Directly adaptive control for robotic manipulators with time varying uncertainties

Hoang Duy, Do Manh Dung, Nguyen Hoai Nam, Nguyen Doan Phuoc*

Hanoi University of Science and Technology.
*Email: phuoc.nguyendoan@hust.edu.vn
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ABSTRACT

The paper proposes an approach of directly adaptive control for manipulators, which is applicable to control the robotics perturbed by time varying uncertain parameters. This approach is established based on combining the well known conventional feedback linearization method and the here in this article proposed model based on online observation of uncertainties. The tracking control performance of this proposed directly adaptive controller has been verified theoretical peer method and numerical simulation.

Keywords: Adaptive control; Robotic manipulator control; Time varying uncertainty observation.

1. INTRODUCTION

Robotic manipulators are known as the most important equipment, which are widely used in industry to perform operating tasks involving their desired speeds and positions [1, 2]. Consequently, many control methods were available for them to ensure that they will achieve the required accuracy. In [1-4] are some of them are presented. And there all existing model based control methods are classified in different types depending on how exact is the available mathematical model of robotic.

In the case that the mathematical model of robotic is sufficiently exact, then the feedback linearization method is useful [1-6]. In the presence of a vector of $\underline{\theta}$ of uncertain constants in the model, then the methods of inverse dynamic [1, 2, 4] or Li-Slotine [5, 6] are applicable, which can be considered as indirect adaptive control approaches. It means that among these approaches the uncertain constant model parameters would not be identified. The main disadvantage of indirect adaptive methods is that they might not be applicable when $\underline{\theta}(t)$ is time varying [7, 8]. In the scenario of time depending uncertainties $\underline{\theta}(t)$ the robust or intelligent control methods seem to be favourite [7, 9]. However, they often require a high capacity of digital device to implement them [2, 9].

This article presents an approach for directly adaptive control of manipulators, which is applicable for manipulators with time varying uncertainties and could be implemented on digital devices with low capacity. This approach is created based on combining the online identification of time varying uncertainties and feedback linearization method. The numerical simulation has been authenticated with its excellent tracking performance.

2. MAIN RESULTS

2.1. Problem formulation and notation

In this article, the identity matrix of dimension $n\times n$ is denoted by I_n, the $n\times n$ matrix with all zero entries is indicated by $\mathbf{0}_n$ and the column vector of zeros is symbolized with $\underline{0}$. A column vector of real numbers $a_1, \dots, a_n$ is denoted by $\underline{a}$. Any p-norm, $1<p<\infty$, of $\underline{a}$ is indicated with $\|\underline{a}\|$. An induced norm of a matrix A is denoted with $\|A\|$. The set of real number is indicated by R.

Consider an industrial robot manipulator of n degree described mathematically by:

$$M(\underline{q},\underline{\theta})\ddot{\underline{q}} + \underline{f}(\underline{q},\dot{\underline{q}},\underline{\theta}) = \underline{u}\,, \tag{1}$$

where $\underline{q} \in \mathrm{R}^n$ is the vector of n joint variables of the robot, $M(\underline{q},\underline{\theta})$ is the symmetric positive definite inertia matrix, $\underline{\theta}(t)$ is the vector of all undetermined system parameters, which may be additionally time varying, $\underline{u} \in \mathrm{R}^n$ is the vector of control inputs, $\underline{f}(\underline{q},\dot{\underline{q}},\underline{\theta})$ is the lumped vector of coriolis, centrifugal terms and gravitational components [3, 5].

The aim of model based output tracking control for manipulator (1) is a $\underline{q},\dot{\underline{q}}$ feedback controller $\underline{u}$ to design, so that its joint variables $\underline{q}$ tends asymptotically to a desired vector of references $\underline{r}(t)$ and this tracking control performance must be free from time varying uncertain system parameters $\underline{\theta}(t)$.

