Prediction of the conjugate depth of the hydraulic jump in the trapezoidal channel using Random Forest regression

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ABSTRACT

Prediction of the sequent depths of the hydraulic jump in the trapezoidal channel using the theoretical equation is a challenging task. Therefore, existing studies have attempted to solve the task by conducting experiments or using semi-empirical calculations. The paper proposes a novel method that applies Buckingham's Pi theory and the Random Forest regression to improve the prediction accuracy of the sequent depths of the hydraulic jump in the trapezoidal channel. The study has shown that Machine Learning models can be efficient for the determination of the geometrical features of the jump and have high ability in many real projects.

Keywords: Machine Learning; Sequent depth; Hydraulic jump; Trapezoidal channel; Pi theory; Random Forest.

1. INTRODUCTION

1.1. Introduction to studies on the sequent depth of hydraulic jump in the trapezoidal channel

The hydraulic jump is a hydraulic phenomenon that occurs on open flows. It appeared when the flow changes from the supercritical flow ($Fr_1 > 1$) to the subcritical flow ($Fr_1 < 1$) [1, 2]. In practice, the jump has many applications, such as energy dissipation, air entrainment, design of sludge compaction tanks in wastewater treatment systems, and so on.

The study of the conjugate depths of the jump in the trapezoidal channel has been carried out by scientists through experimental, semi-empirical, or theoretical methods, such as the research methods on the theory of Wanoschek R. and Hager W. (1989) [3] or Sadiq S.M (2012) [4] or the experimental study of A.N. Rakhmanov (1930), Samir Kateb (2014) [5], Samir Kateb, Mahmoud Debabeche and Ferhat Riguet (2015) [6], Bahador F.N et al. (2019) [10], etc.

These existing studies only consider specific solutions, and it is difficult to provide general solutions. Moreover, the application of machine learning techniques has not been researched and developed. At the same time, the scientists' data are not inherited, which hinders further research. Meanwhile, the Machine Learning method will open up new research trends and can link the databases of hydraulic scientists' networks to build more practical applications.

1.2. Application of machine learning to hydraulic research

Machine Learning solutions have been applied to water resources engineering, hydraulics, and hydrology. As Steven L. Brunton et al. (2019) [16] has reviewed machine learning and basic applications in fluid mechanics, the study analyzed and showed that machine learning applications give good efficiency in hydraulic research, especially research by physical models. Melhem et al. (1996) [18] analyzed a Machine Learning solution in engineering with a suitable database, and it would give quick and high accuracy results.

Researching the geometrical characteristics of the jump has been recognized in the studies for the application of Artificial Intelligence (AI), especially the application of ANN in the study of the jump length, such as the study of Mahdi Naseri and Faridah Othman (2012) [18], L. Houichi et al. (2013) [19], Mohamed F. Sauida (2016) [20], Masoud Karbasi and H. Md. Azamathulla

(2016) [21], P. Khosravinai et al (2018) [22], etc. Among them, Ghorban Mahtabi et al. (2020) [23] applied a Decision tree classifier (J48) to classify the jump, and Akram ABBASPOUR et al. (2013) [24] analyzed the sequent depth by ANN.

Thus, the application of Artificial Intelligence (AI), Machine Learning (ML), and Deep Learning (DL) to the study for the sequent depth is very little and has not been deeply researched. In this study, we apply Buckingham's Pi theory to the basic momentum equation to establish the relationship between the sequent depths with the influencing factors, and then we collect experimental data (the authors and other studies). Finally, the Random Forest algorithm has been conducted to predict the associated sequent depths of the jump in the trapezoidal channel.

