Expectation maximization channel estimation for nonlinear OFDM systems

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ABSTRACT

This paper introduces the expectation maximization (EM)-based channel estimation for the high power amplifier (HPA)-incurred nonlinear orthogonal frequency division multiplexing (OFDM) systems based on the linearization using extended Bussgang decomposition. Analyses and numerical simulations show that the proposed algorithm only requires reasonable computation complexity with relatively small number of iterations while vastly improves the estimation performance compared to the other conventional estimation methods such as least square error (LSE) or minimum mean square error (MMSE) applied to such system. Moreover, the EM-LSE estimator could give almost the same performance as the EM-MMSE counterpart while does not require the channel statisics, forming a robust estimator to both fading and nonlinear channels with reduced computation complexilty. This makes the estimator to be more applicable.

Keywords: Nonlinear distortion; Channel estimation; Expectation maximization; OFDM.

1. INTRODUCTION

Recently, there has been an increasing number of wireless communication applications considering orthogonal frequency-division multiplexing (OFDM) as the core technique by numerous advantages over single-carrier systems. However, this technique also has several drawbacks, among which one of the most detrimental effects could be high sensitivity to nonlinear distortions caused by high power amplifiers (HPAs) at the transmitter [1-3]. For overcoming performance degradation caused by thes effect, there are various solutions at both transmitter and receiver sides. However, transmitter-side approaches may be hard to implement, for example, in low-cost terminals for cellular communication systems or wireless local area networks due to limits of power, complexity or cost. In such a case, the receiver-side nonlinearity compensation solutions may be an attractive alternative for uplink processing, where more computational resources are available at the base station.

Although the nonlinear amplification of OFDM signals combined with maximum-likelihood decoding at the receiver may result in better BER performance than that of linear OFDM transmission [3], true or even approximate maximum-likelihood decoding is still too complex for practical implementations now. Otherwise, there have been several studies devoted to the sub-optimal compensation of nonlinearly distorted OFDM signals at the receiver side. These techniques permit implementation of nonlinear OFDM demodulators/decoders with reasonable complexity. Nonetheless, most studies assume that the receiver knows the transmitter nonlinear transfer function, and that a perfect channel state information (CSI) is available at the receiver. These assumptions are somehow impratical, therefore previous researches have tended to focus on compensation of special types of nonlinearity rather than on realistic distortions introduced in HPA.

Otherwise, OFDM systems require an efficient channel estimation procedure to demodulate the received data coherently [3]. Although differential detection may be used in the absence of channel information, it results in about 3 dB loss in signal to noise ratio (SNR) as compared to coherent detection. Several channel estimation algorithms have been proposed for OFDM systems. Least-squares error (LSE) and minimum mean square error (MMSE) channel estimators are among the most common methods [2]. The MMSE estimator achieves good performance but requires channel statistics and incurs high complexity, while the LSE estimator has low complexity but suffers in estimation performance. Classical method for channel estimation is based on the use of training sequence. Pilot symbols (on pilot subcarriers) are embedded in between the data symbols (on data subcarriers), which provide the channel information at the receiver [11]. Comb type pilot signals, uniformly spaced across subcarriers within each frame with interpolation for the remaining subcarriers have been extensively used in practice [2, 10].

Use of pilot symbols for channel estimation introduces overhead and it is desirable to keep the number of pilot symbols as minimum as possible. In [15], the authors obtained the tight bound for the number of used pilots in channel estimation for OFDM systems. Otherwise, it is shown in [16] that by separating smoothing (filtering out noise at the received pilot subcarriers only) and interpolation, the estimation performance could achieve close to the optimum MMSE estimator, which performs smoothing and interpolation jointly, while the complexity with respect to the counterpart is significantly reduced.

To reduce the bandwidth overhead in pilot-based channel estimation, expectation-maximization (EM) algorithm is an improved way to estimate the channel coefficients in OFDM systems. This algorithm is a technique for finding maximum likelihood estimates of system parameters in a broad range of problems where observed data are incomplete. The EM algorithm consists of two iterative steps: the expectation step and the maximization step. The expectation step is performed with respect to unknown underlying parameters, using the current estimate of the parameters, conditioned upon the incomplete observations. The maximization step then provides a new estimate of the parameters that maximizes the expectation of log likelihood function defined over complete data, conditioned on the most recent observation and the last estimate [4]. These two steps are iterated until the estimated values converge. There have been several EM-based channel estimation proposals for OFDM systems [5-7]. In conventional EM-based techniques for channel estimation, a cost function is defined in terms of received signal, channel information and transmitted signal. The transmitted signal and channel information are unknown at the receiver. In expectation step of EM technique, calculation of cost function is done on all possible values of transmitted data. The cost function is then maximized to estimate the channel parameters in the maximization step. This process is done iteratively until channel estimates converge. Pilot based estimation of channel is used for initialization and then performance is improved iteratively.

