boldsymbol{\lambda}}_{\boldsymbol{d}}(t) = \frac{R}{L}V(t)\boldsymbol{J}\boldsymbol{\lambda_d}(t) \tag{17}$$

with $|\boldsymbol{\lambda_d}(t)| = \beta_d$ and $\boldsymbol{I}$ denoting the identity matrix, guarantees that the following condition is satisfied

$$\lim_{t\to\infty}\big(\boldsymbol{\lambda}(t) - \boldsymbol{\lambda_d}(t)\big) = 0,\ \lim_{t\to\infty}\big(\tau(t) - \tau_d(t)\big) = 0$$

Proof:

Consider the tracking error $\boldsymbol{e_\lambda} = \boldsymbol{\lambda} - \boldsymbol{\lambda_d}$ and its derivative:

$$\dot{\boldsymbol{e}}_{\boldsymbol{\lambda}} = \dot{\boldsymbol{\lambda}} - \dot{\boldsymbol{\lambda}}_{\boldsymbol{d}} = -\frac{R}{L}\boldsymbol{\lambda} + \frac{R}{L}\boldsymbol{u} - \dot{\boldsymbol{\lambda}}_{\boldsymbol{d}} = -\frac{R}{L}\boldsymbol{\lambda} + \frac{R}{L}(\boldsymbol{I} + VJ)\boldsymbol{\lambda_d} - \frac{R}{L}V\boldsymbol{J}\boldsymbol{\lambda_d} = -\frac{R}{L}(\boldsymbol{\lambda} - \boldsymbol{\lambda_d}) = -\frac{R}{L}\boldsymbol{e_\lambda}$$

It follows directly that

$$\lim_{t\to\infty}\boldsymbol{e_\lambda}(t) = 0, \lim_{t\to\infty}(\boldsymbol{\lambda}(t) - \boldsymbol{\lambda_d}(t)) = 0.$$

Substituting the control law (16) into the electromagnetic torque expression (2), we obtain

$$\tau = \frac{n_p}{L}\,\boldsymbol{u}^T\boldsymbol{J}(\boldsymbol{\lambda_d} + \boldsymbol{e_\lambda}) = \frac{n_p}{L}\,\boldsymbol{\lambda_d^T}(\boldsymbol{I} + V\boldsymbol{J}^T)\boldsymbol{J}\boldsymbol{\lambda_d} + \varepsilon_\tau = \frac{n_p}{L}\,V\boldsymbol{\lambda_d^T}\boldsymbol{\lambda_d} + \varepsilon_\tau = \tau_d + \varepsilon_\tau$$

where $\varepsilon_\tau = \frac{n_p}{L}\,\boldsymbol{u}^T\boldsymbol{J}\boldsymbol{e_\lambda}$ is an exponentially decaying term. Note that the identity $\boldsymbol{J}^T\boldsymbol{J} = \boldsymbol{I}$.

Therefore,

$$\lim_{t\to\infty}(\tau(t) - \tau_d(t)) = 0,$$

which completes the proof.■

The nominal control law (16) does not provide the number of integrators required for the synthesis of the unknown parameter estimation algorithm presented in Proposition 1. Therefore, the backstepping design is adopted by introducing the error vector between the implementable control law, which enables the estimation of unknown parameters, and the nominal control law that guarantees the stability of the closed-loop system.

To this end, consider the variable

$$\boldsymbol{e_1} = \boldsymbol{u} - (\boldsymbol{I} + V\boldsymbol{J})\boldsymbol{\lambda_d}$$

and its derivatives.

$$\begin{aligned}
\boldsymbol{e_2} &= \dot{\boldsymbol{e}}_{\boldsymbol{1}} = \boldsymbol{\chi_2} - a(\dot{V}\boldsymbol{J} + V\boldsymbol{J} - V^2\boldsymbol{I})\boldsymbol{\lambda_d}\\
\boldsymbol{e_3} &= \dot{\boldsymbol{e}}_{\boldsymbol{2}} = \boldsymbol{\chi_3} - a^2(\ddot{V}\boldsymbol{J} + \dot{V}\boldsymbol{J} - 3V\dot{V}\boldsymbol{I} - V^2\boldsymbol{I} - V^3\boldsymbol{J})\boldsymbol{\lambda_d}\\
\boldsymbol{e_4} &= \dot{\boldsymbol{e}}_{\boldsymbol{3}} = \boldsymbol{\chi_4} - a^3(V^{(3)}\boldsymbol{J} + \ddot{V}\boldsymbol{J} - 3\dot{V}^{\,2}\boldsymbol{I} - 4V\ddot{V}\boldsymbol{I} - 3V\dot{V}\boldsymbol{I} - 6V^2\dot{V}\boldsymbol{J} - V^3\boldsymbol{J} + V^4\boldsymbol{I})\boldsymbol{\lambda_d}\\
\boldsymbol{e_5} &= \dot{\boldsymbol{e}}_{\boldsymbol{4}} = \boldsymbol{v} - \boldsymbol{q_4}(a, V, \boldsymbol{\lambda_d})
\end{aligned}$$

where v is the auxiliary control input and $q_4(a, V, \lambda_d)$ is a known nonlinear function.

