A ROBUST CONTROLLER DESIGN FOR AIRCRAFT

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Abstract: *This paper is concerned with the design of a robust controller for the longitudinal flight dynamics of an aircraft control system. The design objective is to achieve robust stability and good dynamic performance against the variation of aircraft parameters. The design is based on the framework of linear parameter-varying systems where the aircraft forward speed is considered to be a real uncertainty parameter. The controller synthesis is aimed at maintaining robust dynamic performance for frozen values of the aircraft forward speed in a specified operating range. The proposed robust controller is implemented using the robust control toolbox in Matlab. Simulation results verify the performance of the proposed controller for the aircraft control system with respect to different values of the aircraft forward speeds.*

Keywords: Flight control; Robust controller; Mixed sensitivity approach; Loop-shaping design; Robust performance.

1. INTRODUCTION

Dynamical models of aircraft are highly non-linear and complicated with time-varying parameters. Regardless of the complex model, in flight control systems, the performance requirements should be maintained over the entire range of speeds and altitudes (flight envelope). For improving closed-loop longitudinal performance and robustness, multi-input multi-output controller designs can be employed for both elevator and throttle servos with the linear quadratic regulator (LQR) control [1, 2]. On the other hand, it is well-known in flight control that the actuator dynamics depend on the angle of attack regions [3]. In order to improve the system's closed-loop performance against changes in the machine parameters and exogenous inputs, several controller designs have been proposed for such aircraft in literature. In [2, 4], the H_∞ controller design is proposed for multivariable vertical short take-off and landing (VSTOL) aircraft system. In [1], the same approach is also applied to the generic VSTOL aircraft model. The H_∞ control provides a very powerful tool for controller synthesis of multivariable linear time-invariant systems in the presence of uncertainty. However, the design techniques become more complicated if considering uncertain linear time-varying systems. At the expense of conservativeness and possibly poor performance, the varying parameters can be treated as uncertainties and a single robust parameter-independent LTI controller can be designed for the entire operating range. On the other hand, if the parameter value is measurable online, one might instead try to design a parameter-dependent controller in order to improve performance. Recently, the linear parameter-varying (LPV) control approach, which takes the parameter variations into account directly in the control design, is applied for flight control systems of several types of aircraft and flight conditions [3, 5-7].

A common feature of the above publications is that the mathematical models of aircraft are not provided explicitly for some reasons. Furthermore, the robustness of the controlled system with respect to the changes of parameters is not also given clearly. Therefore, in this work, we present a robust H_∞ controller design to improve the performance of the elevator deflection control loop with respect to machine parameter variations and in view of the throttle servos disturbance. The designed controller has to be able to ensure performance in the wide operating range of the aircraft forward speed. The simulations show that the designed H_∞ controller achieves the required performance specifications over the operating range of the aircraft operation. In addition, the robustness of the controlled system against parameter changes as well as the impact of throttle servos on the robustness of the control system is also considered by means of some substantial simulation results.

2. PRELIMINARIES

In this section, we briefly discuss the multiple-objective H_∞ controller synthesis for a class of linear time-invariant systems. The method presented in this section is especially focused on affine parameter-dependent systems. The synthesis is based on the linear matrix inequality (LMI) approach and the bounded real lemma as a powerful tool for turning H_∞-constraints into LMIs. A subsequent section is devoted to discussing the classical small-gain based robust control for LTI uncertainties. More detail of the approach can be found in the literature, for instance in [8-10].

2.1. The H_∞-norm

Consider a linear input-output system that is described by

$$\begin{aligned} \dot{x} &= Ax + B\eta \\ z &= Cx + D\eta \end{aligned} \tag{1}$$

and whose transfer matrix is given by $G(s) = C(sI - A)^{-1}B + D$. The H_∞-norm of the corresponding transfer matrix G given by $\|G\|_\infty = \sup_{\omega\in\mathbb{R}} \overline{\sigma}(G(j\omega))$ where $\overline{\sigma}(M)$ stands for the largest singular value of the complex matrix M. In fact, we can characterize the stability of G and the validity of the inequality $\| G \|_\infty < \gamma$ as a linear matrix inequality (LMI). Indeed, it can be shown that A is stable and that $\| G \|_\infty < \gamma$ if and only if $D^TD - \gamma^2 I \prec 0$ and there exits some $K^T = K \succ 0$ such that the Riccati inequality

$$A^TK + KA + C^TC + (KB + C^TD)(\gamma^2 I - D^TD)^{-1}(B^TK + D^TC) \prec 0 \tag{2}$$

is satisfied. By the Schur lemma (2), these conditions are equivalent to the following system of LMIs [11]

$$K \succ 0, \quad \begin{pmatrix} A^TK + KA + C^TC & KB + C^TD \\ B^TK + D^TC & D^TD - \gamma^2 I \end{pmatrix} \prec 0. \tag{3}$$

This result is referred to as the bounded real lemma.

