An adaptive sliding mode controller for a class of MIMO Euler-Lagrange systems with variable parameters

Le Van Chuong[1], Nguyen Trung Kien[2*]

[1]Vinh University, No.182 Le Duan, Ben Thuy, Vinh city, Nghe An Province;
[2]Academy of Military Science and Technology, No.17 Hoang Sam, Nghia Do, Cau Giay, Ha Noi.
[*]Corresponding author: kiennt67@gmail.com

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ABSTRACT

This paper presents a method to synthesize the adaptive sliding mode controller for a class of MIMO Euler-Lagarance systems with variable parameters. We perform a Taylor series expansion of a class of MIMO Euler-Lagarance systems into nonlinear state-space equations, considering cases of varying parameters and unmeasured external disturbances. We propose a method to identify and compensate for variable components, external disturbance based on adaptive control theory, and RBF neural network to make the system invariant to these uncertain changing components. We build the control law based on sliding mode control. The research results of the paper obtained are that the adaptive sliding mode controller is adaptive, resistant to interference, and has high control quality.

Keywords: Euler-Lagarance systems; Adaptive sliding-mode control; RBF neural network.

1. INTRODUCTION

The Euler-Lagrange (EL) system is commonly found in industry, transportation, and many other fields. The EL system has high nonlinear characteristics, and there are many uncertain factors; in some cases, the system is affected by unmeasured disturbances. There have been many studies on synthesizing controllers for EL systems in the past decades with many good results. However, in the face of increasing requirements for product quality, the study of solutions to improve control quality for the EL system continues to be an urgent issue. The adaptive control method for the EL system has been studied in [1-5]. The condition for the adaptive control algorithm to converge is to know in advance the limit of the uncertain components; in many cases, this limit is not known in advance. In [6-10], the robust control for the EL system is built based on sliding mode control; the closed system ensures stability when the sliding mode exists on the sliding surface. The problem with these papers is that there exists a significant chattering effect when the uncertainty component varies greatly; this adversely affects the control quality of the system. Below, the article proposes a method to synthesize the adaptive sliding mode controller to overcome some of the remaining problems mentioned above.

2. SYNTHESIS OF THE ADAPTIVE SLIDING MODE CONTROL SYSTEMS

Suppose that a class of MIMO Euler-Lagrange systems has the equation:

$$\ddot{\mathbf{q}} = \mathbf{M}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{N}(\mathbf{q},\dot{\mathbf{q}})\boldsymbol{\tau} \tag{1}$$

where: $\mathbf{q} = [q_1, q_2, ..., q_n]^T$ is state vector; $\boldsymbol{\tau} = [\tau_1, \tau_2, ..., \tau_n]^T$ is input vector; $\mathbf{M}(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^n$ is known vector; $\mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^{n\times n}$ is known matrix.

To facilitate the design process later, we set: $\mathbf{x} = [\mathbf{x_1}, \mathbf{x_2}]^T$, $\mathbf{x_2} = \dot{\mathbf{x}}_\mathbf{1}$ where:

$\mathbf{x_1} = [x_{11}, x_{12}, ..., x_{1n}]^T = [q_1, q_2, ..., q_n]^T$; $\mathbf{x_2} = [x_{21}, x_{22}, ..., x_{2n}]^T = [\dot{q}_1, \dot{q}_2, ..., \dot{q}_n]^T$; $\mathbf{u} = [u_1, u_2, \ldots, u_n]^T = [\tau_1, \tau_2, \ldots, \tau_n]^T$.

Equation (1) is rewritten as: $\dot{\mathbf{x}} = \boldsymbol{\xi}(\mathbf{x}, \mathbf{u})$, (2)

where $\boldsymbol{\xi}(\mathbf{x}, \mathbf{u}) = [\xi_1, \xi_2, \ldots, \xi_{2n}]^T$.

Perform Taylor expansion of equation (2) at the origin equilibrium point $(\mathbf{x}_0, \mathbf{u}_0)$, we have:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \bar{\mathbf{f}}(\mathbf{x}), \tag{3}$$

where $\mathbf{A}$, $\mathbf{B}$ are Jacobian matrices:

$$\mathbf{A} = \left.\frac{\partial \xi}{\partial \mathbf{x}}\right|_{[\mathbf{x}_0, \mathbf{u}_0]} \quad (4) \qquad \mathbf{B} = \left.\frac{\partial \xi}{\partial \mathbf{u}}\right|_{[\mathbf{x}_0, \mathbf{u}_0]}, \quad (5)$$

$\bar{\mathbf{f}}(\mathbf{x})$ is a higher order terms of the Taylor series expansion and unknown nonlinear function vectors.