For solving the aforementioned control problem it is needed here the main assumtion of uncertain robotics (1) that the vector of undetermined system parameters $\underline{\theta}(t)$ is linearly dependent on this model itself. It means that a matrix $F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})$ exists to satisfy:

$$\underline{u} = M(\underline{q},\underline{\theta})\ddot{\underline{q}} + \underline{f}(\underline{q},\dot{\underline{q}},\underline{\theta}) = F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})\underline{\theta}\,. \tag{2}$$

As claimed in [1, 5, 6, 8], this assumption is always fulfilled in practice.

Example: The model of a two-link polar arm robot exhibited in Fig.1 with unknown mass θ in the format of Eq. (1) is as below [8]:

$$\begin{pmatrix} \theta q_2^2 & 0 \\ 0 & \theta \end{pmatrix}\ddot{\underline{q}} + \begin{pmatrix} 2\theta q_2\dot{q}_1\dot{q}_2 + \theta g q_2\cos(q_1) \\ -\theta q_2\dot{q}_1^2 + \theta g\sin(q_1) \end{pmatrix} = \underline{u}$$

where $\underline{q} = (\varphi, l)^T$ and $\underline{u} = (\tau, f)^T$. This model possesses

$$F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}}) = \begin{pmatrix} q_2^2\ddot{q}_1 + 2q_2\dot{q}_1\dot{q}_2 + gq_2\cos(q_1) \\ \ddot{q}_2 + g\sin(q_1) - q_2 q_1^2 \end{pmatrix}.$$

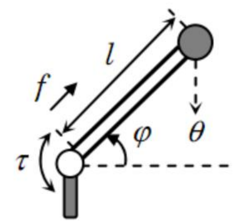

Figure 1. *Two-link polar arm robot.*

2.2. Adaptive controller design

Denote the number of uncertain time varying system parameters $\underline{\theta}(t)$ with p, then if $p \le n$ and the matrix $F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})$ defined in (2) is full rank, the vector $\underline{\theta}(t)$ will be obtained directly from the notation (2) by using the pseudo invert matrix of $F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})$ as below:

$$\underline{\theta}(t) = \left[F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})^T F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})\right]^{-1} F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})^T \underline{u}\,. \tag{3}$$

Note that for being identified of $\underline{\theta}(t)$ it is unavoidable that the collected data have to capture all important dynamics of systems. The condition of full rank of $F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})$ above ensures this.

In the formula above, if only $\underline{q},\dot{\underline{q}}$ are measurable, then the second order derivate $\ddot{\underline{q}}$ needed for determining $F(\underline{q},\dot{\underline{q}},\ddot{\underline{q}})$ will be approximated by using a low pass filter:

$$G(s) = \frac{1}{1+Ts},\ 0 < T \ll 1 \tag{4}$$

with the input $\underline{\dot{q}}$ and the output $\underline{z}$ and an arbitrary small chosen T, peer differential equation, as follows:

$$\underline{\dot{q}} = \underline{z} + T\underline{\dot{z}} \;\;\Rightarrow\;\; \underline{\dot{q}} \approx \underline{z} \text{ since } T \ll 1. \tag{5}$$

Hence

$$\underline{\ddot{q}} \approx \underline{\dot{z}} \; = (\underline{\dot{q}} - \underline{z})/T = \underline{v}. \tag{6}$$

which implies from (3)

$$\underline{\widehat{\theta}}(t) = \left[\hat{F}(\underline{q},\underline{\dot{q}},\underline{v})^T \hat{F}(\underline{q},\underline{\dot{q}},\underline{v}) \right]^{-1} \hat{F}(\underline{q},\underline{\dot{q}},\underline{v})^T \underline{u}, \tag{7}$$

where $\hat{F}(\underline{q},\underline{\dot{q}},\underline{v}) \approx F(\underline{q},\underline{\dot{q}},\underline{\ddot{q}})$ and $\underline{\widehat{\theta}}(t)$ is considered as the approximated value of $\underline{\theta}(t)$. This observed $\underline{\widehat{\theta}}(t)$ will be used hereafter for creating the adaptive controller.