2.2. Small-gain theorem

Consider the simplified closed-loop interconnection shown in fig. 1 with M is a given system and Δ is an uncertainty matrix. The robust stability condition for such a system is stated as the following theorem.

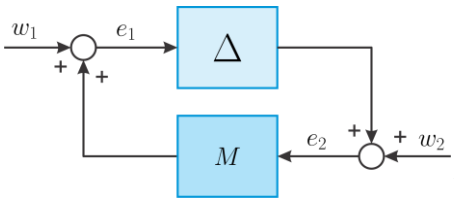


***Fig. 1**. A simplified closed-loop system.*

Theorem 1. (Small-gain theorem) Suppose $M(s)$ is stable. The closed-loop system of fig. 1 is robustly stable for all $\Delta(s) \in \mathcal{RH}_\infty$ when

(i) $\| \Delta \|_\infty \leq 1/\gamma$, it is true that $\| M \|_\infty < \gamma$, or

(ii) $\| \Delta \|_\infty < 1/\gamma$, it is true that $\| M \|_\infty \leq \gamma$.

Proof. See [19].

2.3. H_∞ controller synthesis

Consider a closed-loop control system shown in fig. 2, where w represents the generalized disturbances, z the controlled variable, η the control input, and y the measurement output, while P is a linear time-invariant system described as

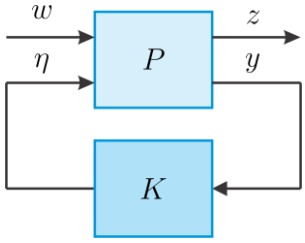


***Fig. 2**. The interconnection of the system.*

$$\begin{pmatrix} \dot{x} \\ z \\ y \end{pmatrix} = \left(\begin{array}{c|cc} A & B_p & B \\ \hline C_p & D_p & E \\ C & F & 0 \end{array}\right) \begin{pmatrix} x \\ w \\ \eta \end{pmatrix}. \tag{4}$$

If the linear time-invariant controller K is expressed as $\begin{pmatrix} \dot{x}_k \\ u \end{pmatrix} = \left(\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right) \begin{pmatrix} x_k \\ y \end{pmatrix}$, the closed-loop system $\mathcal{F}_l(P,K)$ admits the following state-space description $\begin{pmatrix} \dot{\xi} \\ z \end{pmatrix} = \left(\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right) \begin{pmatrix} \xi \\ w \end{pmatrix}$. The H_∞ control problem is rather concerned with finding an LTI controller which renders $\mathcal{A}$ stable and such that

$$\left\| \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right] \right\|_\infty = \left\| \left[\begin{array}{cc|c} A + BD_kC & BC_k & B_p + BD_kF \\ B_kC & A_k & B_kF \\ \hline C_p + ED_k & EC_k & D_p + ED_kF \end{array}\right] \right\|_\infty < \gamma \tag{5}$$

holds true [12], where $\gamma > 0$ is a given number that specifies the performance level. Using the bounded real lemma (3) for (5), the matrix $\mathcal{A}$ is stable and (5) is satisfied if and only if the LMI

$$\begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \hline 0 & I \\ \mathcal{C} & \mathcal{D} \end{pmatrix}^T \left(\begin{array}{cc|cc} 0 & \mathcal{X} & 0 & 0 \\ \mathcal{X} & 0 & 0 & 0 \\ \hline 0 & 0 & -\gamma I & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} I \end{array}\right) \begin{pmatrix} I & 0 \\ \mathcal{A} & \mathcal{B} \\ \hline 0 & I \\ \mathcal{C} & \mathcal{D} \end{pmatrix} \prec 0 \tag{6}$$

holds for some $\mathcal{X} \succ 0$. Note that we can directly minimize γ over these LMIs in order to compute the best possible level γ with (5) that is implemented in the robust control toolbox in Matlab [13-17].

2.4. Weighted H_∞ approach

Consider a simple feedback system shown in fig. 3 where $S = (I + PK)^{-1}$ is the sensitivity function which describes the influence of the external disturbance w on the tracking error y. $T = (I + PK)^{-1}PK$ is the complementary sensitivity function which describes the influence of the reference signal w on the system output z. Finally, KS is the transfer function from w to the control input u that indicates control activity [18]. In general, the performance of the closed-loop system that is specified by H_∞ norm of the channel $w \to z$ can be formulated as a multi-objective problem (see fig. 3). This leads to the minimization of

$$\left\| \begin{pmatrix} W_S S \\ W_P KS \\ W_T T \end{pmatrix} \right\|_\infty \tag{7}$$

where W_S, W_P, W_T are suitably chosen weighting functions. The multi-variable loop shaping with various specifications (7) is the so-called mixed sensitivity H_∞ design approach.