In many cases, system (1) has time-varying parameters and impacts unmeasurable external disturbance. From which we rewrite equation (3) into:

$$\dot{\mathbf{x}} = [\mathbf{A} + \Delta\mathbf{A}]\mathbf{x} + [\mathbf{B} + \Delta\mathbf{B}]\mathbf{u} + \bar{\mathbf{f}}(\mathbf{x}) + \mathbf{d}(t), \tag{6}$$

where $\mathbf{A} \in \mathbb{R}^{2n\times 2n}$, $\mathbf{B} \in \mathbb{R}^{2n\times n}$ are constant parameter matrices; $\Delta\mathbf{A}$, $\Delta\mathbf{B}$ are unknown parameter matrices; $\mathbf{d}(t)$ is external disturbance vector.

Next, we solve the problem of synthesizing an adaptive sliding mode controller for system (6) with variable parameters and external disturbance.

2.1. Algorithm for identification and compensation of uncertain parameters

We use adaptive control and radial basis function (RBF) neural networks to identify the uncertainty components in (6). We set:

$$\mathbf{f}(\mathbf{x}) = \Delta\mathbf{A}\mathbf{x} + \bar{\mathbf{f}}(\mathbf{x}). \tag{7}$$

Substitute (7) into (6) we have: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + [\mathbf{B} + \Delta\mathbf{B}]\mathbf{u} + \mathbf{f}(\mathbf{x}) + \mathbf{d}(t)$. (8)

The identification model for uncertain parameters in (8) can be written:

$$\dot{\mathbf{x}}_m = \mathbf{A}\mathbf{x}_m + \left[\mathbf{B} + \Delta\hat{\mathbf{B}}\right]\mathbf{u} + \hat{\mathbf{f}}(\mathbf{x}) + \hat{\mathbf{d}}(t), \tag{9}$$

where: $\mathbf{x}_m = [x_1^m, x_2^m, \ldots, x_{2n}^m]^T$ is state vector of the model; $\hat{\mathbf{f}}(\mathbf{x}) = [\hat{f}_1(\mathbf{x}), \hat{f}_2(\mathbf{x}), \ldots, \hat{f}_{2n}(\mathbf{x})]^T$ is the estimated vector of $\mathbf{f}(\mathbf{x})$; $\hat{\mathbf{d}}(t) = [\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_{2n}]^T$ is the estimated vector of $\mathbf{d}(t)$, $\Delta\hat{\mathbf{B}}$ is the estimated matrix of $\Delta\mathbf{B}$.

From (8) and (9), we have: $\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \left[\Delta\tilde{\mathbf{B}}\right]\mathbf{u} + \tilde{\mathbf{f}}(\mathbf{x}) + \tilde{\mathbf{d}}(t)$, (10) where: $\mathbf{e}=\mathbf{x}-\mathbf{x}_m$; (11) $\quad \Delta\tilde{\mathbf{B}}=\Delta\mathbf{B}-\Delta\mathbf{B}_m$; (12)

$\tilde{\mathbf{f}}(\mathbf{x})=\mathbf{f}(\mathbf{x})-\hat{\mathbf{f}}(\mathbf{x})$; (13) $\quad \tilde{\mathbf{d}}(t)=\mathbf{d}(t)-\hat{\mathbf{d}}(t)$. (14)

With $\mathbf{f}(\mathbf{x})$ is a smooth function vector, by using a RBF neural network for the approximation. The elements of $\mathbf{f}(\mathbf{x})$ can be written:

$$f_i(\mathbf{x})=\sum_{j=1}^{L} w_{ij}^{*}\phi_{ij}(\mathbf{x})+\varepsilon_i\ , \tag{15}$$

$(i=1,2,...,2n)$, $(j=1,2,...,L)$ where L is number of basis function with a large enough number to guarantee the error $|\varepsilon_i|<\varepsilon_i^m$, $w_{ij}^{*}=const$ is the ideal weights. The basis functions are selected by the following form:

$$\phi_{ij}(\mathbf{x})=\exp\left(\left\|\mathbf{x}-\mathbf{c}_{ij}\right\|^2 \big/ 2\sigma_{ij}^2\right), \tag{16}$$

where: $\mathbf{c}_{ij}$ are the position of the center of the basis functions $\phi_{ij}(\mathbf{x})$, and σ_{ij} are the standard deviation of the basis functions.