2. APPLYING THE PI THEORY IN THE ANALYSIS OF FACTORS AFFECTING THE SEQUENT DEPTHS

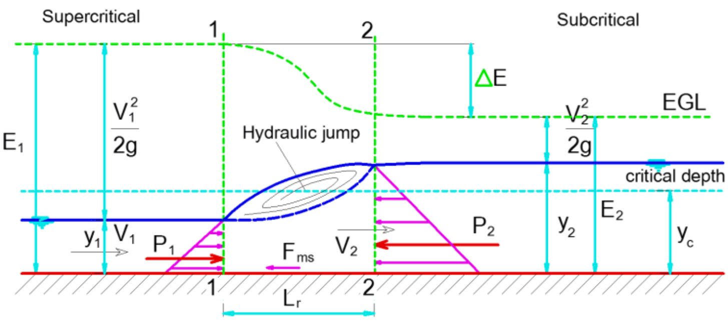

Figure 1. *Schematic analysis of forces acting on the hydraulic jump.*

Considering the momentum equation for the control volume from the cross-section (1-1) to (2-2) in the roller zone of the jump on the horizontal bottom with the friction force [2]:

$$P_1 - P_2 - F_{ms} = \rho Q_2 \alpha_{02} V_2 - \rho Q_1 \alpha_{01} V_1 \quad (1)$$

Where:

- P_1, P_2 as hydrostatic pressure force of the section (1-1), (2-2), respectively;
- F_{ms} as hydraulic resistance (such as bottom friction and turbulence in the roller zone);
- $Q = Q_1 = Q_2$ as discharge (m^3/s);
- V_1, V_2 as velocities of the section (1-1), (2-2), respectively.

The main influencing factors of equation (1) are written as functional relationships:

$$f(y_2, y_1, Q, V_1, V_2, , m, b, \alpha_{01}, \alpha_{02}, \rho, \mu, g, \Sigma\xi) = 0 \quad (2)$$

From equation (2), the number of variables is 9. Choose 3 independent variables (V_1, y_1 and ρ) with the basic dimensions M (Mass), L (Length), and T (Time) for analysis, so the number of dimensionless groups is 6. The values in the function Pi (π_i) are redefined from (2) as follows:

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) = 0 \quad (3)$$

The results of the dimensional analysis according to the Pi function in equation (3) are as follows:

$$\varphi\left(\frac{y_2}{y_1}, \frac{b}{y_1}, \frac{\mu}{V_1.y_1.\rho}, m, \frac{g.y_1}{V_1^2}, \sum\xi\right) = \varphi\left(\frac{y_2}{y_1}, \frac{b}{y_1}, \mathrm{Re}, m, Fr_1, \sum\xi\right) = 0 \quad (4)$$

Transform equation (4) into (ignore Re because this value on experimental model and reality are similar, so it has little impact on research objectives[13]):

$$\beta\left(\frac{y_2}{y_1}, \frac{b}{y_1}, m, Fr_1, \sum\xi\right) = 0 \quad (5)$$

Analyzing according to Buckingham's Pi Theory, equation (5) is expressed as follows:

$$\frac{y_2}{y_1} = \Psi\left(M_1, Fr_1, \sum \xi\right) \tag{6}$$

When m = 0 (the rectangular channel) and ignores the loss, equation (6) becomes a Bélanger equation (1882) [2]. Equation (6) is consistent for the case of the horizontal bottom, smooth bed, and the loss as an empirical coefficient (ignore $\sum \xi$), so equation (6) becomes:

$$\frac{y_2}{y_1} = \Psi\left(M_1, Fr_1\right) \tag{7}$$

The data field of the machine learning model, according to equation (7), is as follows:

- Target variable y_2/y_1
- Analytical data variables: M_1, Fr_1

3. EXPERIMENTAL PHYSICAL MODEL SYSTEM

The experimental model system was carried out at the Vietnam Academy for Water Resources (KLORCE). We conducted experiments on physical models, and the model consists of an ogee spillway, a horizontal trapezoidal channel made of organic glass. Including a static lake ①, and a spillway ogee ②, the flow through the spillway will occur the hydraulic jump phenomenon in an isosceles trapezoidal channel ③ (stilling basin and side slope m = 1), a stable area at the end of the channel ④ and the control gate adjusts to change the water level in channel ⑤. The dimensions of the bed of the channel are arranged in two options: the bed width b = 0.55 m and b = 33.5 cm, height h = 65 cm, and length of trapezoidal channel L = 4 m.