$$\begin{aligned}
\boldsymbol{q_4}(a, V, \boldsymbol{\lambda_d}) = a^4\big(&V^{(4)}\boldsymbol{J} + V^{(3)}\boldsymbol{J} - 10\dot{V}\ddot{V}\boldsymbol{I} - 5VV^{(3)}\boldsymbol{I} - 3\dot{V}^{\,2}\boldsymbol{I} - 4V\ddot{V}\boldsymbol{I} - 15V\dot{V}^{\,2}\boldsymbol{J} - 10V^2\ddot{V}\boldsymbol{J}\\
&- 6V^2\dot{V}\boldsymbol{J} + 10V^3\dot{V}\boldsymbol{I} + V^4\boldsymbol{I} + V^5\boldsymbol{J}\big)\boldsymbol{\lambda_d}
\end{aligned}$$

At the fourth derivative of $\boldsymbol{e_1}$, a new control input $\boldsymbol{v}(\boldsymbol{t})$ appears on the right-hand side. This input is defined in the following proposition.

Proposition 3: [14] The control law (5), with

$$\boldsymbol{v(t)} = -K_{e1}\boldsymbol{e_1} - K_{e2}\boldsymbol{e_2} - K_{e3}\boldsymbol{e_3} - K_{e4}\boldsymbol{e_4} + \boldsymbol{q_4}(a, V, \boldsymbol{\lambda_d}) \tag{18}$$

$$V(t) = \frac{L}{n_p}\frac{\tau_d(t)}{\beta_d^2}, \qquad \dot{\boldsymbol{\lambda}}_{\boldsymbol{d}}(t) = \frac{R}{L}V(t)\boldsymbol{J}\boldsymbol{\lambda_d}(t) \tag{19}$$

$$\tau_d = \frac{1}{p^4 + k_4p^3 + k_3p^2 + k_2p + k_1}[k_0D(\omega^* - \omega) + k_1\tau_L] \tag{20}$$

where the coefficients K_{e_i}, $i = 1, \dots, 4$, and k_j, $j = 0, \dots, 4$, are coefficients of appropriate Hurwitz polynomials

$$D(p) = p^4 + K_{e4}p^3 + K_{e3}p^2 + K_{e2}p + K_{e1},$$
$$M(p) = p^5 + k_4p^4 + k_3p^3 + k_2p^2 + k_1p + k_0,$$

ensures that the control objective (4) is achieved.

Proof:

From the definition of the auxiliary errors,

$$\boldsymbol{e_2} = \dot{\boldsymbol{e}}_{\boldsymbol{1}}, \boldsymbol{e_3} = \dot{\boldsymbol{e}}_{\boldsymbol{2}}, \boldsymbol{e_4} = \dot{\boldsymbol{e}}_{\boldsymbol{3}}, \dot{\boldsymbol{e}}_{\boldsymbol{4}} = \boldsymbol{v} - \boldsymbol{q_4}(a, V, \boldsymbol{\lambda_d}),$$

and the control law (18), one obtains

$$\dot{\boldsymbol{e}}_{\boldsymbol{1}} = \boldsymbol{e_2}$$
$$\dot{\boldsymbol{e}}_{\boldsymbol{2}} = \boldsymbol{e_3}$$
$$\dot{\boldsymbol{e}}_{\boldsymbol{3}} = \boldsymbol{e_4}$$
$$\dot{\boldsymbol{e}}_{\boldsymbol{4}} = -K_{e1}\boldsymbol{e_1} - K_{e2}\boldsymbol{e_2} - K_{e3}\boldsymbol{e_3} - K_{e4}\boldsymbol{e_4}$$

Hence,

$$D(p)\boldsymbol{e_1} = 0$$

where

$$D(p) = p^4 + K_{e4}p^3 + K_{e3}p^2 + K_{e2}p + K_{e1}.$$

Since $D(p)$is Hurwitz, the origin of the error dynamics is exponentially stable, yielding

$$\lim_{t\to\infty} \boldsymbol{e_i}(t) = 0, i = 1, \dots, 4$$

Proposition 2 ensures the convergence of the actual torque to the desired one. In this case, for the speed tracking error $\tilde{\omega} = \omega - \omega^*$, considering the mechanical dynamics of the motor (1) – (3). Substituting (20) into the mechanical equation, yields