The evaluation vector $\hat{\mathbf{f}}(\mathbf{x})$ is defined by (16) with adjusted weights $\hat{w}_{ij}$:

$$\hat{f}_i(\mathbf{x})=\sum_{j=1}^{L}\hat{w}_{ij}\phi_{ij}(\mathbf{x}),\ (i=1,2,...,2n),(j=1,2,...,L). \tag{17}$$

Training of the RBF neural network is implemented by adjustment of the weights $\hat{w}_{ij}$ in comparison with the ideal weights:

$$\tilde{w}_{ij}=w_{ij}^{*}-\hat{w}_{ij}\ . \tag{18}$$

The identification process will converge if $\mathbf{e}\to 0$, $\Delta\tilde{\mathbf{B}}\to 0$, $\tilde{\mathbf{f}}(\mathbf{x})\to 0$, $\tilde{\mathbf{d}}\to 0$ which means that the error equation (10) is stable. For equations (10), the Lyapunov function is selected as follows:

$$V=\mathbf{e}^T\mathbf{P}\mathbf{e}+\sum_{i=1}^{2n}\sum_{j=1}^{n}\Delta\tilde{b}_{ij}^2+\sum_{i=1}^{2n}\sum_{j=1}^{L}\tilde{w}_{ij}^2+\sum_{i=1}^{2n}\tilde{d}_i^2\ , \tag{19}$$

where $\mathbf{P}$ is a positive definite symmetric matrix. Take the derivative of both sides of the equation (19):

$$\dot{V}=\dot{\mathbf{e}}\mathbf{P}\mathbf{e}+\mathbf{e}^T\mathbf{P}\dot{\mathbf{e}}+2\sum_{i=1}^{2n}\sum_{j=1}^{n}\Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij}+2\sum_{i=1}^{2n}\sum_{j=1}^{L}\dot{\tilde{w}}_{ij}\tilde{w}_{ij}+2\sum_{i=1}^{2n}\dot{\tilde{d}}_i\tilde{d}_i\ . \tag{20}$$

Substitute (10) into (20):

$$\begin{gathered}\dot{V}=\mathbf{e}^T\left(\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{e}+2\mathbf{u}^T\Delta\tilde{\mathbf{B}}^T\mathbf{P}\mathbf{e}+2\mathbf{e}^T\mathbf{P}\tilde{\mathbf{f}}(\mathbf{x})+2\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}(t)+\\ +2\sum_{i=1}^{2n}\sum_{j=1}^{n}\Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij}+2\sum_{i=1}^{2n}\sum_{j=1}^{L}\dot{\tilde{w}}_{ij}\tilde{w}_{ij}+2\sum_{i=1}^{2n}\dot{\tilde{d}}_i\tilde{d}_i\ .\end{gathered} \tag{21}$$

For (10) to be stable, from (21) to with draw the condition to $\dot{V}<0$:

$$\mathbf{e}^T\left(\mathbf{A}^T\mathbf{P}+\mathbf{PA}\right)\mathbf{e}+2\mathbf{e}^T\mathbf{P}\varepsilon<0\,; \tag{22}$$

$$2\mathbf{u}^T\Delta\tilde{\mathbf{B}}^T\mathbf{Pe}+2\sum_{i=1}^{2n}\sum_{j=1}^{n}\Delta\dot{\tilde{b}}_{ij}\Delta\tilde{b}_{ij}=0\,; \tag{23}$$

$$2\mathbf{e}^T\mathbf{P}\left[\sum_{j=1}^{L}\tilde{w}_{1j}\phi_{ij}\left(\mathbf{x}\right)...\sum_{j=1}^{L}\tilde{w}_{nj}\phi_{ij}\left(\mathbf{x}\right)\right]^T+2\sum_{i=1}^{2n}\sum_{j=1}^{L}\dot{\tilde{w}}_{ij}\tilde{w}_{ij}=0\,; \tag{24}$$

$$2\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}\left(t\right)+2\sum_{i=1}^{2n}\dot{\tilde{d}}_i\tilde{d}_i=0\,. \tag{25}$$