To the best of our knowledge the problem of joint channel response and nonlinearity compensation based on EM algorithm has not been addressed in previous studies with reasonable complexity. In [8], the Monte-Carlo sequential importance sampling based particle filtering is suggested for nonlinear equalization and signal detection in millimeter-wave communications. Authors in [9] introduce a semi-blind channel estimation scheme using expectation maximisation like for MIMO-OFDM systems. These solutions are inherently too high complexity. Therefore, in this paper, we propose an EM-based channel estimation and nonlinear distortion compensation algorithm for OFDM receivers with reasonable complexity. Our proposal supposes that the receiver has no prior knowledge of PA nonlinearities, and no channel state information. Instead of this, it jointly estimates and compensates the frequency-domain channel transfer function and PA characteristics using regular OFDM signal structure with block-type pilots. Our approach is mainly different to previous studies in that the PA nonlinearity is represented in frequency-domain, facilitating the whole system analysis and the compensation scheme. This allows Gaussian approximation for the nonlinear transmitted OFDM signals, reasonably linearizing the system. The method is computationally less complex and requires less number of iterations, while giving better performance than conventional pilot based methods.

This paper is organized as follows. Section II describes the baseband-equivalent nonlinear OFDM system model with the analysis of the effects of nonlinear PA on transmitted OFDM symbols and the linear approximation for OFDM receiver processing. The classical LSE and MMSE channel estimation and the proposed EM technique for the OFDM systems is described in section III. Validation of the proposed technique is carried out through simulations which form section IV. Finally the paper is concluded in section V.

Throughout the paper the following conventions are adopted: Italic letters (x) denote scalars; Single under-bar italic letters $(\underline{x})$ represent vectors and double under-bar italic letters $(\underline{\underline{x}})$ indicate matrices; Capital letters (X) are associated to the frequency-domain and small letters (x) are related to the time-domain; $|.|$ returns modulus of the input; $\|.\|$ stands for the Euclidean norm, $(.)^T$ and $(.)^H$ denote the transpose and Hermitian transpose operators, respectively; $\underline{\underline{X}}_{n,m}$ designates the element on the *n*-th row of the *m*-th column of matrix $\underline{\underline{X}}$; The probability density function (PDF) $f_x(x)$ of the random variable x is simply denoted by $f(x)$ when there is no risk of ambiguity; $\mathbb{E}\{.\}$ represents the expectation value.

2. SYSTEM DESCRIPTION

2.1. CP-OFDM transmitter

The baseband-equivalent system model is depicted in figure 1, where, in the CP-OFDM transmitter, data bits are first mapped into complex baseband symbols $\left\{S_k^{(m)}\right\}_{m=1}^{M}$ using phase-shift keying (PSK) or quadrature-amplitude modulation (QAM) format (here, subscript *k* represents subcarrier index, $0 \le k \le N-1$, and superscript *m* denotes constellation's signal index, $1 \le m \le M$, with N is the number of subcarriers, and M is the signal constellation size or modulation order). A transmit frame consists of N_S active CP-OFDM symbols. During each active symbol interval a block of N complex baseband symbols $\underline{S} = \left[S_0, S_1, ..., S_{N-1}\right]^T$ is transformed by means of inverse discrete Fourier transform (IDFT) and digital-to-analog conversion (DAC) to the baseband OFDM signal as

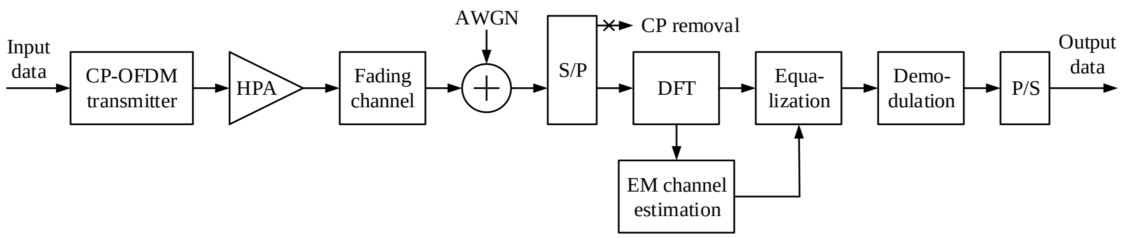


Figure 1. *Nonlinear OFDM system model with channel estimation and equalization.*

$$x(t) = \sum_{k=0}^{N-1} S_k e^{j2\pi k\Delta f t}, \quad 0 < t < T_S, \tag{1}$$

where Δf is the subcarrier spacing, and $T_S = N / \Delta f$ is the symbol interval. A cyclic prefix (CP) is then appended to each symbol $x(t)$. Moreover, to facilitate channel estimation at receiver assume that before IDFT, pilot symbols are distributed uniformly (and scaled power up by factor

K_P) at every D_P subcarriers. The receiver hence could estimate the whole channel with a certain interpolation for the remaining data subcarriers.

2.2. HPA models

As shown in figure 1, the resulting CP-OFDM signal is then passed through a nonlinear HPA. After propagating through this HPA, both amplitude and phase of the output signal will experience serious distortions, which can be commonly characterized respectively by the amplitude modulation-amplitude modulation (AM-AM) function $F_A(|.|)$ and the amplitude modulation-phase modulation (AM-PM) function $F_P(|.|)$ as[1]

$$y(t) = F_A\left(|x(t)|\right)e^{j\left(\arg\left(x(t)\right)+F_P\left(|x(t)|\right)\right)}. \tag{2}$$