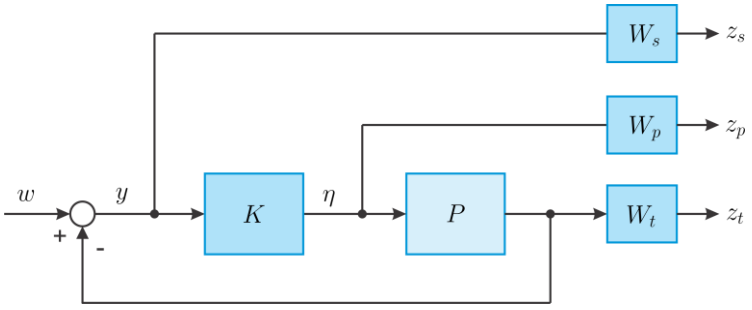


Fig. 3. *Mixed sensitivity H_∞ control.*

3. LONGITUDINAL DYNAMICS MODEL OF AIRCRAFT

Consider the aircraft body axes, (i, j, k) and the north, east, down (NED) local horizon frame, (I, J, K) as shown in fig. 4. Note that longitudinal motion is normally represented by a small displacement from equilibrium (unaccelerated) flight condition in the longitudinal plane. The flight variables in such an equilibrium are denoted with a subscript e. In this fashion, the pitch angle can be represented as $\Theta = \theta_e + \theta$. Similarly, the forward speed $U = U_e + u$, the downward (or plunge) velocity $W = v$, the pitch rate $Q = q$, the forward force $X = X_e + \overline{X}$, the downward force $Z = Z_e + \overline{Z}$, and the pitching moment $M = \overline{M}$ where θ, u, w, q, $\overline{X}$, $\overline{Z}$, $\overline{M}$ are the perturbation quantities. Let $\mathbf{J} = \left(J_{ik}\right)\big|_{i,k=\{x,y,z\}}$ be the inertia tensor, where J_{xx}, J_{yy}, J_{zz} are the moments of inertia, and J_{xy}, J_{yz}, J_{xz} are the products of inertia. Note that, for a symmetrical plane, $J_{xy} = J_{yz} = 0$. Let α be the angle of attack, m be the aircraft's mass, X be the forward force, Z be the downward force, M be the pitching moment, U be the forward speed. Denote $F_x = \frac{\partial F}{\partial x}\big|_e$, where $F \in \{X, Z, M\}$, as the first-order of a Taylor series expansion at the equilibrium point.

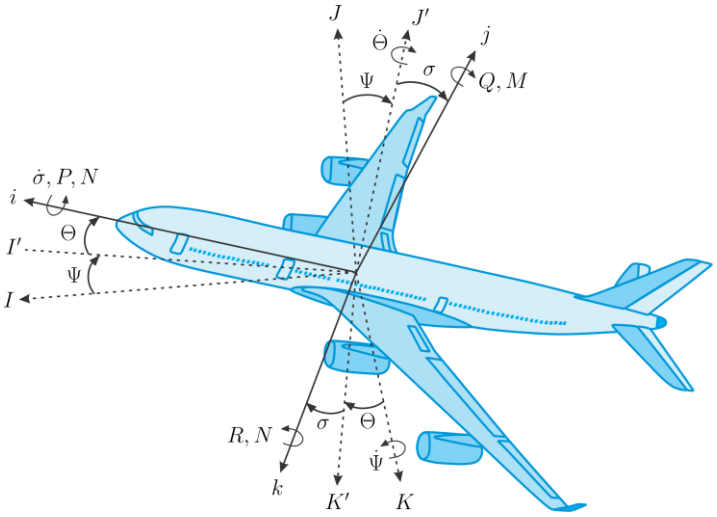


Fig. 4. *The aircraft body axes [1].*

By neglecting products of small perturbation quantities we obtain the longitudinal dynamics model of the aircraft in the state-space form as [1]

$$\dot{x}_l = A_l x_l + B_l w_l \tag{8}$$

$$y_l = C_l x_l \tag{9}$$

where

$$A_l = \begin{bmatrix} \frac{X_u}{m} & \frac{X_\alpha}{m} & -g\cos\theta_e & 0 \\ \frac{Z_u}{mU} & \frac{Z_\alpha}{mU} & -\frac{g\sin\theta_e}{U} & 1+\frac{Z_q}{mU} \\ 0 & 0 & 0 & 1 \\ \frac{M_u}{J_{yy}}+\frac{M_{\dot\alpha}Z_u}{mUJ_{yy}} & \frac{M_\alpha}{J_{yy}}+\frac{M_{\dot\alpha}Z_\alpha}{mUJ_{yy}} & -\frac{M_{\dot\alpha}g\sin\theta_e}{UJ_{yy}} & \frac{M_q}{J_{yy}}+\frac{M_{\dot\alpha}(mU+Z_q)}{mUJ_{yy}} \end{bmatrix}, \tag{10}$$

$$B_l = \begin{bmatrix} \frac{X_\delta}{m} & \frac{X_T}{m} \\ \frac{Z_\delta}{mU} & \frac{Z_T}{mU} \\ 0 & 0 \\ \frac{M_\delta}{J_{yy}}+\frac{M_{\dot\alpha}Z_\delta}{mUJ_{yy}} & \frac{M_T}{J_{yy}}+\frac{M_{\dot\alpha}Z_T}{mUJ_{yy}} \end{bmatrix}, \quad C_l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{11}$$