Transform the left side of the inequality (22):

$$\mathbf{e}^T\left(\mathbf{A}^T\mathbf{P}+\mathbf{PA}\right)\mathbf{e}+2\mathbf{e}^T\mathbf{P}\varepsilon=-\mathbf{e}^T\mathbf{Qe}+2\sum_{i=1}^{2n}\varepsilon_i\bar{\mathbf{P}}_i\mathbf{e}<0\,; \tag{26}$$

where $\mathbf{Q}=-\left(\mathbf{A}^T\mathbf{P}+\mathbf{PA}\right)$, suppose that $\mathbf{A}$ is Hurwitz matrix; $\bar{\mathbf{P}}_i$ is the i-th row of the matrix $\mathbf{P}$. Using inequality transformations [11]:

$$-\mathbf{e}^T\mathbf{Qe}+2\sum_{i=1}^{2n}\varepsilon_i\bar{\mathbf{P}}_i\mathbf{e}<-r_{\min}\left(\mathbf{Q}\right)\left\|\mathbf{e}\right\|^2+2\sum_{i=1}^{2n}\varepsilon_i\left\|\bar{\mathbf{P}}_i\right\|\left\|\mathbf{e}\right\|<0\,; \tag{27}$$

$r_{\min}\left(\mathbf{Q}\right)$ is the smallest eigenvalue of the matrix $\mathbf{Q}$. From (27):

$$\left\|\mathbf{e}\right\|>2\sum_{i=1}^{2n}\varepsilon_i\left\|\bar{\mathbf{P}}_i\right\|/\,r_{\min}\left(\mathbf{Q}\right). \tag{28}$$

Solving equations (23-25) we have:

$$\Delta\dot{\tilde{b}}_{ij}=-u_j\bar{\mathbf{P}}_{\mathbf{i}}\mathbf{e}\,,\left(i=1,2,...,2n\right),\ \left(j=1,2,...,n\right); \tag{29}$$

$$\dot{\tilde{w}}_{ij}=-\bar{\mathbf{P}}_i\mathbf{e}\phi_{ij}\left(\mathbf{x}\right),\ \left(i=1,2,...,n\right),\ \left(j=1,2,...,L\right); \tag{30}$$

$$\dot{\tilde{d}}_i=-\bar{\mathbf{P}}_i\mathbf{e}\,,\ \left(i=1,2,...,2n\right). \tag{31}$$

If simultaneously satisfied (28-31), then the derivative $\dot{V}<0$, so the system (10) is stable.

From (12) and (29), with the attention that the matrix $\Delta\mathbf{B}$ contains slowly variable elements, i.e. $\Delta\dot{b}_{ij}^0\approx 0$. Identify law for the uncertain parameters in the matrix ΔB:

$$\Delta b_{ij}\simeq\Delta\dot{\hat{b}}_{ij}=u_j\bar{\mathbf{P}}_i\mathbf{e}\rightarrow\Delta\hat{b}_{ij}=\int u_j\bar{\mathbf{P}}_i\mathbf{e}dt+\Delta b_{ij}^0 \tag{32}$$

Δb_{ij}^0 is initialization value.

From (13), (17), (18) and (30), because of $w_{ij}^*=const$, we have $\dot{w}_{ij}^*=0$. The nonlinear function vector identification law $\mathbf{f}\left(\mathbf{x}\right)$ has the following elements:

$$f_i(\mathbf{x})\simeq\hat{f}_i\left(\mathbf{x}\right)=\sum_{j=1}^{L}\hat{w}_{ij}\phi_{ij}\left(\mathbf{x}\right),\ \dot{\hat{w}}_{ij}=\bar{\mathbf{P}}_i\mathbf{e}\phi_{ij}\left(\mathbf{x}\right),\ \left(i=1,2,...,2n\right). \tag{33}$$

From (14) and (31), because of slow-varying external disturbance $\dot{\mathbf{d}}\left(t\right)\approx 0$. The unmeasurable external disturbance vector identification law $\mathbf{d}(t)$ has the following elements:

$$d_i(t) \simeq \hat{d}_i(t) = \int \overline{\mathbf{P}}_i \mathbf{e} dt, \ (i = 1, 2, ..., 2n). \tag{34}$$

The identification results in this section are used to synthesize the control law presented in the next section.

