Investigating the effects of various input beam profiles on the propagations of light in two-dimensional interfaced binary waveguide arrays

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ABSTRACT

In this work, we numerically study how various input beam profiles influence the linear and nonlinear light propagation at the interface of two-dimensional (2D) binary waveguide arrays. It is revealed that, due to the presence of the central homogeneous interfaced waveguides, light beams are effectively steered into the preferred direction. Interestingly, the formation of discretely localized states in nonlinear modes can be intentionally utilized to optimize the stability and intensity of the signals at the central interfaced channels. This study thus opens alternative possibilities to achieve reliable distant beam propagation through discrete optical systems.

Keywords: Binary waveguide; Localized state; Beam profile; Interfaced channel; Beam propagation.

1. INTRODUCTION

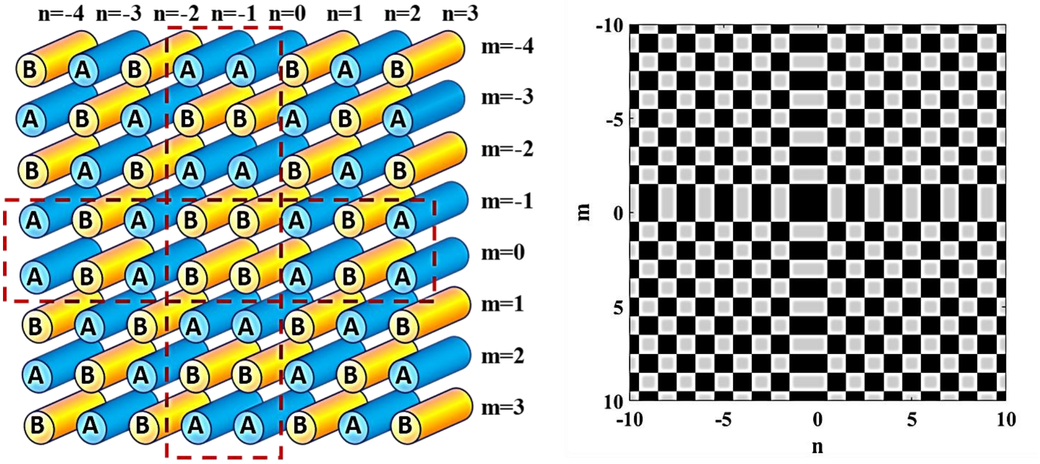
With the advance in optical telecommunication, the requirements for quality, reliability, and functionality of integrated optics devices have increased. Advanced fabrication technologies have now promoted the manufacturing of nanostructured materials with optimized properties which can be tailored to a large extent [1–4]. In these nanostructures, beam propagation can be diversely manipulated and controlled. One of the common and practical ways to implement this idea is to use ordered waveguide arrays, where beam propagation can be deliberately regulated compared with that in bulk materials [3, 5]. In these waveguide lattices, light is confined to individual local channels by design. The inherent dynamics and evolution of the discretized light are dominated by the linear and nonlinear coupling effects among respective adjacent guided modes [6–12]. Some studies of inhomogeneous waveguide arrays revealed that the localized optical modes present photonic Bloch-oscillations in the waveguides [13]. Recently, waveguide arrays have also been progressively used to mimic relativistic phenomena typical of quantum field theory, such as Klein tunneling [14–16], the Zitterbewegung [17], and fermion pair production [14, 18]. Recently, one-dimensional interfaced binary waveguide arrays (BWAs) were systematically studied in the article by Truong et al. [9]. This work has shown the significant potential to stabilize the intense localized states during propagation, especially in the case with a negative coefficient for the cubic nonlinearity term. The extended research on the 2D interfaced binary waveguide arrays is expected to provide more practical and useful data to promote remote optical data transferring devices.

In this work, we investigate the light propagation of various input beam profiles at the neighboring regions to the interfaces between four adjacent 2D flipped waveguide arrays, in both linear and nonlinear optical modes. First, we develop the theoretical models of 2D interfaced binary waveguide arrays based on a dimensionless coupled-mode equation. Second, we systematically simulate the beam propagation of several input profiles, i.e., single mode gaussian and top-hat beams of varied parameters, to determine outputs of the interaction between light and the interfaced waveguide lattices, in both linear and nonlinear modes. Third, from the simulated results, we provide some optimized system parameters to achieve stable intensified signals at the central interfaced channels.