$x_l = \begin{bmatrix} u & \alpha & \theta & q \end{bmatrix}^T$ is the state variable, $w_l = \begin{bmatrix} \delta_E & \beta_T \end{bmatrix}^T$ is the input of the system, δ_E is the elevator deflection, β_T is the throttle servos.

Let $v_a = \frac{1}{U}$ and express $\mathsf{v_a}$ as an uncertainty element $v_a = v_n(1 + p_v\delta_v)$, where v_n is the nominal value of v_a, $p_v \in \mathbb{R}$ indicates the variation of v_a around its nominal value, $\delta_v \in \mathbb{R}$, $-1 \leq \delta_v \leq 1$, we can write

$$A_l = A_{ln} + \delta_v A_{lv} \tag{12}$$

$$B_l = B_{ln} + \delta_v B_{lv} \tag{13}$$

in which

$$A_{ln} = \begin{bmatrix} \frac{X_u}{m} & \frac{X_\alpha}{m} & -g\cos\theta_e & 0 \\ \frac{Z_u}{m}v_n & \frac{Z_\alpha}{m}v_n & -g\sin\theta_e v_n & 1 + \frac{Z_q}{m}v_n \\ 0 & 0 & 0 & 1 \\ \frac{M_u}{J_{yy}} + \frac{M_{\dot\alpha}Z_u}{mJ_{yy}}v_n & \frac{M_\alpha}{J_{yy}} + \frac{M_{\dot\alpha}Z_\alpha}{mJ_{yy}}v_n & -\frac{M_{\dot\alpha}g\sin\theta_e}{J_{yy}}v_n & \frac{M_q}{J_{yy}} + \frac{M_{\dot\alpha}}{J_{yy}} + \frac{M_{\dot\alpha}Z_q}{mJ_{yy}}v_n \end{bmatrix} \tag{14}$$

$$A_{lv} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{Z_u}{m}v_n p_v & \frac{Z_\alpha}{m}v_n p_v & -g\sin\theta_e v_n p_v & \frac{Z_q}{m}v_n p_v \\ 0 & 0 & 0 & 0 \\ \frac{M_{\dot\alpha}Z_u}{mJ_{yy}}v_n p_v & \frac{M_{\dot\alpha}Z_\alpha}{mJ_{yy}}v_n p_v & -\frac{M_{\dot\alpha}mg\sin\theta_e}{mJ_{yy}}v_n p_v & \frac{M_{\dot\alpha}Z_q}{mJ_{yy}}v_n p_v \end{bmatrix} \tag{15}$$

$$B_{ln} = \begin{bmatrix} \frac{X_\delta}{m} & \frac{X_T}{m} \\ \frac{Z_\delta}{m}v_n & \frac{Z_T}{m}v_n \\ 0 & 0 \\ \frac{M_\delta}{J_{yy}} + \frac{M_{\dot\alpha}Z_\delta}{mJ_{yy}}v_n & \frac{M_T}{J_{yy}} + \frac{M_{\dot\alpha}Z_T}{mJ_{yy}}v_n \end{bmatrix} \quad B_{lv} = \begin{bmatrix} 0 & 0 \\ \frac{Z_\delta}{m}v_n p_v & \frac{Z_T}{m}v_n p_v \\ 0 & 0 \\ \frac{M_{\dot\alpha}Z_\delta}{mJ_{yy}}v_n p_v & \frac{M_{\dot\alpha}Z_T}{mJ_{yy}}v_n p_v \end{bmatrix} \tag{16}$$

Equation (8) can now be expressed as

$$\begin{aligned} \dot{x}_l &= (A_{ln} + \delta_v A_{lv})x_l + (B_{ln} + \delta_v B_{lv})w_l \\ &= \begin{pmatrix} A_{ln} & B_{ln} \end{pmatrix}\begin{pmatrix} x_l \\ w_l \end{pmatrix} + \delta_v \begin{pmatrix} A_{lv} & B_{lv} \end{pmatrix}\begin{pmatrix} x_l \\ w_l \end{pmatrix} = \begin{pmatrix} A_{ln} & B_{ln} \end{pmatrix}\begin{pmatrix} x_l \\ w_l \end{pmatrix} + w_v \end{aligned} \tag{17}$$

where

$$w_v = \delta_v \begin{pmatrix} A_{lv} & B_{lv} \end{pmatrix}\begin{pmatrix} x_l \\ w_l \end{pmatrix} = \delta_v z_v, \quad z_v = \begin{pmatrix} A_{lv} & B_{lv} \end{pmatrix}\begin{pmatrix} x_l \\ w_l \end{pmatrix}. \tag{18}$$