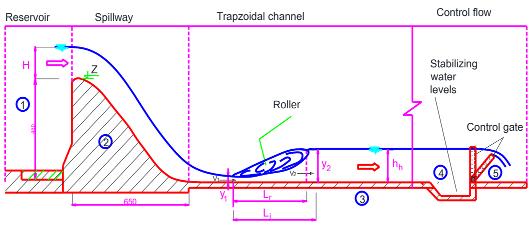

Figure 2. *Plan of the experimental equipment.*

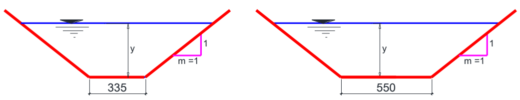

Figure 3. *The cross section of trapzoidal channel.*

Figure 4. *Panoramic view of* the *experimental model from upstream to downstream.*

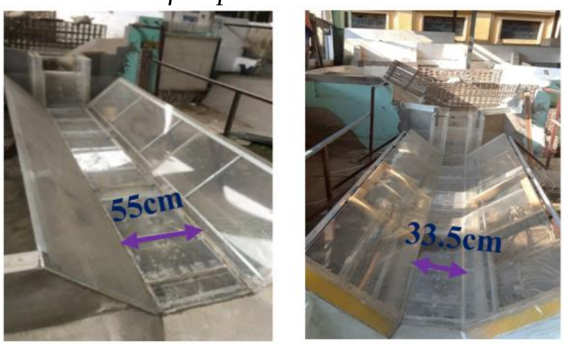

Figure 5. *Ogee spillway and energy dissipation channel.*

The flow over the Ogee spillway ② from the tank ①, measuring the height of the water level on the spillway crest to determine the discharge (Q), the flow through the spillway with the largest energy, will create a supercritical flow at the toe of spillway, then use the control gate ⑤ to adjust the water level at the end of the model to change the water level in the trapezoidal channel, so making the hydraulic jump in trapezoidal channel. Other hydraulic parameters were measured by the leveling staff and the automatic level surveying (figure 7).

Figure 6. *Hydraulic jump in trapezoidal channel (side slope 1:1) with Q = 118 l/s and Fr_1 = 4.19.*

Figure 7. *Measuring water level data by a leveling staff.*

The experimental results of the jump in the trapezoidal channel are shown as follows:

Table 1. *Experimental data range with b = 55 cm (25 values).*

No	**Parameter**	Symbol	Unit	Max	Min
1	Discharge	Q	l/s	60	201
2	Initial depth of hydraulic jump	y_1	mm	40	79
3	Secondary depth of hydraulic jump	y_r	mm	182	488
4	The ratio of Sequent depth	y_r/y_1		9.40	4.26
5	Sidewall constant in trapezoidal channel	M		0.15	0.07
6	Upstream Froude number	Fr_{D1}		8.25	4.18

Table 2. *Experimental data range with b = 33.5 cm (36 values).*

No	**Parameter**	Symbol	Unit	Max	Min
1	Discharge	Q	l/s	40	158
2	Initial depth of hydraulic jump	y_1	mm	40	92
3	Secondary depth of hydraulic jump	y_r	mm	186	488
4	The ratio of Sequent depth	y_r/y_1		8.61	4.32
5	Sidewall constant in trapezoidal channel	M_1		0.27	0.12
6	Upstream Froude number	Fr_{D1}		8.40	4.10

Table 3. *Experimental data of Wanoschek R. & Hager W. (1989) with b = 0.2 m (39 values).*

No	**Parameter**	Symbol	Unit	Max	Min
1	Discharge	Q	l/s	98	7.5
2	Initial depth of hydraulic jump	y_1	mm	81.2	20.3
3	Secondary depth of hydraulic jump	y_r	mm	441	80
4	The ratio of Sequent depth	y_r/y_1		12.44	2.35
5	Sidewall constant in trapezoidal channel	M_1		0.41	0.1
6	Upstream Froude number	Fr_{D1}		14.7	2.35

Combining data from different empirical models and the other scientists, a set of data (about 100 data) can use to study Machine Learning to determine the features of the jump. It can initially meet the trend and effectiveness of research methods. Actually, we have prepared the data samples that are efficient for predicting by Machine Learning models as mentioned in the related studies of the fields [16, 17].