2. INTERFACED 2D BINARY WAVEGUIDE ARRAY STRUCTURE AND MODELING

2.1. Designed structure

In our investigated system, 2D interfaced binary waveguide arrays comprise four adjacent flipped lattices with alternating waveguides of types A and B. A geometrical representation of the structure is given in Fig. 1(a). Coupling effects among adjacent waveguides are defined by propagation mismatch and coupling coefficient. The nonlinearity of the waveguide lattices is specified by the nonlinear coefficient which is positive for self-focusing, but negative for self-defocusing media. The central homogeneous waveguides are intentionally introduced into the design to promote coupling effects, hence providing effective beam steering channels. The simulated waveguide type distribution at the center of the 2D interfaced BWA is magnified and presented in Fig. 1(b).



(a) (b)

__Figure 1.__ (a) Illustrative sketch of the 2D interfaced BWA created by combining four flipped waveguide lattices, each consisting of alternating waveguides of types A and B. The interfaced waveguides are indicated by the red dashed boundaries. (b) The simulated waveguide type distribution at the center of the 2D interfaced BWA.

2.2. Theoretical modeling

The theoretical model of the 2D interfaced BWAs can be described by the following coupled-mode equation [9, 14]:

$$\begin{aligned} & i\frac{da_{m,n}}{dz}+\kappa_1\left[\left(a_{m+1,n}+a_{m-1,n}\right)+\left(a_{m,n+1}+a_{m,n-1}\right)\right] \\ & +\kappa_2\left[\left(a_{m+1,n+1}+a_{m-1,n-1}\right)+\left(a_{m+1,n-1}+a_{m-1,n+1}\right)\right]-(-1)^{m+n}\sigma a_{m,n}+\gamma\left|a_{m,n}\right|^2 a_{m,n}=0 \end{aligned} \qquad (1)$$

In Eq (1), $a_{m,n}$ is electric field amplitude in the waveguide of *m*th row and *n*th column, z is the longitudinal spatial coordinate, 2σ is the propagation mismatch between two different types of waveguides, κ_1 and κ_2 are coupling coefficients between neighboring waveguides of the lattice, and γ is the nonlinear coefficient of waveguides. For simplicity, here we suppose all waveguides have the same nonlinear coefficient. Due to the distances between neighboring waveguides, it is evident that the coupling coefficients $\kappa_1 > \kappa_2$.

3. NUMERICAL SIMULATION OF BEAM PROPAGATION

In this section, we investigate the propagation of various input beam profiles in the 2D interfaced BWAs. The numerical simulation of the 2D interfaced binary waveguide arrays was implemented using MATLAB ver. R2018b. First, single-mode input beams with a conventional Gaussian profile are examined. The complex amplitude of the Gaussian beam is obtained from the wave theory of light as a particular case of the complex amplitude of paraxial waves [19]:

$$a(\vec{r}) = A(\vec{r})e^{-ikz} \tag{2}$$

where k represents the wave number, z corresponds to the optical axis-z and the amplitude $A(\mathbf{r})$ satisfies the following form:

$$A(\vec{r}) = Ae^{\frac{-jk\rho^2}{2w^2}} \tag{3}$$

where ρ represents the radial distance from the center axis of the beam, w is the radius at which the field amplitudes fall to $1/e$ of their axial values.

In a discrete waveguide system, the input profile of the single-mode Gaussian beam can be generated from the equation:

$$a_{m,n} = Ae^{\frac{-jk(m^2+n^2)}{2w^2}} \tag{4}$$

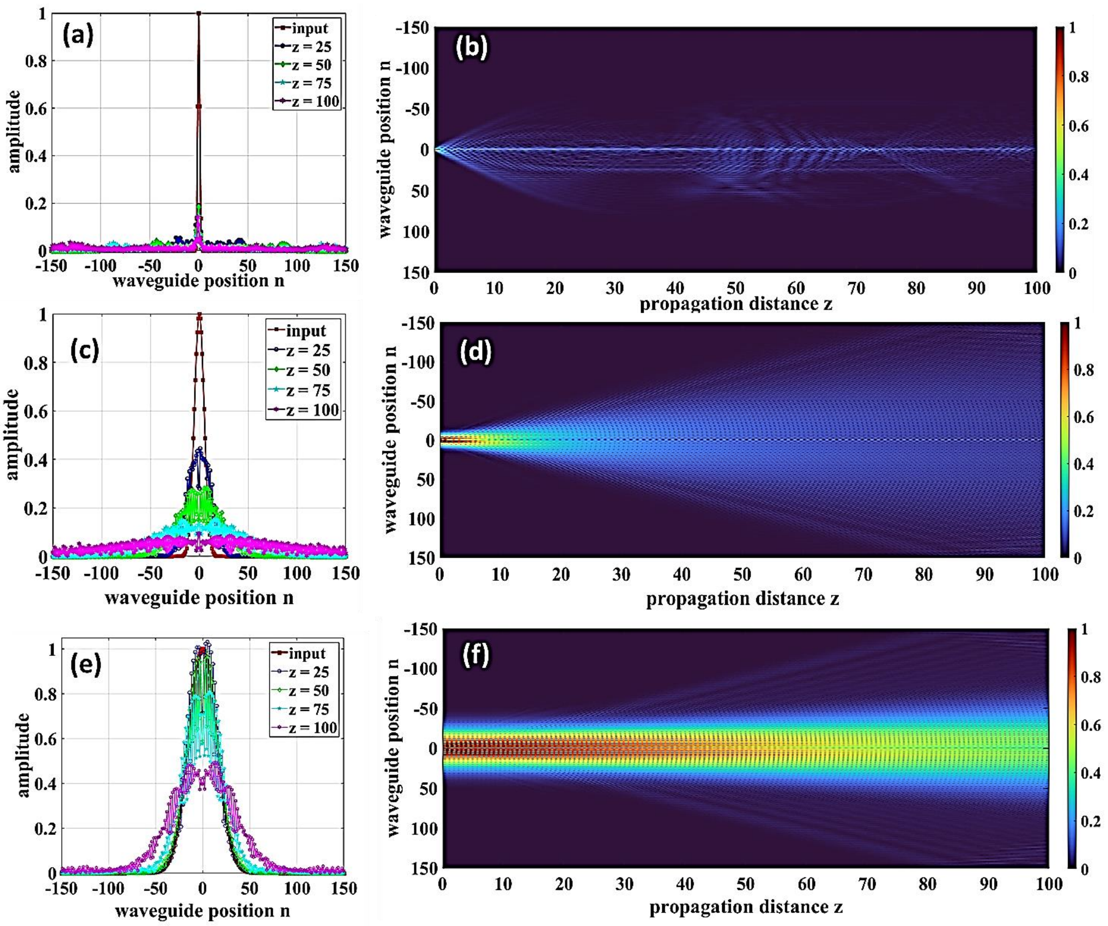


Figure 2. *Amplitudes of the input Gaussian beam illuminating (a) 3; (c) 5; and (e) 15 central waveguides at input and outputs at z = 25, 50, 75, and 100, respectively. On the right side, the simulated side views of the Gaussian beam illuminating (b) 3; (d) 5; and (f) 15 input central waveguides, respectively. Parameters are* $-\sigma_1 = \sigma_2 = 1$, $\kappa_1 = 1$, $\kappa_2 = 0.5$ *and* $\gamma = 0$.

As a first step, we investigate the influence of the beam width of an input single-mode Gaussian profile on the beam propagation in the waveguide array in the linear regime. Parameters are $-\sigma_1 = \sigma_2 = 1$, $\kappa_1 = 1$, $\kappa_2 = 0.5$ and $\gamma = 0$. The beam is excited into 3, 5, and 15 central waveguides at the input of the BWA. Fig. 2(a), (c), and (e) present the amplitudes of the Gaussian beam of 3, 5, and 15 irradiated central waveguides at the input and at the distances z = 25, 50, 75, and 100, respectively. The corresponding 2D side views of the Gaussian beam in the lattices are shown in Fig. 2(b), (d), and (f). It is interesting to note that, in Fig. 2(b), most of the beam's energy, though gradually dissipated, remains concentrated at its central axis due to the effective confinement of the homogeneous central waveguides. As the Gaussian beam size grows wider than the central interfaced waveguides, energy in discrete waveguide arrays quickly expands during propagation due to the dominance of the diffraction effect (see Fig. 2(d)). Additionally, as the input beam illuminates as many as 15 central waveguides, the discreteness of the waveguide lattice becomes moderate. We can observe in Fig. 2(f) that the diffraction phenomenon in the BWA*s* is analogous to that in continuous bulk materials.