In (18), w_v and z_v represent the input and output signals of the disturbance channel corresponding to the time-varying parameter v_a. Rewrite equations (17), (18), and (9) in a matrix form as

$$\begin{pmatrix} \dot{x}_l \\ z_v \\ y_l \end{pmatrix} = \begin{pmatrix} A_{ln} & B_{lw} & B_{ln} \\ A_{lv} & B_{lz} & B_{lv} \\ C_{ln} & D_{lw} & D_{lu} \end{pmatrix}\begin{pmatrix} x_l \\ w_v \\ w_l \end{pmatrix}, \tag{19}$$

$$w_v = \Delta_v z_v, \quad \Delta_v = \delta_v I_4 \tag{20}$$

where $B_{lw} = I_4$ is a 4×4 unity matrix, $B_{lz} = Z_4$ is a 4×4 zero matrix, $D_{lw} = D_{lu} = 0$. Δ_v is also called the perturbation block. Let G_{la} be the transfer function with the state-space realization (19), i.e.

$$G_{la} \triangleq \left[\begin{array}{c|c} A_{la} & B_{la} \\ \hline C_{la} & D_{la} \end{array}\right] = \left[\begin{array}{c|cc} A_{ln} & B_{lw} & B_{ln} \\ \hline A_{lv} & B_{lz} & B_{lv} \\ C_{ln} & D_{lw} & D_{lu} \end{array}\right]. \tag{21}$$

The system can then be generally described by

$$\begin{pmatrix} z_v \\ y_l \end{pmatrix} = G_a \begin{pmatrix} w_v \\ w_l \end{pmatrix} = \begin{pmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{pmatrix} \begin{pmatrix} w_v \\ w_l \end{pmatrix}, \tag{22}$$

where G_{yu} is the transfer function mapping w_l to y_l.

4. H_∞ CONTROLLER DESIGN

In this section we start with H_∞-synthesis for the above mentioned frozen values of the aircraft forward speed. Then the performance of the linear time-invariant (LTI) controller designed for a fixed value of v_a is evaluated with other constant values of its.

4.1. H_∞ loop shaping design

A standard control structure for the synthesis of an H_∞-controller is depicted in fig. 5. Here, G_{ln} is the LTI part of the plant as given in (21), Δ_v is the uncertainty block as given in (20), K_{le} is the H_∞ controller that is to be designed. In this configuration, the airspeed u_d is the reference input, δ_E is the controller output, y is the controlled output, and $e_{le} = u_d - y$ is the controller input which is equal to the tracking error.

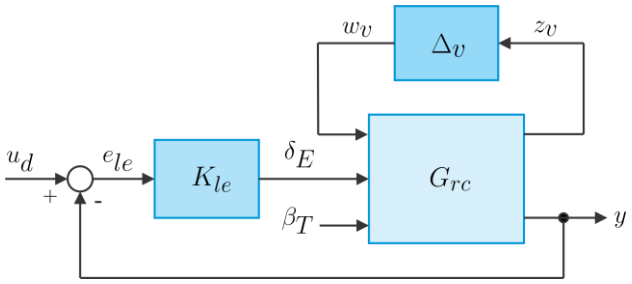


Fig. 5. *Structure of the closed-loop system in H_∞ design.*

The interconnection of the system used for the controller synthesis is shown in fig. 6. The external control input $\mathbf{w_{le}}$ consists of throttle servo and reference elevator deflection $w_{le} = \begin{pmatrix} u_d & \beta_T \end{pmatrix}^T$. The controlled variable is $z_{le} = \begin{pmatrix} z_t & z_s \end{pmatrix}^T$. Note that the component β_T of the external control inputs are considered disturbances and their influences on the controlled outputs must be reduced as much as possible.

The weighting function W_s is used to shape the transfer function from the external control input w_{le} to the tracking error e_{le}. W_s is kept large over the low-frequency range for tracking. The weighting function W_t is used to shape the transfer function from the external control input w_{le} to the controlled output y. The selection of the weighting function W_t is not only intended to keep the closed-loop bandwidth at the desired value but also to reject the effects of the component β_T on the controlled outputs as discussed above. Note that a large bandwidth corresponds to a faster rise time but the system is more sensitive to noise and to parameter variations [18].