For solid-state power amplifiers (SSPAs), GaAs pHEMT (Gallium arsenide pseudomorphic High-electron-mobility transistor) HPAs are increasingly dominant in terms of production technologies and market shares compared to other power semiconductor technologies [13]. These amplifiers could operate at frequencies up to 28 GHz (at K band, largely proposed for the 5G millimeter wave cells) and are modeled by modified Ghorbani model with the AM-AM and AM-PM characteristics fixed from measurements as:

$$F_a(r) = \frac{\alpha_1 r^{\alpha_2} + \alpha_3 r^{\alpha_2+1}}{1+\alpha_4 r^{\alpha_2}}, \tag{3}$$

$$F_p(r) = \frac{\beta_1 r^{\beta_2} + y_3 r^{\beta_2+1}}{1+\beta_4 r^{\beta_2}}, \tag{4}$$

where $r = |x|$ is the input signal's amplitude (here, without risk of confusion we omit the time variable *t* for concision) and a set of model parameters used in this work is given by $\alpha_1 = 7.851$, $\alpha_2 = 1.5388$, $\alpha_3 = -0.4511$, $\alpha_4 = 6.3531$, $\beta_1 = 4.6388$, $\beta_2 = 2.0949$, $\beta_3 = -0.0325$, $\beta_4 =$ 10.8217 [14]. Further note that for optimal operation, the HPA is often set at specific input/output backoff (IBO/OBO) which is defined as the input/output power level pulling back from full saturation in the AM-AM curve as:

$$BO = -10\log10\left(P_{Average}/P_{Saturation}\right). \tag{5}$$

On the other hand, for convertablity yet more generality, the memoryless complex polynomial model (Taylor series expansion) could be used to represent (arbitrary) HPA nonlinearity [2]:

$$y(t) = \sum_{p=1,3,5,...}^{P} \beta_p |x(t)|^{(p-1)} x(t) = \sum_{p=1,3,5,...}^{P} \beta_p [x(t)]^{(p+1)/2} [x^*(t)]^{(p-1)/2}, \tag{6}$$

where P is the highest order of nonlinearity, and $\{\beta_p\}$ are complex baseband power-series coefficients. It can be seen that the power-series (6) represents a general nonlinear transfer function and its odd-order coefficients could be extracted from any AM-AM and AM-PM characteristics (3) and (4). Otherwise, note that only the odd-order terms produce in-band distortions, which are considered in this work.

2.3. Effects of nonlinear PA on transmitted OFDM symbols

Considering the baseband-equivalent OFDM signal at the output of nonlinear HPA modeled by (6). Substituting (1) into (6), $y(t)$ could be recast as

[1] With long enough CP, the multipath fading effect is then fully canceled out at the receiver when removing CP (refer to Figure 1). Thus, we ignore the CP part in the analysis.

$$y(t)=\beta_1\sum_{k=0}^{N-1}S_k e^{j2\pi k\Delta ft}+\beta_3\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}\sum_{n_3=0}^{N-1}S_{n_1}S_{n_2}S_{n_3}^{*}e^{j2\pi(n_1+n_2-n_3)\Delta ft}+... \tag{7}$$

Thus, it can be seen that the OFDM transmitter with memoryless nonlinear HPA (6) is equivalent to a linear OFDM transmitter that emmits modified baseband symbols S'_k, $k=0,1,...,N-1$:

$$S'_k=\beta_1 S_k+\beta_3\sum_{\substack{n_1+n_2\\-n_3=k}}S_{n_1}S_{n_2}S_{n_3}^{*}+\beta_5\sum_{\substack{n_1+n_2+n_3\\-n_4-n_5=k}}S_{n_1}S_{n_2}S_{n_3}S_{n_4}^{*}S_{n_5}^{*}+... \tag{8}$$

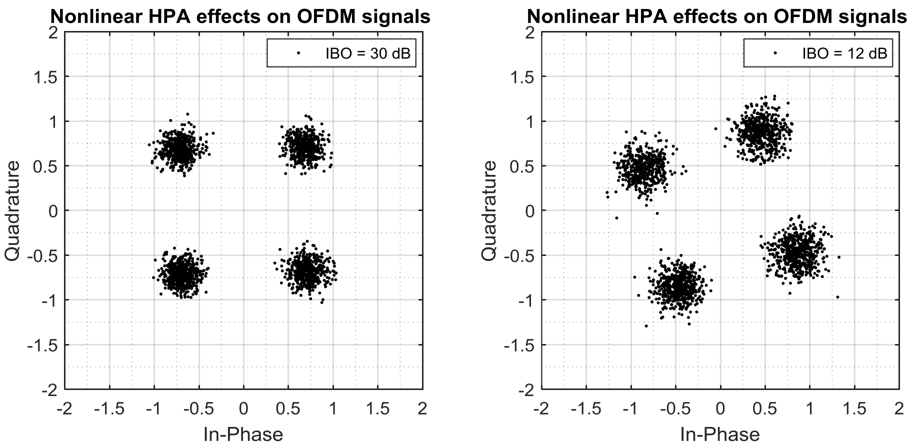
Note that the out-of-band emissions are respresented in terms of $S'_k\neq 0$ for $k<0$ and $k>N-1$. However, these components are not concerned here and are assumed to be filtered out by transmitter and/or receiver zonal filters. Moreover, it is not difficult to see that the OFDM sub-carriers after nonlinear transformation are no longer orthogonal; therefore the optimal maximum-likelihood (ML) receiver requires a joint detection of transmitted vector $\underline{S}$ [3]. However, for large and even medium number of subcarriers, this optimal ML solution is (extremely) high complexity and thus has little practical value. We then try to enhance the channel estimation quality to better compensate for the nonlinear effects at receiver to improve the overall BER performance.