$$D\dot{\tilde{\omega}} = \frac{-k_0D\tilde{\omega} + k_1\tau_L}{p^4 + k_4p^3 + k_3p^2 + k_2p + k_1} - \tau_L + \varepsilon_\tau$$

where $\tilde{\omega} = \omega - \omega^*$. Applying the operator

$$P(p) = p^4 + k_4p^3 + k_3p^2 + k_2p + k_1$$

to both sides and using the fact that $\tau_L$is constant, i.e.,

$$P(p)\tau_L = k_1\tau_L$$

gives

$$M(p)\tilde{\omega} = \bar{\varepsilon}_\tau$$

where

$$M(p) = p^5 + k_4p^4 + k_3p^3 + k_2p^2 + k_1p + k_0$$

is Hurwitz and $\bar{\varepsilon}_\tau \to 0$.

Therefore,

$$\lim_{t\to\infty} \tilde{\omega}(t) = 0$$

which proves that

$$\lim_{t\to\infty}(\omega(t)-\omega^*)=0$$

This completes the proof. ■

It should be noted that the control law (18) - (20) contains the values of the resistance parameter R, inductance L, and the load torque τ_L, which are unknown a priori. Since the estimates of these parameters converge to their true values independently of the controller (i.e., in open loop), substituting these estimates into the control law that ensures satisfaction of the control objective (4) yields

$$\boldsymbol{u}(t)=\boldsymbol{h}_{\boldsymbol{u}}^{T}\boldsymbol{\chi}(t),\dot{\boldsymbol{\chi}}(t)=\boldsymbol{\Gamma_0\chi}(t)+\boldsymbol{b_0v}(t)$$
$$\boldsymbol{v}(t)=-K_{e1}\boldsymbol{e_1}-K_{e2}\boldsymbol{e_2}-K_{e3}\boldsymbol{e_3}-K_{e4}\boldsymbol{e_4}+\boldsymbol{q_4}(\hat{a},V,\hat{\boldsymbol{\lambda}}_{\boldsymbol{d}})$$
$$V(t)=\frac{\hat{L}}{n_p}\frac{\tau_d}{\beta_d^2},\dot{\hat{\boldsymbol{\lambda}}}_{\boldsymbol{d}}(t)=\frac{\hat{R}}{\hat{L}}V\boldsymbol{J}\hat{\boldsymbol{\lambda}}_{\boldsymbol{d}}(t)$$
$$\tau_d=\frac{1}{p^4+k_4p^3+k_3p^2+k_2p+k_1}[k_0D(\omega^*-\omega)+k_1\hat{\tau}_L]$$

where the estimates $\hat{R}$, $\hat{L}$ and $\hat{\tau}_L$ are defined in (15).

3. RESULTS AND DISCUSSION

Figures 1 – 6 present simulation results illustrating the performance of the proposed control algorithm with unknown resistance, inductance and load torque. The motor parameters are as follows:

$$R=2.76, L=0.42, \tau_L=0.1, D=0.06, n_p=2$$

with initial conditions $\boldsymbol{\lambda}(0)=[1\ 0]^{\top}$ and $\omega(0)=0$.

The controller parameters are selected as

$$q_0=1.2, k_0=q_0, k_1=5q_0^2, k_2=10q_0^3, k_3=12q_0^4, k_4=3q_0^5$$
$$c_e=10, K_{e4}=4c_e, K_{e3}=6c_e^2, K_{e2}=4c_e^3, K_{e1}=c_e^4, \text{and } \beta_d=1$$

The simulation results demonstrate the effectiveness of the proposed adaptive controller under the considered operating conditions. The convergence of the estimated parameters is consistent with the theoretical analysis presented in Propositions 2 and 3. It can be observed from Figures 2 and 6 that relatively large transient deviations occur at the initial stage. This is mainly due to the initial mismatch between the estimated and actual motor parameters, together with the initial tracking errors. As the DREM estimator progressively updates the parameter estimates, the control accuracy improves, leading to the subsequent convergence of the tracking errors. Experimental validation and robustness evaluation under time-varying load disturbances will be investigated in future work.