The standard H_∞ control problem is to find a stabilizing LTI controller K_{le} at fixed frozen values of v_a such that the H_∞-norm of the channel $w_{le} \to z_{le}$ is smaller than a given number γ:

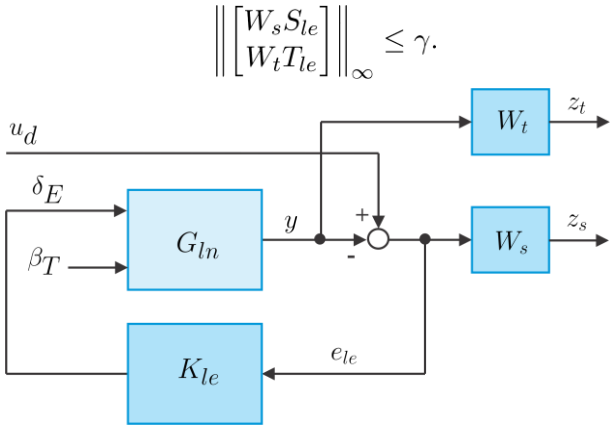


Fig. 6. *The interconnection of the system.*

4.2. Simulation results with the H_∞ controller

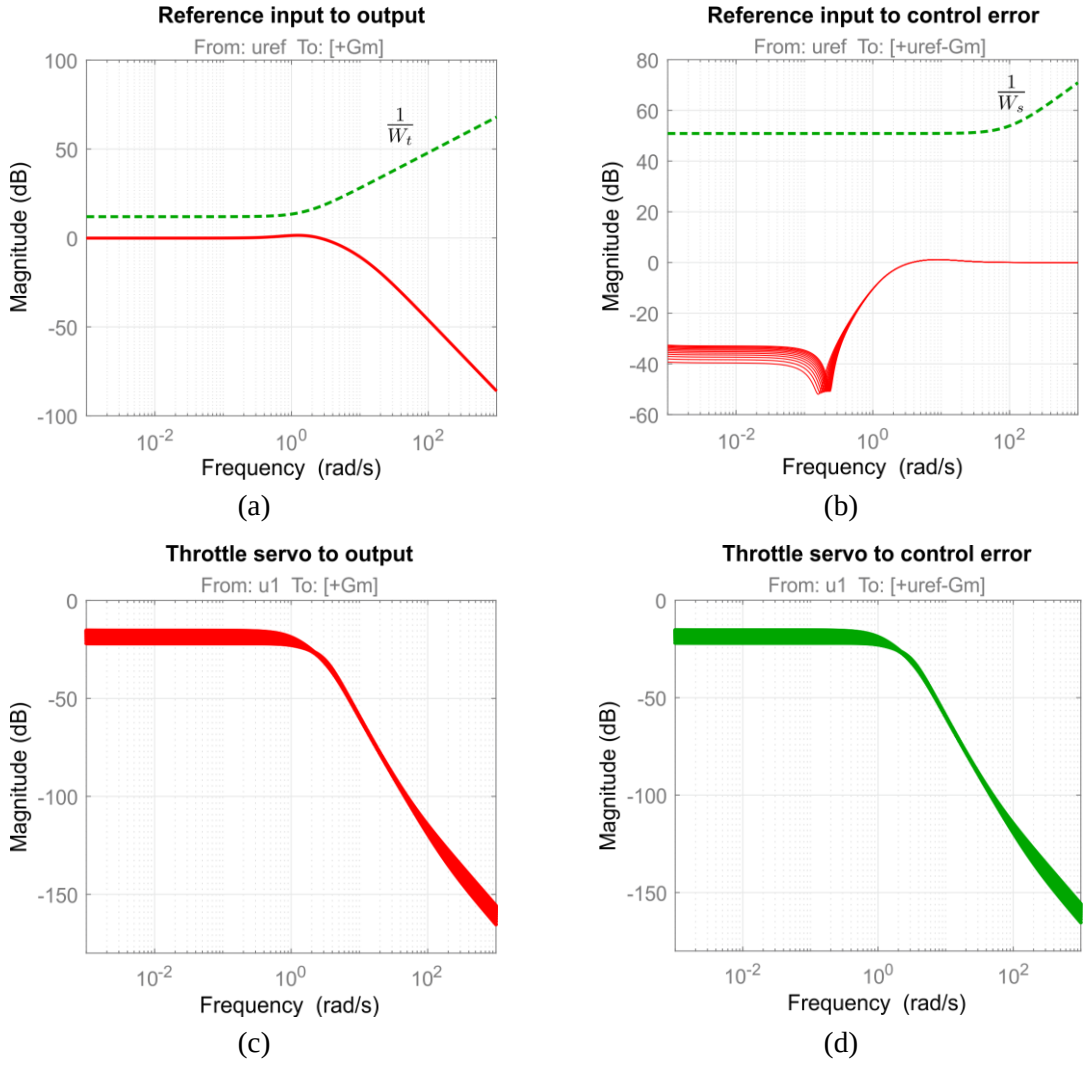


Fig. 7. *Performance of the controlled system with H_∞ controller in the frequency domain at different aircraft forward speeds.*