Supposing that N is sufficiently large, each frequency-domain OFDM symbol distorted in nonlinear HPA (8) can be represented on the basis of extended Bussgang theorem [1] as

$$\underline{S}'=\alpha\underline{S}+\underline{D}, \tag{9}$$

where $\underline{S}'=\left[S'_0,S'_1,...,S'_{N-1}\right]^T$, α is the complex attenuation factor, and $\underline{D}=\left[D_0,D_1,...,D_{N-1}\right]^T$ is the (frequency-domain) uncorrelated nonlinear distortion term.

2.4. Linear approximation for OFDM receiver processing



(a) (b)

Figure 2. *Effects of nonlinear HPA on OFDM signals.*

Assume the use of linear OFDM transmitter with equivalent symbol vector $\underline{S}'$, at receiver, after the transformation from time domain to frequency domain based on discrete Fourier transform (DFT) (refer to figure 1), the received signal vector $\underline{R}=\left[R_0,R_1,...,R_{N-1}\right]^T$ can be expressed as

$$\underline{R} = \underline{\underline{H}}'\underline{S}' + \underline{W}' = \alpha\underline{\underline{H}}'\underline{S} + \underline{\underline{H}}'\underline{D} + \underline{W}' = \underline{\underline{H}}\underline{S} + \underline{W}, \tag{10}$$

where $\underline{\underline{H}} = \alpha\underline{\underline{H}}'$, $\underline{W} = \underline{\underline{H}}'\underline{D} + \underline{W}'$, $\underline{W}' = \left[W_0', W_1', ..., W_{N-1}'\right]^T$ is the (frequency domain) complex white Gaussian noise vector with i.i.d. components having zero-mean and variance $\sigma_{w'}^2$, and $\underline{\underline{H}}' = \text{diag}\left(\left[H_0, H_1, ..., H_{N-1}\right]\right)$ is the diagonal matrix containing the frequency domain channel coefficients (i.e. the DFT of the length-L channel impulse response $\underline{h} = \left[h_0, h_1, ..., h_{L-1}\right]^T$)

$$H_k = \sum_{l=0}^{L-1} h_l e^{-j2\pi\frac{kl}{N}}, \qquad 0 \le k \le N-1. \tag{11}$$

As observed from (10), the nonlinear OFDM system now could be considered as a linear one with the effective channel $\underline{\underline{H}}$, which is an α-scaled version of $\underline{\underline{H}}'$, and the equivalent additive noise $\underline{W}$, which is a combination of an AWGN $\underline{W}'$ and a nonlinear distortion term $\underline{\underline{H}}'\underline{D}$. This approximation is denmonstrated in figures 2.(a) and 2.(b) by simulating the received signal $\underline{R}$ with perfect equalization $\underline{\underline{H}}'$. The main system parameters are QPSK modulation, number of subcarriers $N = 2048$, exponential decay power-delay profile (PDP) for multipath channel of length $L_{\text{CH}} = 40$, modified Ghorbani HPA model (3) and (4) with input backoff $IBO = 30$ dB (figure 2.(a), demonstrating the almost linear case) and $IBO = 12$ dB (figure 2.(b), demonstrating the strongly nonlinear case). For the latter case, it is not difficult to recognize the effects of complex scaling factor α that rotates (and just slightly compresses) the signal constellation and the effect of equivalent noise $\underline{W}$ that evenly scatters the signal points. Moreover, this noise term could be well approximated as Gaussian. Using the approximated linear model (10) vastly facilitates the expection maximization channel estimation described in the following section.

3. EM-BASED CHANNEL ESTIMATION ALGORITHM

As mentioned above, in this work pilot based method is used for channel estimation. Equally spaced pilot symbols (on pilot subcarriers at every D_p subcarriers) are embedded in between the data symbols (on data subcarriers) which, according to the Gaussian approximation (10), provide the best channel information at the receiver [11]. The channel estimates are then interpolated over the data subcarriers for equalization. With the availability of such pilot symbols, the least square (LS) and minimum mean square error (MMSE) techniques are widely used for channel estimation [2] and could be used as the initial for the EM-based estimator.

Let's start this section with a brief discussion for the LS and MMSE channel estimators. Using the assumption of comb-type pilot pattern, without the risk of ambiguity, the system equation (10) could be used to represent the transmission of N_{P} pilot terms as:

$$\underline{R} = \underline{\underline{S}}\underline{H} + \underline{W}, \tag{12}$$

where $\underline{\underline{S}}$ is the diagonal matrix of N_{P} pilots symbols, $\underline{H}$ is the column vector including N_{P} entries of $\left\{H_k\right\}_{k=0}^{N-1}$ in (11) with indexes corresponding to the pilot subcarrier indexes.