4. RANDOM FOREST REGRESSION MODEL TO CALCULATE THE CONJUGATE DEPTHS

Random Forest (RF) is a supervised learning method [14] and has been early used since 1990. The model can handle problems of classification and regression. In the work, the Random Forest regression is applied to determine the conjugate depths**.** In theory, the RF consists of a collection of decision trees [15]. Compared to individual decision trees, the RF allows for obtaining higher prediction accuracy. However, the RF structure is more complex, and it consists of more parameters in the training process.

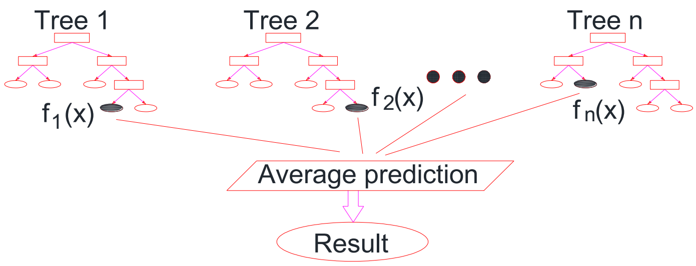

Figure 8. *A structure of random forest algorithm.*

The algorithm of the random forest model can be described as follows:

$$g(x) = \sum_{i=1}^{n} \alpha_i f_i(x) \tag{8}$$

Where:

- $f_i(x)$ as the "Decision tree" at No "i";

- α_i as the influence coefficient of the " Decision tree" in the "Random Forest", $\sum \alpha_i = 1$.

This is the "bagging" method, and it is widely used and obtains high predictive accuracy.

In the work, we implement the Random Forest in the Matlab environment. The Random Forest applies the Least-Squares Boosting (LS_Boost) algorithm for the regression of the sequent depths of the hydraulic jump.

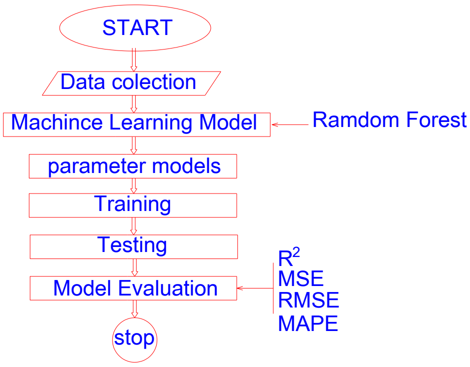

Figure 9. *Flowchart of the Random Forest Model.*

ALGORITHM 2 (LS_Boost).
$F_0(\mathbf{x}) = \bar{y}$
For $m = 1$ to M do:
$\tilde{y}_i = y_i - F_{m-1}(\mathbf{x}_i), \quad i = 1, N$
$(\rho_m, \mathbf{a}_m) = \arg\min_{\mathbf{a},\rho} \sum_{i=1}^{N} [\tilde{y}_i - \rho h(\mathbf{x}_i; \mathbf{a})]^2$
$F_m(\mathbf{x}) = F_{m-1}(\mathbf{x}) + \rho_m h(\mathbf{x}; \mathbf{a}_m)$
endFor
end Algorithm

Figure 10. *Structure of the Least-Squares Boost algorithm in Friedman (2001)'s "Random Forest" [25].*

The process of the prediction of the sequent depths of the hydraulic jump consists of two main steps:

(1) Training process: The number of decision trees is carefully selected during the training process to obtain the highest accuracy. We have compared the performance of the Random

Forest with various numbers of the decision trees and selected the optimal number. Actually, when the number of decision trees increases from 10 to 800, the prediction error sharply decreases from 0.30 to 0.05. Then, when the number of trees increases from 800 to 1000, the prediction error slowly decreases from 0.05 to 0.03. After that, the prediction error is almost convergent. So, the optimal number of trees is selected at 1000. The training algorithm for the random forest is Least-Squares Boosting (LS_Boost) [16]. The model is trained on the training dataset obtained by our experimental model. The parameters are illustrated in figure 14.

(2) Testing process: The trained model is applied to determine the sequent depths of the hydraulic jump using the testing data.

The Random Forest model was implemented in Matlab 2019b, and the computer environment is Windows 10 operating system, 8GB RAM, Core i5, 2.67 GHz CPU.

5. DISCUSSION OF OBTAINED RESULTS

Datasets for training and testing the Random Forest regression model for the sequent depths contain 100 items representing 03 different sized physical models. In the dataset, about 14% of the data is used (table 4) to test the trained model.

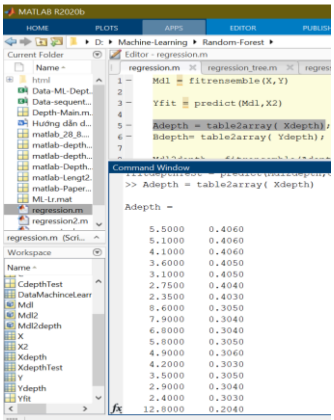

Figure 11. *Data for analyzing Machine learning.*

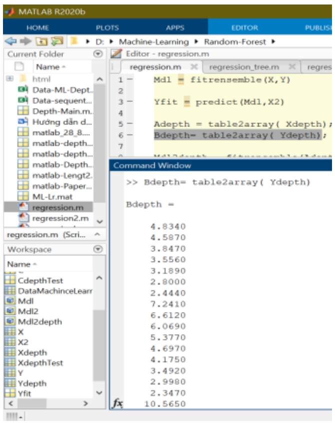

Figure 12. *Data of the target variable.*

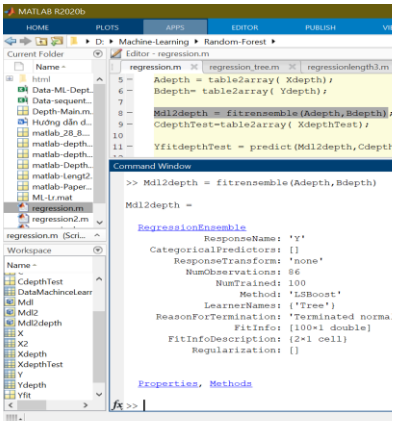

Figure 13. *Parameters for training in the Machine Learning model.*

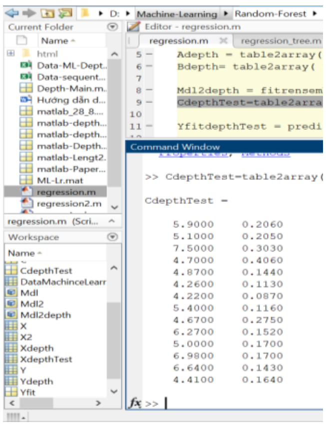