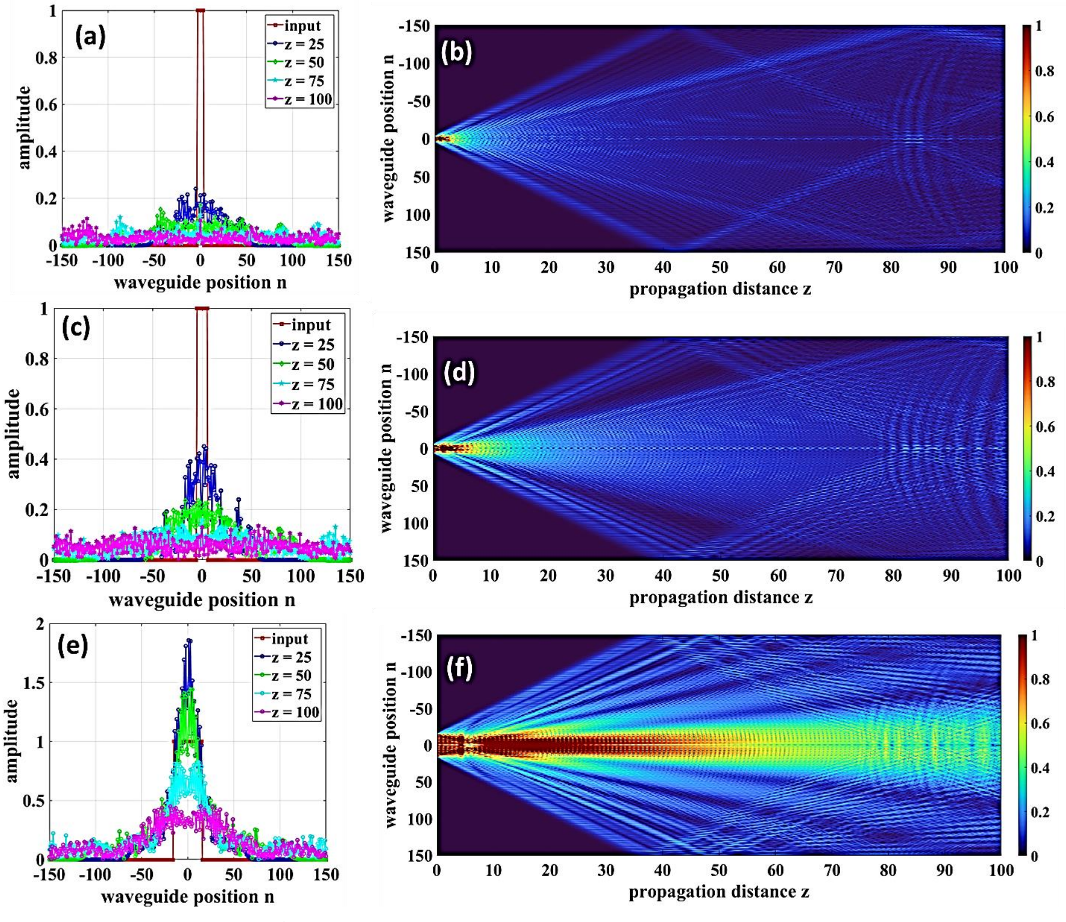


Figure 3. *Amplitudes of the input top-hat beam illuminating (a) 3; (c) 5; and (e) 15 central waveguides at input and outputs at z = 25, 50, 75 and 100, respectively. On the right side, the simulated side views of the top-hat beam illuminating (b) 3; (d) 5; and (f) 15 input central waveguides, respectively. Parameters are* $-\sigma_1 = \sigma_2 = 1$, $\kappa_1 = 1$, $\kappa_2 = 0.5$ *and* $\gamma = 0$.