3.1. LSE Channel estimation

The least-square error (LSE) channel estimation method finds the channel estimate $\underline{\hat{H}}_{LS}$ in such a way that the cost function $J_{LS}(\underline{\hat{H}}_{LS}) = \| \underline{R} - \underline{\underline{S}}\underline{\hat{H}}_{LS} \|^2$ is minimized [11]:

$$\underline{\hat{H}}_{LS} = \left(\underline{\underline{S}}^H \underline{\underline{S}}\right)^{-1} \underline{\underline{S}}^H \underline{R} = \underline{\underline{S}}^{-1} \underline{R}. \tag{13}$$

Denote each component of the LSE channel estimate $\underline{\hat{H}}_{LS}$ by $\hat{H}_{LS}(k),\ k = 0,1,2,...,N_{\mathrm{P}} - 1$. Since $\underline{\underline{S}}$ is assumed to be diagonal due to approximation (10), the LSE channel estimate $\underline{\hat{H}}_{LS}$ can be written for each subcarrier as

$$\hat{H}_{LS}(k) = \frac{R(k)}{S(k)},\ k = 0,1,2,...,N_{\mathrm{P}} - 1. \tag{14}$$

The mean-square error (MSE) of this LSE channel estimate is given as

$$MSE_{LS} = \mathbb{E}\left\{\left(\underline{H} - \underline{\hat{H}}_{LS}\right)^H \left(\underline{H} - \underline{\hat{H}}_{LS}\right)\right\} = \frac{\sigma_w^2}{\sigma_s^2}, \tag{15}$$

where σ_w^2 and σ_s^2 are the signal and noise variances, correspondingly.

Note that the MSE in Equation (15) is inversely proportional to the $SNR = \sigma_s^2 / \sigma_w^2$, which implies that it may be subject to noise enhancement, especially when the channel is in a deep null. Hence, without using any knowledge of the statistics of the channels, the LSE estimators are calculated with very low complexity, but they suffer from a high mean-square error. Due to its simplicity, however, the LSE method has been widely used for channel estimation.

3.2. MMSE channel estimation

The MMSE channel estimation method finds a better (linear) estimate in terms of the weight matrix $\underline{\underline{Q}}$ for $\underline{\hat{H}}_{MMSE} = \underline{\underline{Q}}\underline{\hat{H}}_{LS}$ in such a way that the cost function $J_{MMSE}(\underline{\hat{H}}_{MMSE}) = \mathbb{E}\left\{\| \underline{e} \|^2\right\} = \mathbb{E}\left\{\| \underline{H} - \underline{\hat{H}}_{MMSE} \|^2\right\}$ is minimized:

$$\underline{\underline{Q}} = \underline{\underline{R}}_{H\tilde{H}} \underline{\underline{R}}_{\tilde{H}\tilde{H}}^{-1}, \tag{16}$$

where $\underline{\underline{R}}_{\tilde{H}\tilde{H}}$ is the autocorrelation matrix of the LSE channel estimate $\underline{\tilde{H}} = \underline{\hat{H}}_{LS}$, given as $\underline{\underline{R}}_{\tilde{H}\tilde{H}} = \mathbb{E}\left\{\underline{\hat{H}}\underline{\hat{H}}^H\right\} = \mathbb{E}\left\{\underline{H}\underline{H}^H\right\} + (\sigma_w^2 / \sigma_s^2)\underline{\underline{I}},$ and $\underline{\underline{R}}_{H\tilde{H}}$ is the cross-correlation matrix between the true channel vector and temporary channel estimate vector in the frequency domain. The MMSE estimator yields much better performance than LSE estimators, especially under the low SNR values. A major drawback of the MMSE estimator is its high computational complexity with the requirement of knowing channel statistics.

3.3. EM-based channel estimation algorithm

As is known from the general convergence property of the EM algorithm, there is no guarantee that the iterative steps converge to a global maximum [4]. For a likelihood function with multiple local maxima the convergence point may be one of these local maxima, depending on the initial estimate $H_n^{(0)}$. We, therefore, propose to use pilot symbols distributed at certain locations in the OFDM time-frequency lattice with conventional LSE- or MMSE-based channel estimation to obtain an appropriate initial value $H_n^{(0)}$, which is more likely to converge to the true maximum point.

Again, taking (10) into account with the assumption that each subcarrier is undergoing independent fading, we regard $\underline{\underline{H}}_{n,n} = \alpha H_n,\ 0 \le n \le N-1$, in (11) as a deterministic parameter to be estimated from the observed data R_n. Hence, in the expectation maximization context, R_n is the insufficient observed information, while S_n is the unknown information. The log likelihood (cost) function for OFDM system can be defined as $\log f\left(R_n, S_n \mid H_n\right)$, where incomplete and complete data sets are $\{R_n\}_{n=0}^{N-1}$ and $\{R_n, S_n\}_{n=0}^{N-1}$, respectively.

Each iterative process $i = 0,1,2,...,$ in the EM algorithm for estimating $\underline{H}$ from $\underline{R}$ consists of the following two steps: Expectation (E) step and Maximization (M) step [4]. Let's $H_n^{(i)}$ denotes the estimated parameter at $(i)^{th}$ iteration. At $(i+1)^{th}$ iteration, in the expectation step, the cost function is averaged over all possible values of transmitted data given the received symbol and channel estimate at $(i)^{th}$ iteration.