First we put $\underline{\widehat{\theta}}(t)$ obtained from (6) and (7) in the robotic model (1):

$$M(\underline{q},\underline{\widehat{\theta}})\underline{\ddot{q}} + \underline{f}(\underline{q},\underline{\dot{q}},\underline{\widehat{\theta}}) = \underline{u}, \tag{8}$$

and then by using the following feedback linearization controller:

$$\underline{u} = M(\underline{q},\underline{\widehat{\theta}})\left[\underline{\ddot{r}} + A_1\underline{e} + A_2\underline{\dot{e}}\right] + \underline{f}(\underline{q},\underline{\dot{q}},\underline{\widehat{\theta}}), \tag{9}$$

with the tracking error $\underline{e} = \underline{r} - \underline{q}$ and A_1, A_2 are two arbitrarily chosen matrices, the closed loop system as illustrated in Fig.2 becomes:

$$\underline{\dot{x}} = A\underline{x} + B\underline{\delta}_\theta \tag{10}$$

where $\underline{\delta}_\theta(t)$ indicates the approximation error of (9), i.e. of $\hat{F}(\underline{q},\underline{\dot{q}},\underline{v}) \approx F(\underline{q},\underline{\dot{q}},\underline{\ddot{q}})$ and

$$\underline{x} = \begin{pmatrix} \underline{e} \\ \underline{\dot{e}} \end{pmatrix},\; A = \begin{pmatrix} \mathbf{0}_n & I_n \\ -A_1 & -A_2 \end{pmatrix},\; B = \begin{pmatrix} \mathbf{0}_n \\ I_n \end{pmatrix}. \tag{11}$$

Furthermore, the approximation (5) shows that if $\underline{\dot{z}}$ bounded, then the smaller $0 < T \ll 1$ is chosen, the smaller $\underline{\delta}_\theta$ will be.

Finally, it is clearly realizable from (10) that if A is Hurwitz, what is definitely fulfilled through choosing suitably both matrices A_1, A_2, and $\underline{\delta}_\theta = \underline{0}$, then $\underline{x} \to \underline{0}$. For choosing simply A_1, A_2 it is possible to apply the well known pole assignment method of theory of linear control systems [10]. The lower negative poles are assigned, the better convergence rate to zero of $\underline{x}$ will be.

2.3. Analysis of control performance

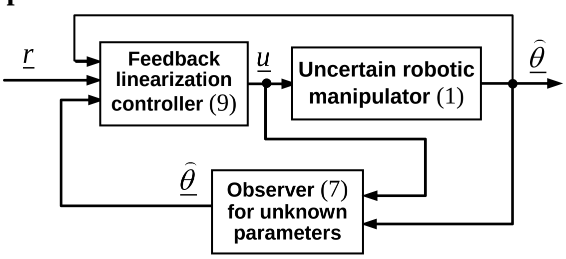

Figure 2. *Directly adaptive control system.*

Fig.2 depicts the structure of closed-loop control system by using proposed directly adaptive controller, of which the tracking performance is confirmed below.

Theorem: The directly adaptive controller (7),(9), as illustrated in Fig.2, with appropriately chosen matrices $A_1 = \mathrm{diag}(a_{1i}),\ A_2 = \mathrm{diag}(a_{2i}),\ a_{2i}^2 > a_{1i} > 0$, will drive the joint variables $\underline{q}$ and their velocity $\underline{\dot{q}}$ of the uncertain robotic manipulator (1), to an arbitrarily small neighbourhood $\mathcal{O}$ of desired references $\underline{r}, \underline{\dot{r}}$, determined by:

$$\mathcal{O} = \left\{ \underline{x} \in \mathrm{R}^{2n} \;\middle|\; \lim_{t\to\infty} |\underline{x}(t)| \le \frac{a_{\max}}{a_{\min}} \lim_{t\to\infty} |\underline{\delta}_\theta(t)| \right\} \tag{12}$$

where $\underline{x} = \mathrm{col}(\underline{e}, \underline{\dot{e}}),\ \underline{e} = \underline{r} - \underline{q},\ a_{\min} = \min_i(a_{1i}^2, a_{2i}^2 - a_{1i}),\ a_{\max} = \max_i(a_{1i}, a_{2i})$ **.**