Next, we investigate the influence of the beam width of an input top-hat profile on the beam propagation in the waveguide array in the linear regime. Parameters are $-\sigma_1 = \sigma_2 = 1$, $\kappa_1 = 1$, $\kappa_2 = 0.5$ and $\gamma = 0$. The top-hat beam is also illuminated into 3, 5, and 15 central waveguides at the input of the BWA. Fig. 3(a), (c), and (e) present the amplitudes of the top-hat beam of 3, 5, and 15 irradiated central waveguides at the input and at propagation distances z = 25, 50, 75, and 100, respectively. The corresponding 2D side views of the top-hat beam in the lattices are shown in Fig. 3(b), (d), and (f). Compared to the case of the Gaussian profile, the evolution of the top-hat beam is relatively similar during the propagation.

When the beam intensity is sufficiently high, which can be easily achieved with powerful laser beams, nonlinear effects will occur. As is demonstrated in Fig 4(a), the beam transmission is particularly stable in the BWA while the peak amplitude and n-axis width of the beam almost remain steady during transmission along the z-axis in BWA. The stability of the beam propagation in nonlinear mode can be attributed to a trade-off between the Kerr nonlinear effect and the diffraction effect. Indeed, the diffraction effect tends to expand the light beam during transmission, while the self-focusing Kerr nonlinear effect (when $\gamma > 0$) tends to contract the beam. As $\gamma = 1.5$, these two diametrically opposite effects balance each other and form stable peak amplitude and beam width during transmission in BWA as clearly shown in Fig. 4.

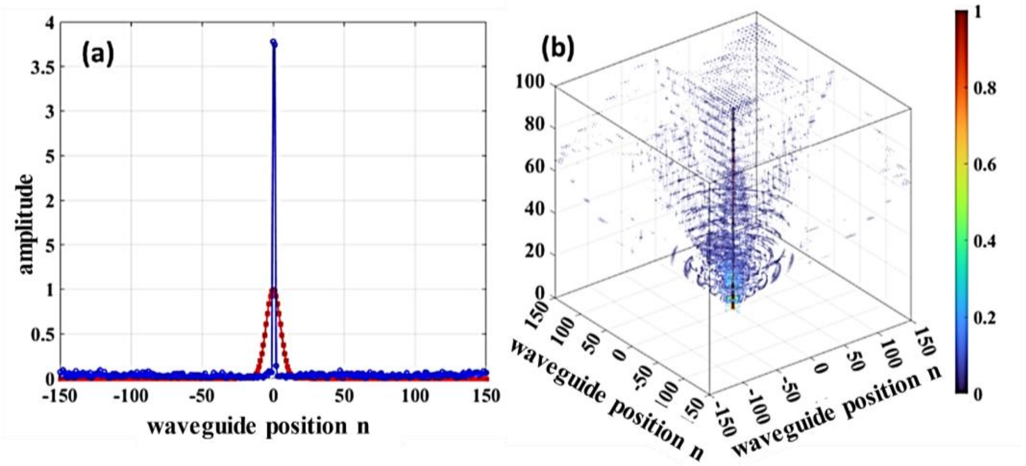


Figure 4. *(a) Amplitudes of the input Gaussian beam illuminating 5 central waveguides at input and outputs at z =100, respectively. (b) The simulated 3D views of the Gaussian beam illuminating 5 input central waveguides. Parameters are* $-\sigma_1 = \sigma_2 = 1$, $\kappa_1 = 1$, $\kappa_2 = 0.5$ *and* $\gamma = 1.5$.