E step:

$$\begin{aligned} Q\left(H_n \mid H_n^{(i)}\right) &= \mathbb{E}_S\left\{\log f\left(R_n, S_n \mid H_n\right) \middle| R_n, H_n^{(i)}\right\} \\ &= \frac{1}{M}\sum_{m=1}^{M}\log\left\{\frac{1}{M} f_m\left(R_n \middle| H_n\right)\right\}\frac{f_m\left(R_n \middle| H_n^{(i)}\right)}{f\left(R_n \middle| H_n^{(i)}\right)}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} f_m\left(R_n \middle| H_n\right) &= \frac{1}{\sqrt{2\pi\sigma_w^2}}\exp\left\{-\frac{1}{2\sigma_w^2}\left\|R_n - H_n S_n^{(m)}\right\|^2\right\}, \\ f_m\left(R_n \middle| H_n^{(i)}\right) &= \frac{1}{\sqrt{2\pi\sigma_w^2}}\exp\left\{-\frac{1}{2\sigma_w^2}\left\|R_n - H_n^{(i)} S_n^{(m)}\right\|^2\right\}, \\ f\left(R_n \middle| H_n^{(i)}\right) &= \frac{1}{\sqrt{2\pi\sigma_w^2}}\sum_{m=1}^{M}\exp\left\{-\frac{1}{2\sigma_w^2}\left\|R_n - H_n^{(i)} S_n^{(m)}\right\|^2\right\}. \end{aligned} \tag{18}$$

M step:

Putting the above values of conditional PDFs from (18) in (17), and maximizing the Q function, the estimated channel parameter at $(i+1)^{th}$ iteration is obtained as,

$$\hat{H}_n^{(i+1)} = \left[\sum_{m=1}^{M} S_n^{(m)}\left(S_n^{(m)}\right)^* \frac{f_m\left(R_n \middle| H_n^{(i)}\right)}{f\left(R_n \middle| H_n^{(i)}\right)}\right]^{-1} \times \sum_{m=1}^{M} R_n^{(m)}\left(R_n^{(m)}\right)^* \frac{f_m\left(R_n \middle| H_n^{(i)}\right)}{f\left(R_n \middle| H_n^{(i)}\right)}. \tag{19}$$

For initial estimation of $H_n^{(i)}$ LSE- or MMSE-based technique could be used as mentioned above.

4. SIMULATION RESULTS AND DISCUSSION

4.1. Simulation setup

Computer simulations have been run to check and extend the analytical results of the previous section. The simulated system is as follows.

Fading channel parameters

The block fading channel has $L_{\text{CH}} = 40$ paths, with path delays of $0,1,2,...,L_{\text{CH}}-1$ samples.

The amplitude of each path A_k varies independently of the others, according to a Rayleigh distribution with an exponential power-delay profile, i.e.,

$$E\left\{A_k^2\right\} = \exp\left(-k/\gamma\right),\ \ k = 0,1,2,...,L_{\mathrm{CH}} - 1, \tag{20}$$

where $\gamma = 10$ is the channel exponential power decay factor. The phase shift on each path is uniformly distributed over $[0, 2\pi)$. The channel is static over the OFDM symbol duration. A new channel is generated at each OFDM frame, and the system performance is averaged over the channel impulse response (CIR) realizations, which is given by

$$h(n) = \frac{1}{\sqrt{2C}} \sum_{k=0}^{L_{\mathrm{CH}}} \exp\left(-k/2\gamma\right)\left(h_k^{(r)} + jh_k^{(i)}\right)\delta(n-k), \tag{21}$$

where $C = \sqrt{\sum_{k=0}^{L_{\mathrm{CH}}} \exp\left(-k/\gamma\right)}$ is the normalization constant and $h_n^{(r)}$ and $h_n^{(i)}$ are Gaussian random variables with zero mean and unit variance.

OFDM system parameters

The number of subcarriers used in the simulation is rather large to guarantee a good Gaussian approximation (9) and thus the linearization (10). The CP length is larger than the channel impulse length to prevent ISI and ICI caused by multipath fading. The OFDM system main parameters are listed in table 1.

Table 1. *Parameters of the OFDM system.*

Parameter	Value
Number of subcarriers, N	2048
Subcarrier spacing, Δf (kHz)	15
Pilot interval, D_{P}	32
Number of pilots, N_{P}	64
Pilot power scale factor, K_{P}	1
Cyclic prefix length, L_{CP}	64
Channel length, L_{CH}	40
Channel exponential power decay factor, γ	10
Modulation	QPSK

HPA parameters

For broadband signals such as 5G, the AM-PM nonlinearity could not be ignored [13]. Although this nonlinearity severely distorts the OFDM signal in terms of phase rotations as shown in figure 2, it is however, not much discussed in previous related publications [2, 3, 8]. Thus, we use modified Ghorbani HPA model having both AM-AM (3) and AM-PM (4) characteristics with their parameters given in Section 2.2. It is then shown in our simulations results that using the proposed EM-based channel estimation algorithm, the phase rotation is well estimated and compensated for, yielding improved BER performance.

4.2. Simulation results and comments

Number of EM algorithm's iterations

Figure. 3 compares numbers of iterations required for the estimates $\underline{H}$ to converge at two nonliearities, IBO = 30 dB for the almost linear case, and IBO = 12 dB for the strongly nonlinear case using the EM-MMSE estimator. Here, the estimator exploits the EM algorithm (17), (19) with initials $\{H_n^{(0)}\}_{n=0}^{N-1}$ given by the MMSE estimator (16). It can be observed that number of necessary iterations is relatively small (less than 5) for pratical values of E_b / N_0 (larger than 15 dB). This guarantees a reasonable complexity for pratical applications. Moreover, the strong nonlinearity causes almost no increase in the number of iterations required to converge. This implies that in the strong nonlinearity conditions, the algorithm can achieve a substantial performance improvement without increasing in computational complexity.