The set of the aircraft parameters that is given as follows [1]: $\theta_e = 0$, $\frac{Z_u}{m} = -0.36/s$, $\frac{X_\alpha}{m} = 1.96m/s^2$, $\frac{Z_\alpha}{m} = 108m/s^2$, $\frac{M_\alpha}{J_{yy}} = -8.6/s^2$, $\frac{M_{\dot{\alpha}}}{J_{yy}} = -0.9/s$, $\frac{M_q}{J_{yy}} = -2/s$, $\frac{X_\delta}{m} \approx 0$, $\frac{Z_\delta}{m} = 0.3m/s^2/rad$, $\frac{M_\delta}{J_{yy}} = 0.1243s^{-2}$, $\frac{X_T}{m} = 0.2452m/s^2/rad$, $\frac{Z_T}{m} \approx 0$, and $\frac{M_T}{J_{yy}} \approx 0$. During the controller design stage, a trial-and-error-repetition technique is used in order to achieve the desired performance specifications by adjusting the weighting functions. The design steps are repeated until we are able to meet the required performance specifications. Finally, the following weighting functions were obtained:

$$W_s = \frac{1}{3.5s + 350}, \tag{23}$$

$$W_t = \frac{0.5}{1.25s + 1.98}. \tag{24}$$

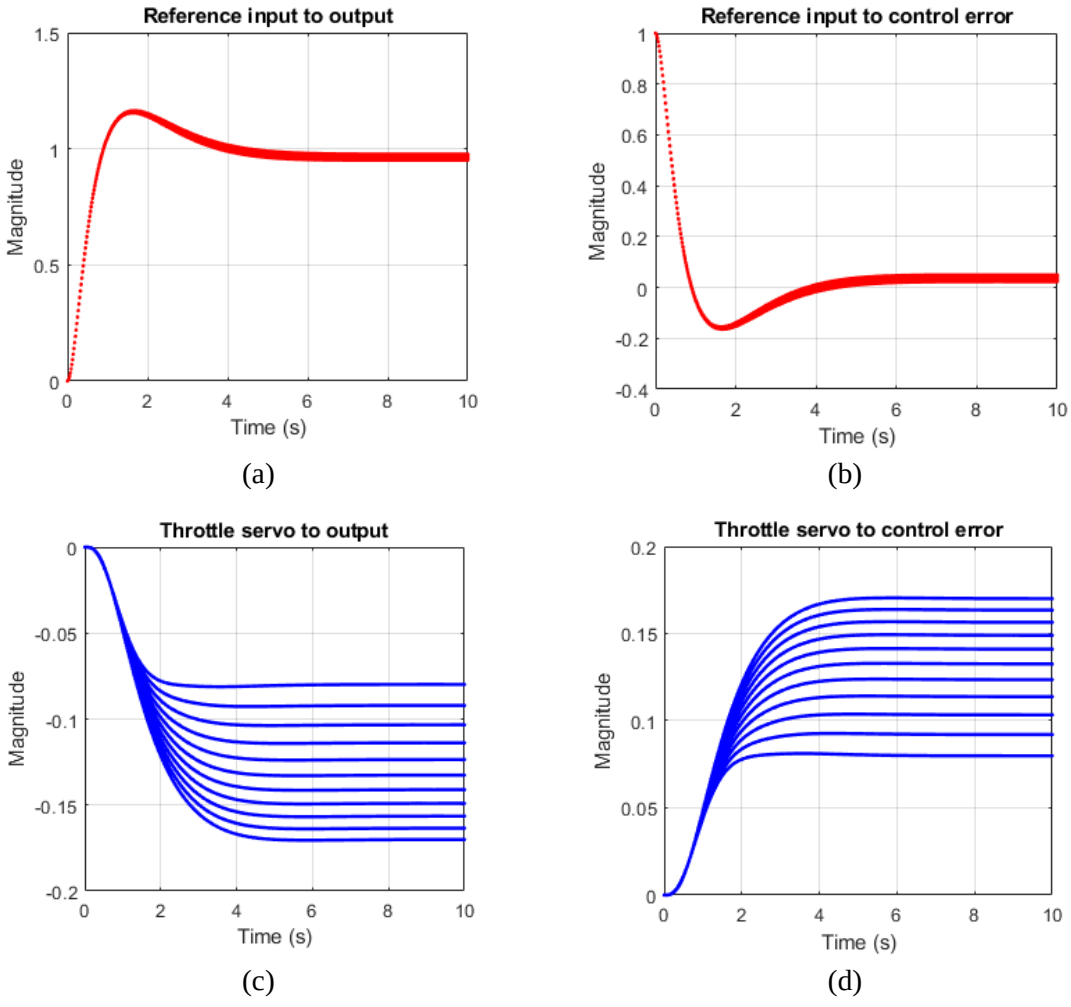


Fig. 8. *Performance of the controlled system with H_∞ controller in the frequency domain at different aircraft forward speeds.*