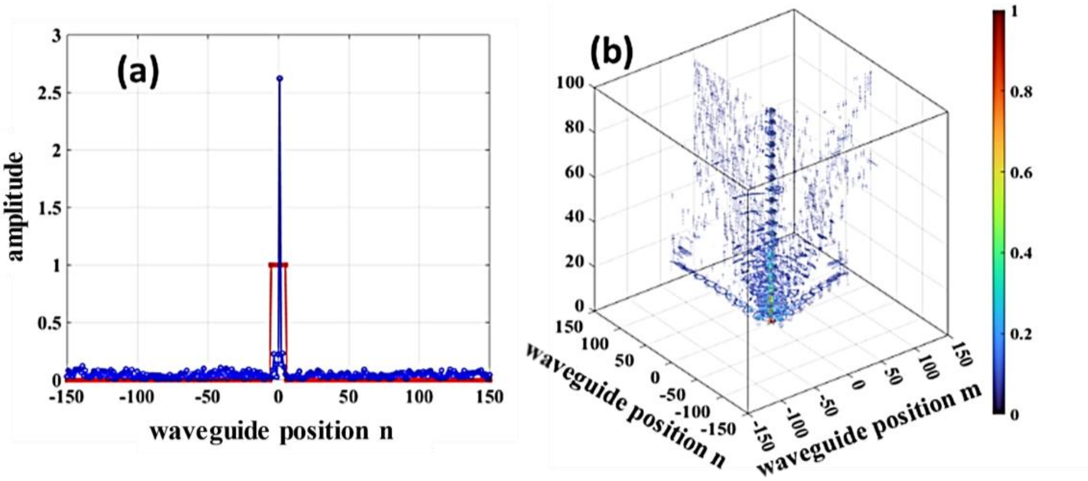


Figure 5. *(a) Amplitudes of the input top-hat beam illuminating 5 central waveguides at input and outputs at z =100, respectively. (b) The simulated 3D views of the top-hat beam illuminating 5 input central waveguides. Parameters are* $-\sigma_1 = \sigma_2 = 1$, $\kappa_1 = 1$, $\kappa_2 = 0.5$ *and* $\gamma = 1.5$.

Next, we investigate the nonlinearity (γ = 1.5) of top-hat beam propagation in the 2D interfaced BWAs. As shown in Fig. 5, a trade-off between the Kerr nonlinear effect and the diffraction effect also results in stable beam transmission in the lattice. In both cases of Gaussian and top-hat beams, interestingly, the peak amplitude of the signals at the central interfaced channels is remarkably intensified because the augmented energy from adjacent waveguides is effectively confined at central homogeneous waveguides. It is evident that the high nonlinearity of the lattice improves the stability and intensity of the central channels.

4. CONCLUSIONS

To conclude, we have systematically investigated the influence of various input beam profiles on the light propagation at the interface of 2D BWAs in both linear and nonlinear regimes. The obtained results in the linear regime show that the beam width of the input profile noticeably affect the diffraction phenomenon of the beam during propagation. A wider input beam diminishes the discreteness of the lattice, hence, the diffraction effect in the interfaced BWA becomes analogous to that in continuous bulk materials. In the nonlinear regime, the simulation shows that the stability of the beam propagation can be achieved by balancing the trade-off between the Kerr nonlinear effect and the diffraction effect. This study has revealed that nonlinear modes can be purposely exploited to achieve stable and intensified signal transmission at the central interfaced channels.

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TÓM TẮT

Nghiên cứu sự ảnh hưởng của dạng chùm tia đầu vào lên sự truyền sáng trong hệ ống dẫn sóng nhị phân tiếp giáp 2 chiều

Trong bài báo này, chúng tôi trình bày nghiên cứu về sự ảnh hưởng của các dạng chùm tia đầu vào khác nhau lên sự lan truyền ánh sáng tại vùng tiếp giáp của hệ ống dẫn sóng nhị phân hai chiều. Kết quả cho thấy các ống dẫn sóng đồng nhất trung tâm có tác dụng điều chỉnh hiệu quả chùm sáng lan truyền theo hướng ưu tiên định trước. Sự hình thành các trạng thái rời rạc cục bộ ở chế độ phi tuyến có thể được sử dụng để tối ưu hóa sự ổn định và cường độ tín hiệu ở các kênh tiếp giáp trung tâm. Nghiên cứu này do đó mở ra các khả năng ứng dụng truyền sáng cự ly xa trong các hệ quang học rời rạc.

Từ khoá: Ống dẫn sóng nhị phân; Trạng thái cục bộ; Dạng chùm tia; Kênh tiếp giáp; Lan truyền chùm sáng.