2.2. Synthesis of the control law

Equation (8) is rewritten as: $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \Delta\mathbf{Bu} + \mathbf{f}(\mathbf{x}) + \mathbf{d}(t)$. (35)

We set: $\mathbf{f}^* = \Delta\mathbf{Bu} + \mathbf{f}(\mathbf{x}) + \mathbf{d}(t)$, (36)

$\mathbf{f}^* = \left[f_1^*, f_2^*, ..., f_{2n}^* \right]^T$. Substitute (36) into (35), we have:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{If}^*, \tag{37}$$

where $\mathbf{I}^{2n\times 2n}$ matrix has main diagonal elements $I_{ij} = 1, (i, j = 1, 2, ..., 2n)$ are rows which corresponds to the vector $\mathbf{f}^*$ in the case $\left| f_i^* \right| \neq 0$; other elements $I_{ij} = 0$ in the case $i \neq j$ and $\left| f_i^* \right| = 0$.

The control law (37) can be considered as follow: $\mathbf{u} = \mathbf{u}_{smc} + \mathbf{u}_c$, (38)

where $\mathbf{u}_{smc}$ is the sliding mode control law, $\mathbf{u}_c$ is a control law for the compensation of uncertain components. Substitute (38) into (37):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}_{smc} + \mathbf{Bu}_c + \mathbf{If}^*, \tag{39}$$

From (39), we can see those uncertain elements in $\mathbf{f}^*$ will be compensated with the condition:

$$\mathbf{Bu}_c + \mathbf{If}^* = 0. \tag{40}$$

Next, we have to generate signal vector $\mathbf{u}_c$ satisfying equation (40); we choose:

$$\mathbf{u}_c = -\mathbf{Kf}^*, \tag{41}$$

where $\mathbf{K}$ is the gain matrix.

From the identification results in (32), (33) and (34), we replace $\mathbf{f}^*$ with $\hat{\mathbf{f}}^*$(36):

$$\hat{\mathbf{f}}^* = \Delta\hat{\mathbf{B}}\mathbf{u} + \hat{\mathbf{f}}(\mathbf{x}) + \hat{\mathbf{d}}(t), \tag{42}$$

From (41) and (42), we have the signal vector fed to the input of (39) to compensate for the uncertain elements as: $\mathbf{u}_c = -\mathbf{K}\hat{\mathbf{f}}^*$, (43)

Substitute (43) into (39), we have: $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}_{smc} - \mathbf{K}\hat{\mathbf{f}}^* + \mathbf{If}^*$. (44)

From (44) we can see that $\mathbf{f}^*$ will be compensated with the condition:

$$-\mathbf{BK}\hat{\mathbf{f}}^* + \mathbf{If}^* = 0. \tag{45}$$

To generate a satisfactory compensation signal (45), we must have: $\mathbf{BK} = \mathbf{I}$. (46)

From (46), we choose $\mathbf{K}=\mathbf{B}^{+}$, where $\mathbf{B}^{+}$ is the pseudo-inverse matrix of $\mathbf{B}$ [11].

From (42) and (43), with attention to (45) and (46), we have the control law for the compensation of the uncertain elements.:

$$\mathbf{u}_c=-\mathbf{K}\left[\left[\Delta\hat{b}_{ij}\right]\mathbf{u}+\left[\hat{\mathbf{f}}_i\left(\mathbf{x}\right)\right]^T+\left[\hat{\mathbf{d}}_i\left(t\right)\right]^T\right];\ \left(i=1,2,...,2n\right);\ \left(j=1,2,...,n\right). \quad (47)$$

With the signal vector $\mathbf{u}_c$ (47) fed to the input of the plant, the uncertainty elements are compensated, and then (39) becomes:

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}_{smc}. \quad (48)$$

Next, the system (48) of control rules is built based on sliding mode control. The error vector between the state vector and the desired state vector $\mathbf{x}_d$:

$$\tilde{\mathbf{x}}=\mathbf{x}-\mathbf{x}_d \rightarrow \mathbf{x}=\tilde{\mathbf{x}}-\mathbf{x}_d. \quad (49)$$