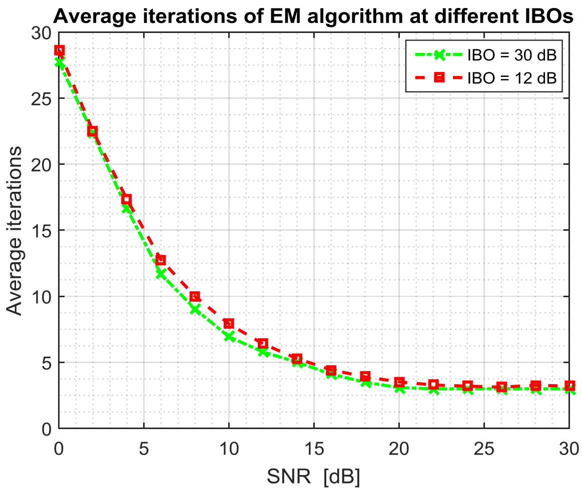


Figure 3. *Average iterations of EM-MMSE algorithm.*

Channel estimation performance

The channel estimation performance is quanlitatively evaluated by considering the LSE-, MMSE-, and EM-MMSE-based estimated frequency responses depicted in figure 4. The LSE estimator uses (14), while the MMSE estimator folows (16). Simulations are carried out in a moderate nonlinearity condition, IBO = 20 dB. Roughly, the LSE does worst while the EM-MMSE outperforms the rest, always closely tracks to the true response at all subcarrier indexes. This is quantified using MSE measures denmonstrated in figure 5. From this figure, it can be seen that by using the channel statistics in (16), the MMSE estimator surpasses the LSE. Moreover, the EM-based estimators could yields much higher quality of estimation than others do, especially at mild nonlinearity conditions. The MSE of the EM-based methods could gain an order of magnitude better than the others. Another interesting result obtained from our simulation is that compared to the EM-MMSE estimator, the performance degradation is quite small when we use the EM-LSE algorithm that does not use the channel statistics. Thus, this algorithm is robust to both the channel fading and HPA nonlinearity conditions.

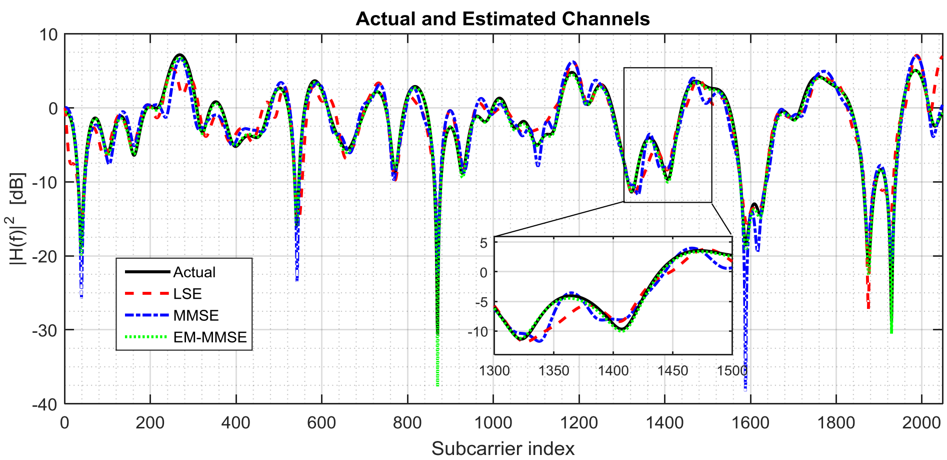
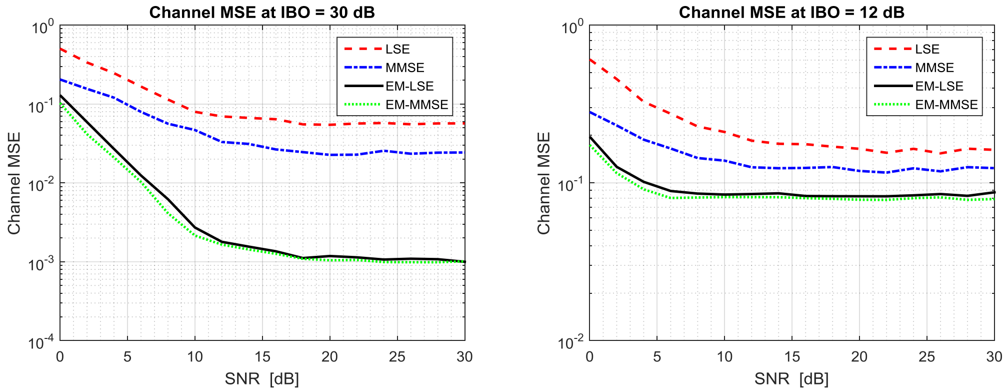