Proof: All matrices

$$A = \begin{pmatrix} \mathbf{0}_n & I_n \\ -A_1 & -A_2 \end{pmatrix},\ Q = \begin{pmatrix} 2A_1^2 & \mathbf{0}_n \\ \mathbf{0}_n & 2(A_2^2 - A_1) \end{pmatrix},\ P = \begin{pmatrix} 2A_1A_2 & A_1 \\ A_1 & A_2 \end{pmatrix}$$

satisfy the Lyapunov equation $A^T P + PA = -Q$, where both P, Q are positive definite. Hence A is Hurwitz [11]. Using positive definite function $V(\underline{x}) = (\underline{x}^T P \underline{x})/2$ we have

$$\begin{aligned} \dot{V} &= -\underline{x}^T Q \underline{x} + \underline{\delta}_\theta^T (A_1\ , A_2)\underline{x} \\ &= -\sum_{i=1}^n a_{1i}^2 x_i^2 - \sum_{i=1}^n (a_{2i}^2 - a_{1i}) x_i^2 + \left(a_{11}x_1,\ \dots\ , a_{1n}x_n, a_{21}x_{n+1},\ \dots\ , a_{2n}x_{2n}\right)\underline{\delta}_\theta \\ &\le -a_{\min}|\underline{x}|^2 + a_{\max}|\underline{x}|\cdot|\underline{\delta}_\theta| = -\left(a_{\min}|\underline{x}| - a_{\max}|\underline{\delta}_\theta|\right)|\underline{x}| \end{aligned}$$

which implies that $\dot{V} < 0$ as long as $a_{\min}|\underline{x}| - a_{\max}|\underline{\delta}_\theta| > 0$. Hence, the vector of tracking errors $\underline{x}(t)$ will decrease until the condition $a_{\min}|\underline{x}| \le a_{\max}|\underline{\delta}_\theta|$ is reached, which coincides with (12). ♦

3. NUMERICAL SIMULATION

To verify the control performance of the proposed adaptive controller we will apply this to tracking control of the uncertain 3-link cylindrical arm robot as exhibited physically in Fig.3, which possesses the following mathematical model [1, 5, 12]:

$$\begin{pmatrix} J + m_2 q_3 & 0 & 0 \\ 0 & m_1 + m_2 & 0 \\ 0 & 0 & m_2 \end{pmatrix} \underline{\ddot{q}} + \begin{pmatrix} 2m_2 q_3 \dot{q}_3 \dot{q}_1 \\ (m_1 + m_2)g \\ -m_2 q_3 \dot{q}_1^2 \end{pmatrix} = \underline{u}$$

where both arm masses m_1, m_2 and the inertia of the base link J are additionally assumed to be unknown. Denote all of them as system uncertainties with $\theta_1 = m_1,\ \theta_2 = m_2$ and $\theta_3 = J$, respectively, we obtain with $\underline{u} = (u_1, u_2, u_3)^T$

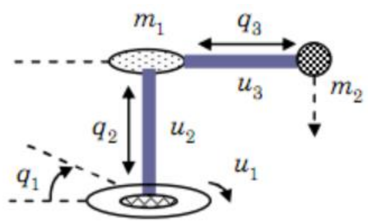

Figure 3. *Three-link cylindrical arm robot.*

$$\begin{cases} \theta_2\left(2\dot{q}_3\dot{q}_1+\ddot{q}_1\right)q_3+\theta_3\ddot{q}_1=u_1 \\ (\theta_1+\theta_2)(\ddot{q}_2+g)=u_2 \\ \theta_2(\ddot{q}_3-q_3\dot{q}_1^2)=u_3 \end{cases}$$