Substitute (49) into (48): $\dot{\tilde{\mathbf{x}}}=\mathbf{A}\tilde{\mathbf{x}}+\mathbf{B}\mathbf{u}_{smc}+\mathbf{A}\mathbf{x}_d-\dot{\mathbf{x}}_d$. (50)

For (50), the hyper sliding surface is chosen as follows [12]: $\mathbf{s}=\mathbf{C}\tilde{\mathbf{x}}$, (51)

where $\mathbf{C}$ is the parameter matrix of hyper sliding surface and choose $\mathbf{C}$ such that $\det\left(\mathbf{CB}\right)\neq 0$; $\mathbf{s}=\left[s_1,s_2,...,s_n\right]^T$. (52)

The next problem is to define the control law $\mathbf{u}_{smc}$ which ensures movement of the system (50) towards the hyper sliding surface (51) and keep it there.

The control signal $\mathbf{u}_{smc}$ can be written by: $\mathbf{u}_{smc}=\begin{cases}\mathbf{u}_s & \text{if} \quad \mathbf{s}\neq 0\\ \mathbf{u}_{eq} & \text{if} \quad \mathbf{s}=0\end{cases}$, (53)

$\mathbf{u}_s$ is the control signal that moves the system (50) towards the hyper sliding surface (51); $\mathbf{u}_{eq}$ is the equivalent control signal that keeps the system (50) on the hyper sliding surface (51).

From (53), we can rewrite: $\mathbf{u}_{smc}=\mathbf{u}_{eq}+\mathbf{u}_s$. (54)

$\mathbf{u}_{eq}$ is defined in [12]: $\dot{\mathbf{s}}=\mathbf{C}\dot{\tilde{\mathbf{x}}}=0$, (55)

From (50) and (55), we have: $\mathbf{C}\left(\mathbf{A}\tilde{\mathbf{x}}+\mathbf{B}\mathbf{u}_{eq}+\mathbf{A}\mathbf{x}_d-\dot{\mathbf{x}}_d\right)=0$. (56)

From (56), the equivalent control signal can be defined as follows:

$$\mathbf{u}_{eq}=-\left[\mathbf{CB}\right]^{-1}\left[\mathbf{CA}\tilde{\mathbf{x}}+\mathbf{CA}\mathbf{x}_d-\mathbf{C}\dot{\mathbf{x}}_d\right]. \quad (57)$$

Next, we define the control signal $\mathbf{u}_s$ that moves the system (50) towards the hyper sliding surface (51). For the hyper sliding surface (51), the Lyapunov function can be selected by:

$$V=\mathbf{s}^T\mathbf{s}/2. \quad (58)$$

Condition for the existence of slip mode can be written: $\dot{V}=\mathbf{s}^T\dot{\mathbf{s}}<0$. (59)

From (50), (51), (54) and (59), we have:

$$\dot{V} = \mathbf{s}^T \left[\mathbf{C}\left(\mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\mathbf{u}_{eq} + \mathbf{C}\mathbf{A}\mathbf{x}_d - \mathbf{C}\dot{\mathbf{x}}_d \right) + \mathbf{C}\mathbf{B}\mathbf{u}_s \right] < 0 . \tag{60}$$

Attention to (56), inequality (60) can be written as: $\mathbf{s}^T \left[\mathbf{C}\mathbf{B}\mathbf{u}_s \right] < 0$. (61)

So to satisfy the condition (59), from (61) can be defined as follows:

$$\mathbf{u}_s = -[\mathbf{CB}]^{-1}[\delta \operatorname{sgn}(s_1), \delta \operatorname{sgn}(s_2), \ldots, \delta \operatorname{sgn}(s_i)]^T, \ (i = 1, 2, ..., n); \tag{62}$$

δ is a small positive coefficient. Substituting (57) and (62) into (53), the control signal can be defined by $\mathbf{u}_{smc}$ as follows:

$$\mathbf{u}_{smc} = \begin{cases} -[\mathbf{CB}]^{-1}[\delta \operatorname{sgn}(s_1), \delta \operatorname{sgn}(s_2), \ldots, \delta \operatorname{sgn}(s_i)]^T & \text{if } \ \mathbf{s} \neq 0 \\ -[\mathbf{CB}]^{-1}[\mathbf{CA}\tilde{\mathbf{x}} + \mathbf{CA}\mathbf{x}_d - \mathbf{C}\dot{\mathbf{x}}_d] & \text{if } \ \mathbf{s} = 0 \end{cases}, \tag{63}$$

$$(i = 1, 2, ..., n).$$

Thus, the article has synthesized the control law for the class of EL systems with a model (1). Control law (38), where $\mathbf{u}_c$ and $\mathbf{u}_{smc}$ are defined in expressions (47) and (63).