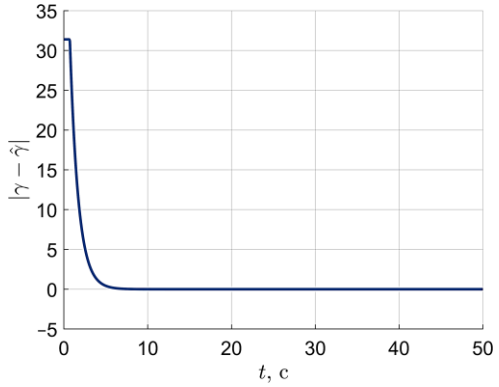


Figure 1. *Parameter estimation error.*

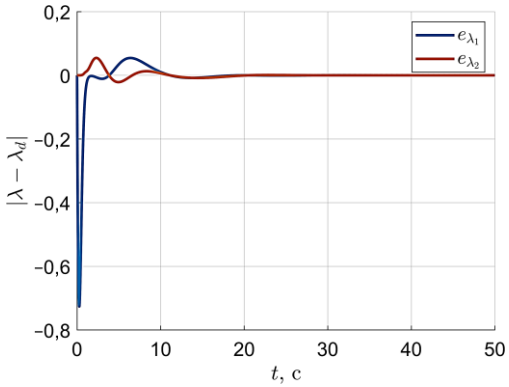


Figure 2. *Rotor flux tracking error.*

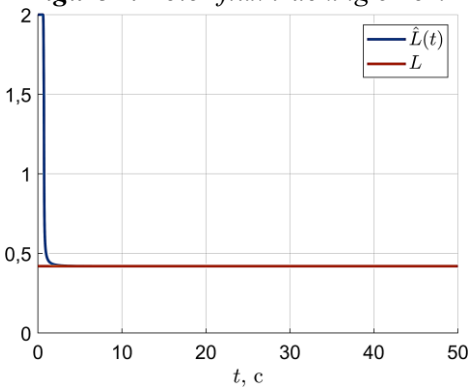


Figure 3. *Inductance estimates L.*

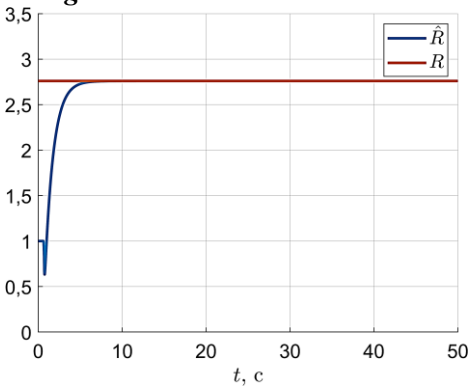


Figure 4. *Resistance estimates R.*

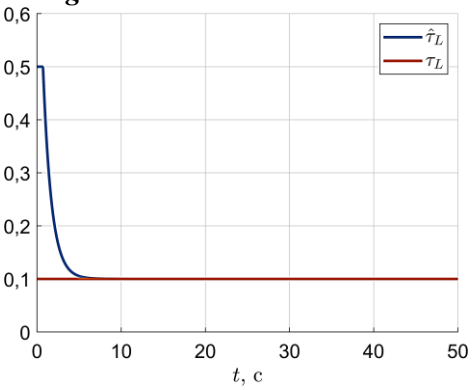


Figure 5. *Load torque estimates* τ_L.

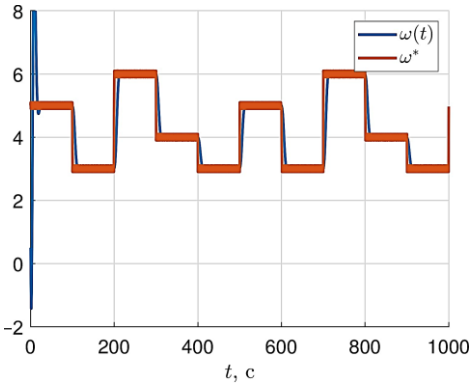


Figure 6. *Transient of rotor speed $\omega(t)$.*

4. CONCLUSIONS

This study has presented an output-feedback control framework for induction motors operating under unknown electrical and mechanical parameters. By integrating a DREM-based estimation scheme with a dynamic regulator, the proposed approach enables online parameter estimation while ensuring closed-loop stability and asymptotic convergence of the speed and rotor-flux tracking errors. The simulation results demonstrate the effectiveness and robustness of the proposed method under parameter uncertainties. The current limitation of the model is that mechanical friction and backlash associated with varying loads are not considered. Future work will incorporate these nonlinear factors to further improve control accuracy and reliability under a wider range of operating conditions.

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TÓM TẮT

Điều khiển thích nghi theo đầu ra động cơ cảm ứng với thông số chưa biết

Bài báo xem xét mô hình động cơ cảm ứng điều khiển bằng dòng điện stator với các tham số điện trở, độ tự cảm và mô-men tải không xác định. Một mô hình hồi quy tuyến tính hóa tham số được thiết lập dựa trên động lực học của tín hiệu điều khiển, cho phép áp dụng phương pháp mở rộng và trộn hồi quy động (DREM). Trên cơ sở phép đo vận tốc góc rotor, thuật toán ước lượng các tham số bất định và bộ quan sát từ thông rotor được tổng hợp. Luật điều khiển động được đề xuất đảm bảo ổn định vận tốc tại giá trị đặt và tính ổn định toàn cục cho hệ thống kín.

Từ khoá: Tham số hóa động; Điều khiển đầu ra; Điều khiển thích nghi; DREM; Động cơ cảm ứng.