which implies

$$F(\underline{q},\underline{\dot{q}},\underline{\ddot{q}})=\begin{pmatrix} 0 & \left(2\dot{q}_3\dot{q}_1+\ddot{q}_1\right)q_3 & \ddot{q}_1 \\ \ddot{q}_2+g & \ddot{q}_2+g & 0 \\ 0 & \ddot{q}_3-q_3\dot{q}_1^2 & 0 \end{pmatrix}$$

what has full rank as long as $\ddot{q}_1(\ddot{q}_2+g)(\ddot{q}_3-q_3\dot{q}_1^2)\neq 0$. This requirement is obligatory for the vector of uncertainties $\underline{\theta}=\left(\theta_1,\theta_2,\theta_3\right)^T$ being identified from collected system data of the robot.

In the simulation three desired references $\underline{r}(t)=\left(r_1(t),r_2(t),r_3(t)\right)^T$ are assigned with the following different periodic function

$$\begin{aligned} r_1(t)&=\cos(0.2\pi t)+0.5\sin(0.6\pi t) \\ r_2(t)&=0.5\sin(0.2\pi t)+0.75\sin(0.6\pi t) \\ r_3(t)&=0.2\cos(0.2\pi t)-0.2\sin(0.6\pi t). \end{aligned} \tag{13}$$

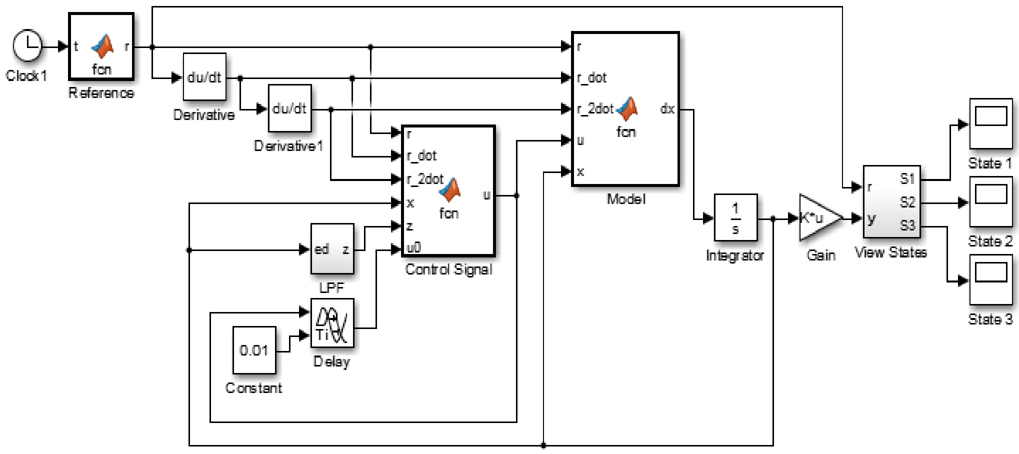

Figure 4. *Simulation program.*

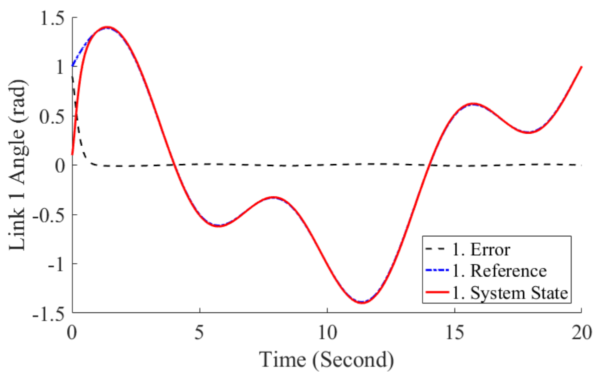

Figure 5. *Tracking performance of the first joint variable* q_1.