3. RESULTS AND DISCUSSION

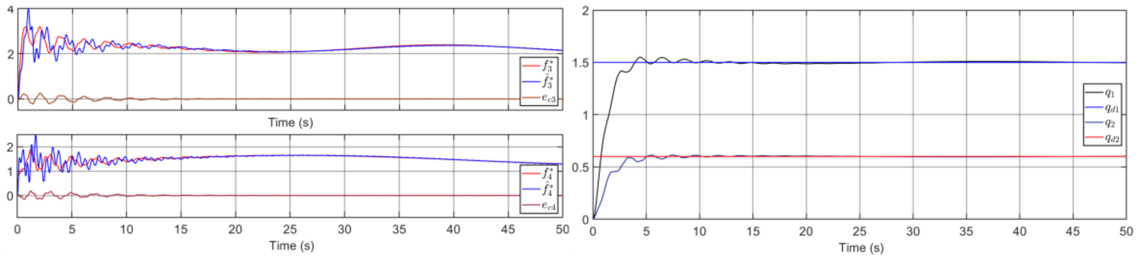
Suppose that a MIMO Euler-Lagrange system has the equation:

$$\begin{aligned} \ddot{q}_1 &= \left(q_1\dot{q}_2 + q_2^2 + 0.2 \right) + \left(1 + q_1^2 \right)\tau_1 + \tau_2 \\ \ddot{q}_2 &= \left(q_2\dot{q}_2 + 0.4 \right) + \left(1 + \dot{q}_1^2 \right)\tau_2 \end{aligned}, \tag{64}$$

Let $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]^T$, $\mathbf{x}_2 = \dot{\mathbf{x}}_1$, where: $\mathbf{x}_1 = [x_{11}, x_{12}]^T = [q_1, q_2]^T$; $\mathbf{x}_2 = [x_{21}, x_{22}]^T = [\dot{q}_1, \dot{q}_2]^T$; $\mathbf{u} = [u_1, u_2]^T = [\tau_1, \tau_2]^T$. Perform Taylor series expansion of equation (64) at the origin equilibrium point $(\mathbf{x}_0, \mathbf{u}_0) = (\mathbf{0}, \mathbf{0})$. Assuming there are higher-order terms of the Taylor series expansion, uncertain nonlinear function vectors, and external disturbance. We have:

$$\begin{aligned} \dot{\mathbf{x}} = &\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1.5 & 0 & -2.5 & 0 \\ 0 & -1.5 & 0 & -2.5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\mathbf{u} + \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0.05x_{21} + 0.075x_{12} + 0.05x_{22} + 0.05\sin(x_{11}) + 1.5x_{11} - 1.5x_{12} + 2.5x_{21} - 2.5x_{22} \\ 0.075x_{12} + 0.05x_{11}x_{22} + 0.05\sin(-x_{21}) + 1.5x_{12} + 2.5x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.15\sin(0.2t) - 0.12 \\ 0.2\sin(0.1t) + 0.5 \end{bmatrix}. \end{aligned} \tag{65}$$

In the case of $\Delta\mathbf{A} = 0$, $\Delta\mathbf{B} = 0$, and the desired signal $q_{d1} = x_{d11} = 1.5$, $q_{d2} = x_{d12} = 0.6$, using the controller (38), simulation results are shown in figure 1.