Figure 14. *Testing data.*

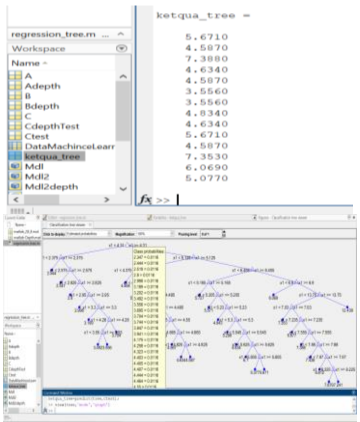

Figure 15. *Calculation results and diagram of "Decision Tree".*

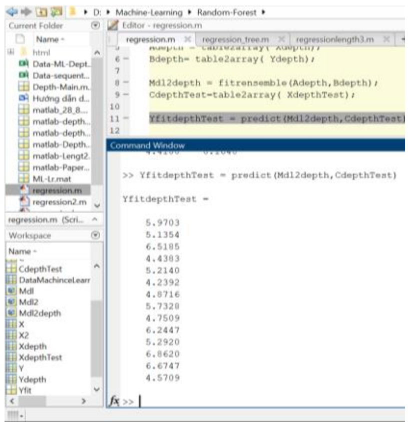

Figure 16. *Forecasting results of the "Random Forest".*

The comparison results of "Decision Tree" and "Random Forest" are shown in table 4.

Table 4. *The sequent depths obtained by the machine learning model.*

No.	Observed values				Calculated values			
	b (cm)	Fr_{D1}	M_1	y_r/y_1	Decision tree		Random Forest	
					y_r/y_1	ε (%)	y_r/y_1	ε (%)
1	20	5.90	0.206	5.718	5.671	0.8	5.970	4.2
2		5.10	0.205	5.086	4.587	10.9	5.135	1.0
3		7.50	0.303	6.518	7.388	11.8	6.518	0.0
4		4.70	0.406	4.335	4.634	6.5	4.438	2.3
5	55	4.87	0.144	5.038	4.587	9.8	5.214	3.4
6		4.26	0.113	4.452	3.556	25.2	4.239	5.0
7		4.22	0.087	4.708	3.556	32.4	4.872	3.4
8		5.40	0.116	5.750	4.834	18.9	5.733	0.3
9	33.5	4.67	0.275	4.543	4.634	2.0	4.751	4.4
10		6.27	0.152	6.431	5.671	13.4	6.245	3.0
11		5.00	0.170	5.018	4.587	9.4	5.292	5.2
12		6.98	0.170	7.053	7.353	4.1	6.862	2.8
13		6.64	0.143	7.000	6.069	15.3	6.675	4.9
14		4.41	0.164	4.691	5.077	7.6	4.571	2.6
Max						**32.4**		**5.2**
Min						**0.8**		**0.0**

Table 4 shows that the maximum error between the actual measurement and the calculation of the model "Random Forest" (5.2%) is much better than that of "Decision Tree" (32.4%). As observed in table 4 and figure 17, the computed data are close to the agreement line and indicate ± 5% difference from the observed data. This shows the efficiency of the proposed method.

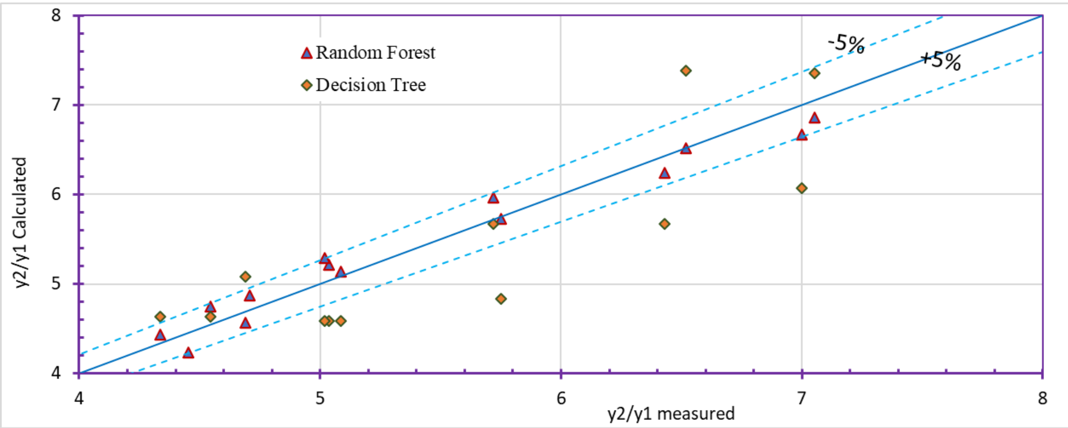

Figure 17. *Comparison between the observed values with calculated values.*

Table 5. *Statistical parameters of the testing data by models. The highest results are in bold.*