For the chosen frozen value of $U = 55$m/s, the controlled system with the H_∞ controller for the above given weighting functions achieves a norm of 0.836. Since we assume that $-1 \leq \delta_v \leq 1$, the perturbation block Δ_v in (20) satisfies $\| \Delta_v \|_\infty < 1$. Therefore, by Theorem 1, the closed-loop

system is robustly stable against the variations of aircraft forward speed in the specified range because the H_∞-norm of the channel $w_{le} \rightarrow z_{le}$ is smaller than 1 (see fig. 1 and fig. 5).

Fig. 7 shows the frequency responses of the controlled system with the H_∞ controller and the inverse of the weighting functions W_t (see equations 23) and (24) with 11 frozen values of the aircraft forward speed from 50% up to 150% of its nominal magnitude. Figs. 6a, b show the relevant magnitude plots of the complementary sensitivity and sensitivity functions of the closed-loop system with the performance requirements achieved by W_t and W_s. In Fig. 6a, the solid curve shows the response of the output y with respect to the reference inputs u_d. The inverse of the weighting function W_t (see Fig. 7) is depicted by the dashed line in Fig. 7a and the inverse of the weighting function W_s is depicted by the dashed lines in fig. 7b, respectively. The performance of the reference input to the control error as well as the influence of the throttle servos β_T on the controlled output are shown in Figs. 7c,d.

It is clear from fig. 7 that the sensitivity and complementary sensitivity functions are below the inverse of the performance weighting functions. The gains of the frequency responses of the throttle servos to controlled output and controller error are all smaller than -15db. This indicates that the controlled system has good disturbance rejection with respect to the throttle servos β_T. The overshoot of the channel $u_d \rightarrow y$ is about 15% and hence satisfies the performance requirement as well. Note that these performance curves are obtained for 11 values of the aircraft forward speeds as mentioned above. The simulation results confirm that if the H_∞-norms of the uncertain and LTI blocks are smaller than 1, the closed-loop system is robustly stable and the closed-loop performance is maintained when aircraft forward speeds vary in the given range as it is stated in Theorem 1.

Fig. 8 shows the time responses of the controlled system for a step input in the consistency to the curves in the frequency domain as shown in fig. 7 with 11 values of the aircraft forward speed as shown above.

5. CONCLUSIONS

This paper briefly presented an LMI-based loop-shaping design of the robust H_∞ controller for the linear simplified longitudinal model of an aircraft, in which the aircraft forward speed is considered as an uncertain parameter. The robust H_∞ controller is then synthesized to guarantee that the H_∞-norm of the closed-loop system is smaller than some given number for different frozen values of aircraft forward speed. Next, the robust performance of the robust controller with respect to the other aircraft forward speeds is investigated in the range from 50% up to 150% of its nominal values. Some simulation results are given to demonstrate the performance and robustness of the control algorithm.

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TÓM TẮT

MÔ HÌNH HÓA PHƯƠNG PHÁP XÁC ĐỊNH ĐIỆN MÔI PHỨC VÀ TỪ THẨM PHỨC CỦA VẬT LIỆU SỬ DỤNG TRUYỀN SÓNG ĐIỆN TỪ TRONG KHÔNG GIAN TỰ DO Ở BĂNG TẦN X

Bài báo này liên quan đến việc thiết kế bộ điều khiển bền vững cho mô hình động học chuyển động dọc của hệ thống điều khiển máy bay. Mục tiêu của bài toán thiết kế là nhằm đạt được tính ổn định bền vững với chất lượng động học mong muốn để chống lại các biến đổi tham số trong mô hình máy bay. Việc thiết kế bộ điều khiển được thực hiện dựa trên cơ sở hệ thống tuyến tính có các tham số biến đổi, trong đó, vận tốc của máy bay được coi là một tham số bất định. Việc tổng hợp bộ điều khiển là nhằm duy trì chất lượng động học bền vững của hệ thống khi vận tốc của máy bay nằm trong một khoảng xác định trước. Bộ điều khiển bền vững được thực hiện nhờ hộp công cụ điều khiển bền vững của phần mềm Matlab. Các kết quả mô phỏng đã kiểm chứng chất lượng của bộ điều khiển đối với các giá trị khác nhau của vận tốc máy bay.

Từ khóa: Điều khiển bay; Bộ điều khiển bền vững; Phương pháp độ nhạy hỗn hợp; Thiết kết loop-shaping; Chất lượng bền vững.

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