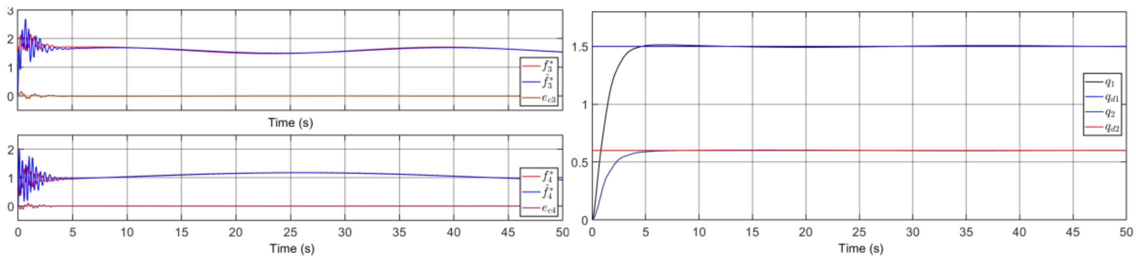
(a) (b)

Figure 1. *Simulation results for the case* $\Delta\mathbf{A}=0,\Delta\mathbf{B}=0$.

Attention to (6), the uncertain parameters matrices are assumed:

$$\Delta\mathbf{A}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\-0.6&0&-1&0\\0&-0.6&0&-1\end{bmatrix}; \qquad (66) \qquad \Delta\mathbf{B}=\begin{bmatrix}0&0\\0&0\\0.4&0.4\\0&0.4\end{bmatrix}. \qquad (67)$$

In the case $\Delta\mathbf{A}$, $\Delta\mathbf{B}$ varies according to (66), (67) and with the desired signal $q_{d1}=x_{d11}=1.5$, $q_{d2}=x_{d12}=0.6$, using the controller (38), simulation results are shown in figure 2.



(a) (b)

Figure 2. *Simulation results for case* $\Delta\mathbf{A}$, $\Delta\mathbf{B}$ *change according to (66), (67).*

Simulation of the case of variable parameter components $\Delta\mathbf{A}=0$, $\Delta\mathbf{B}=0$: Fig. 1(a) shows that the identification results are asymptotic to the uncertain elements, and the adaptive compensation error is close to zero. Fig. 1(b) shows that the tracking quality of the desired signal meets the requirements. Simulate the case when there are variable parameter components $\Delta\mathbf{A}$, $\Delta\mathbf{B}$: Fig. 2(a) shows the result of identification - compensation of uncertain elements; Fig. 2(b) shows the state vectors of the system tracking to the desired signal vector, control quality is guaranteed. The simulation results have proved the correctness of the algorithm proposed by the article.

4. CONCLUSIONS

The article has synthesized an adaptive sliding mode controller for a class of MIMO Euler-Lagarance systems with variable parameters. The EL equation is converted into a nonlinear equation of state by the Taylor series expansion method. Synthesized algorithms identify the components of uncertainty and thereby generate signals to compensate for their effects. The convergence domain of the algorithm is the entire state space except for the neighborhood of origin whose radius is close to zero. When the adaptive algorithm converges, the uncertainty elements are compensated so that the chattering effect in the sliding mode control law is reduced to a minimum. The proposed control system is adaptive, resistant to interference, has high control quality, and overcomes some disadvantages in [1-10]. Simulation results have confirmed the correctness and effectiveness of the proposed control systems.

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TÓM TẮT

Tổng hợp bộ điều khiển thích nghi trượt cho một lớp hệ Euler-Lagrange mimo có tham số thay đổi

Bài báo giới thiệu một phương pháp tổng hợp bộ điều khiển thích nghi trượt cho một lớp hệ Euler-Lagarance MIMO có tham số thay đổi. Chúng tôi thực hiện khai triển Taylor lớp hệ Euler-Lagarance MIMO thành dạng hệ phương trình trạng thái phi tuyến, có tính đến trường hợp có tham số thay đổi và chịu của nhiễu ngoài không đo được. Chúng tôi đề xuất phương pháp nhận dạng và bù trừ các thành phần thay đổi, nhiễu ngoài trên cơ sở lý thuyết điều khiển thích nghi và mạng nơ ron RBF làm cho hệ trở nên bất biến với các thành phần thay đổi bất định này. Luật điều khiển được chúng tôi xây dựng trên cơ sở điều khiển trượt. Kết quả nghiên cứu của bài báo thu được là bộ điều khiển thích nghi trượt có khả năng thích nghi, kháng nhiễu và có chất lượng điều khiển cao.

Từ khoá: Euler-Lagarance; Điều khiển thích nghi trượt; Mạng nơron RBF.