No.	Model	MEA	MSE	RMSE	R^2	MAPE (%)
1	**Random Forest**	**0.163**	**0.035**	**0.187**	**0.959**	**3.047**
2	Decision Tree	0.573	0.439	0.663	0.484	10.609
3	SVM [16]	0.540	0.355	0.550	0.675	5.555

Table 5 demonstrates the comparison between different models: Random Forest, Decision Tree, and Support Vector Machine (SVM) [16] for the prediction of the parameter. Compared to the Decision Tree and SVM, the calculation method by "Random forest" with LS_Boost algorithm of Breiman, L (2001) has gained the highest results in analyzing the conjugate depths of the jump.

6. CONCLUSIONS

The paper presents a novel approach to predict the conjugate depths of the jump in rectangular and isosceles triangle channels. The study analyzed the factors affecting the sequent depth from Buckingham's theory, thereby identifying the influencing variables as a basis data for collection in experimental models. Obtained results on the sequent depth employed the "Random Forest" regression have shown a small error and the agreement of the measured and calculated values.

In the future, advanced machine learning models (e.g., Artificial Neural Networks) can be applied to improve the prediction accuracy of the sequent depth. Moreover, some data augmentation techniques can be applied to enlarge datasets for training machine learning models efficiently.

NOTATIONS

Fr_1	Inflow Froude number with y_1	m	Sidewall slope of the trapezoidal channel
Fr_D	Inflow Froude number with D	A	Cross-sectional area (m^2)
y	Depth of flow (m)	M_1	Sidewall constant in trapezoidal channel ($M_1 = m.y_1 / b$)
b	Bed width of the channel (m)		
y_c	Critical depth (m)	y_1	Upstream depth in a hydraulic jump (m)
LS_Boost	Least-Squares Boost	y_2	Downstream depth in a hydraulic jump (m)
Q	Flow rate/ Discharge (m^3/s)	RMSE	Root mean square error
Re	Reynolds number	MAPE	Mean absolute percentage error
MEA	Mean absolute error	ε (%)	The error between the measured and calculated values $\varepsilon = \left(\left| Lr_{measured} - Lr_{calculated} \right| \times 100 \right) / Lr_{calculated}$ (%)
MSE	Mean squared error		
R^2	A goodness-of-fit measure for linear regression models		
π_i	Pi group (Non-dimensional parameters)		

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TÓM TẮT

Dự báo độ sâu liên hiệp của nước nhảy trong kênh hình thang bằng mô hình Rừng ngẫu nhiên

Dự báo độ sâu liên hiệp của nước nhảy trên kênh hình thang vẫn là bài toán khó xác định được công thức lý thuyết, nên các nghiên cứu đã có chủ yếu thông qua phương pháp thực nghiệm hoặc bán thực nghiệm. Tuy vậy, tính tổng quát của các nghiên cứu đã có vẫn chưa được đảm bảo. Bài báo này trình bày một phương pháp mới dựa trên lý thuyết Pi của Buckingham và áp dụng mô hình học máy "Rừng ngẫu nhiên" nhằm nâng cao độ chính xác trong dự báo độ sâu liên hiệp của nước nhảy trên kênh hình thang. Kết quả thực nghiệm của bài báo đã cho thấy, phương pháp học máy trong nghiên cứu nước nhảy là một phương pháp phù hợp và hiệu quả cao trong nghiên cứu xác định đặc trưng hình học của nước nhảy và cũng có khả năng cao ứng dụng trong thiết kế công trình thực tế.

Từ khóa: Học máy; Độ sâu liên hiệp; Nước nhảy; Kênh hình thang; Lý thuyết Pi; Rừng ngẫu nhiên.