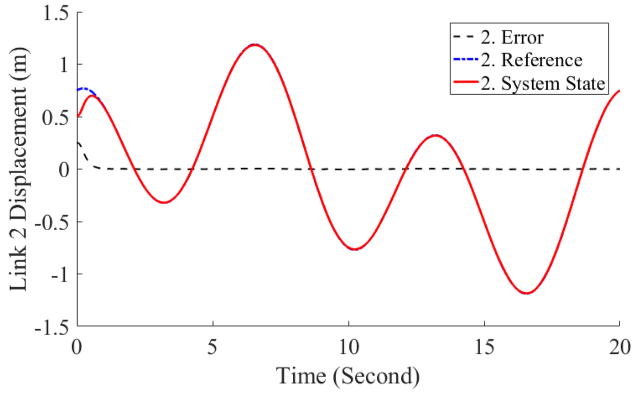

Figure 6. *Tracking performance of the second joint variable* q_2.

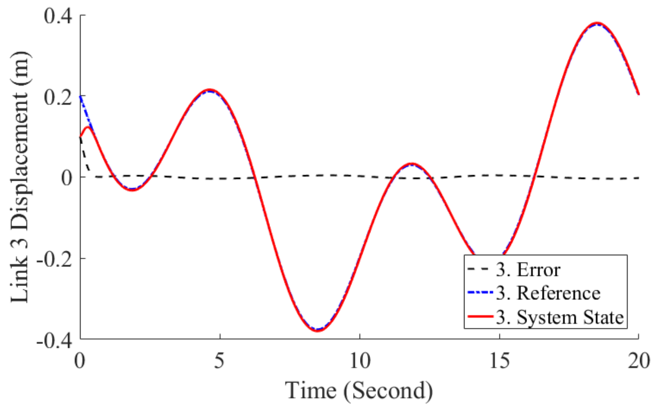

Figure 7. *Tracking performance of the third joint variable* q_3.

The simulation program is created in MatLab-Simulink with a chosen time constant $T = 0.01s$ for low-pass filter (4), as well as for derivation approximating. This program is given particularity in Fig.4 with all function blocks in it written in m-file.

All with this program obtained simulation results are depicted in Fig.5 – Fig.7, where each figure demonstrates separately a pair q_i, r_i, $i = 1,2,3$. They illustrate that all robot joint variables $\underline{q}$ have tracked quickly to their references $\underline{r}$ (just after $2s$ from the beginning) as expected, in spite of the presence of uncertain robotic parameters $\underline{\theta}$.

4. CONCLUSIONS

The main novelty of this paper is here a simple approach of the directly adaptive control method for robotic manipulators with time varying uncertain parameters had been proposed, in comparison with other existing adaptive control methods for uncertain, non-autonomous systems. In this method, first the time varying uncertain parameters of robotics are identified online from collected data of the plant and then they are used as control parameters for the feedback linearization controller. Based on this, the proposed controller could be seen as a data-driven one. It means that in such systems the identification of the process model and/or the design of the controller are based entirely on experimental data collected from the plant [13, 14]. The smaller data collecting time T is chosen, the precise approximation of signal derivation will be.

The numerical simulation affirmed the tracking performance of the proposed adaptive control method as desired. In consequence, this adaptive controller could now be applicable in practice.

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TÓM TẮT

Điều khiển thích nghi trực tiếp cho tay máy robot có nhiễu bất định theo thời gian

Bài báo đề xuất một cách tiếp cận điều khiển thích nghi trực tiếp cho tay máy robot, nó có thể ứng dụng để điều khiển cho tay máy robot có các tham số nhiễu bất định theo theo thời gian. Cách tiếp cận này dựa trên việc kết hợp phương pháp phản hồi tuyến tính hóa kinh điển và đề xuất quan sát trực tuyến dựa trên mô hình cho các nhiễu bất định. Chất lượng điều khiển bám của bộ điều khiển thích nghi trực tiếp đề xuất này đã được kiểm chứng bằng phương pháp lý thuyết và mô phỏng số.

Từ khóa: Điều khiển thích nghi; Điều khiển tay máy Robot; Quan sát bất định theo thời gian.