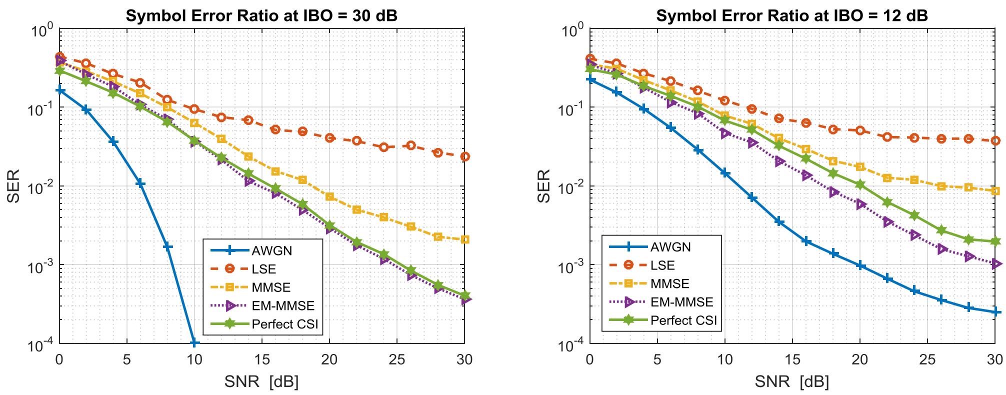
Figure 4. *Actual channel and estimated values based on LSE, MMSE, EM-MMSE algorithms.*



(a) (b)

Figure 5. *Channel estimation perforamance in terms of MSE.*

SER performance



(a) (b)

Figure 6. *Symbol error performance of OFDM system with different channel estimators.*

Figure 6 compares the SER performance of EM-based channel estimation algorithm to the other conventional ones (LSE, MMSE). The "AWGN" curve is the SER of system with non fading channel, while the "Perfect CSI" curve is the SER of system with fading channel known at receiver for channel equalization however, the phase rotation is not known and not conpensated for. It is obvious that the implementation of EM-based channel estimation vastly improves the SER performance compared to the conventional channel estimation methods. It is the higher accuracy of channel estimation performance that greatly improves the channel equalization for the better SER.

It can be seen that in figure 6.(a), EM-MMSE shows worse SER performance than Perfect CSI at low SNRs i.e. from 0 to around 10 dB. In this case the system is almost linear. Thus, AWGN is the dominance disturbing both pilots and data. Channel estimations then are worse affected such that the perfect CSI-based equalization outperforms that of EM-MMSE estimated values. Refereing to the same range of SNR in figure 6.(b), the system here is heavily affected by nonlinear noise further, such that perfect CSI-based and EM-MMSE-based equalizations give almost the same results. This addtion of nonlinear noise is what could be seen from figure 2.

More generaly, for weak nonlinearity (in figure 6.(a)) the EM-MMSE estimator could yield almost perfect channel state information resulting in the ideal channel equalization and the BER closes to the perfect CSI case. It can be seen the diversity order 1 for SER curve this case. Moreover, in strongly nonlinear behavior (figure 6.(b)), the EM-MMSE even better compensates for the fading channel and nonlinear effects than the perfect CSI case. The reason is that phase rotation as illustraed in figure 2 (b) is well estimated and compensated for as quantified in (10) while the "pefect CSI" system only compensates for the fading channel. The SER improvement in this case is significant, further showing the advantages of our proposal.

5. CONCLUSIONS

In this paper, the EM-based channel estimation has been proposed for the nonlinear OFDM system based on the linearization using extended Bussgang decomposistion. It is shown that the proposed algorithm only requires reasonable computation complexity with relatively small number of iterations while vastly improves the estimation performance compared to the other conventional estimation methods. Moreover, the EM-LSE estimator could give almost the same performance as the EM-MMSE counterpart while does not require the channel statisics, forming a robust estimator to both fading and nonlinear channels with reduced computation complexilty. This makes the estimator to be more applicable. The proposed technique may be extended to multiple input multiple output (MIMO) and will be addressed in the following researches.

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TÓM TẮT

Ước lượng kênh cực đại kỳ vọng cho các hệ thống OFDM có méo phi tuyến

Bài báo đề xuất việc sử dụng bộ ước lượng kênh dựa trên thuật toán kỳ vọng-cực đại EM cho các hệ thống ghép kênh phân chia theo tần số trực giao OFDM có méo phi tuyến trên cơ sở xấp xỉ tuyến tính hóa sử dụng phân tích Bussgang mở rộng. Các kết quả phân tích và mô phỏng chứng minh rằng thuật toán đề xuất chỉ yêu cầu độ phức tạp tính vừa phải với số lần giải lặp nhỏ trong khi cải thiện rất đáng kể chất lượng ước lượng kênh so với các phương pháp ước lượng thông thường khác như sai số nhỏ nhất LSE hay sai số bình phương trung bình cực tiểu MMSE. Điều này cho phép thực hiện san bằng hiệu quả hơn và hệ thống do đó cải thiện đáng kể được tỉ lệ lỗi bit BER. Ngoài ra, bộ ước lượng EM-LSE có thể bảo đảm chất lượng ước lượng gần tương đương như bộ ước lượng EM-MMSE trong khi không yêu cầu thông tin đặc trưng thống kê của kênh pha-đinh, cho phép xây dựng bộ ước lượng mạnh trên cả kênh pha-đinh và kênh phi tuyến với độ phức tạp tính giảm thiểu.

Từ khoá: Méo phi tuyến; Ước lượng kênh; Cực đại kỳ vọng; OFDM.