Terminal sliding mode control for longitudinal stabilization of fixed-wing UAV

Nguyen Vu*

Faculty of Electrical Engineering, Hanoi University of Industry (Minh Khai, Bac Tu Liem, HN).
*Corresponding author: vutudonghoa@yahoo.com.vn
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ABSTRACT

Terminal sliding mode control is one of the modern control methods and has wide application in practice. In order to apply this method in UAV control, this paper presents an algorithms of applying non-singular terminal sliding mode control for longitudinal stability of fixed-wing UAV. Sliding modes occur on both the sliding surface and its derivative, the convergence time of the sliding variables is also calculated explicitly to ensure that the method can be applied to synthesize controllers in systems that require fixed stabilization time. The process of synthesizing control laws is strictly mathematically guaranteed. The simulation in Matlab shows the research results visually.

Keywords: Sliding mode control; Terminal sliding mode control; Non-singular TSM; UAV.

1. INTRODUCTION

Sliding mode control, also known as variable structure control, is a modern non-linear control method. Currently, the sliding mode control is widely interested and used in practice [1] due to its stability and invariance against system parameter changes and external interference. Sliding mode occurs when control commands are synthesized to bring the system's state variables towards the sliding surface and keep it on this surface. The equations of the sliding surface are chosen so that when the states of the system are on the sliding surface, they will converge to the origin stably. However, in the tracking control systems, the asymptotic stability does not guarantee the system quality once the value to follow changes rapidly. This requires a fast convergence of state variables. To meet this requirement, a fast suboptimal sliding surface selection method has been proposed [2]. However, this method uses a linear sliding surface, so it only ensures the asymptotic stability of the system. As the state of the system approaches the origin, the convergence rate decreases, and the time to the equilibrium point is also unknown. To overcome this phenomenon, sliding mode control systems using nonlinear sliding surfaces have been proposed, including terminal sliding mode control (TSM). Terminal sliding mode control will ensure that the convergence time of the state variables is limited, thus greatly improving the system transition. TSM control has been applied widely and mentioned in literature [3-6]. However, the disadvantage of TSM control is that the system can be degraded because the control instruction contains a state variable in the denominator. To overcome this drawback, the non-singular terminal sliding mode control (NSTMS) was introduced. Currently, NSTSM control has been applied in many fields and published in many works [7-10]. With its advantages, NSTSM control can also be applied to control of fixed-wing UAV.

This paper will briefly present TSM control and NSTSM control, and apply NSTSM control to longitudinal stability control for fixed -wing UAV.

2. PROBLEM

2.1. Terminal slide mode control concept

The terminal sliding mode control is a sliding mode control that uses a nonlinear sliding surface [4- 6]. To illustrate the TSM control below, a quadratic system will be used.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + gu \end{cases} \quad \text{with } g \neq 0 \tag{1}$$

To synthesize the controller for system (1), use a nonlinear sliding variable:

$$s = x_2 + c x_1^{p/q} \tag{2}$$

Where, *c* is a positive coefficient and *q>p>0*. For convenience of taking derivatives of s, p and q are chosen to be odd numbers.

In order to bring the system states to the sliding surface and keep them on this sliding surface, the selected control command consists of two components, the discontinuous component responsible for bringing the system states to the sliding surface and the continuous component responsible for keeping them on this surface:

$$u = u_c + u_d \tag{3}$$

Where:

$$\begin{cases} u_c = -g^{-1}\left(f(x) + c\dfrac{p}{q}\dfrac{\dot{x}_1}{x_1^{(q-p)/q}}\right) \\ u_d = -K sign(s) \end{cases} \tag{4}$$

By using controls from (3) and (4), system (1) will operate in sliding mode on sliding surface (2). This can be proved by Lyapunov function:

$$V = \frac{1}{2}s^2 \tag{5}$$

Take derivative of (5):

$$\dot{V} = s\dot{s} = s\left(\dot{x}_2 + c\frac{p}{q}\frac{\dot{x}_1}{x_1^{(q-p)/q}}\right) = s\left(f(x) + gu + c\frac{p}{q}\frac{\dot{x}_1}{x_1^{(q-p)/q}}\right) \tag{6}$$

Apply (3) and (4) to (6), one can obtain:

$$\dot{V} = s\dot{s} = s(-K sign(s)) \tag{7}$$

Necessary condition for the existence of sliding mode, or $\dot{V} < 0$, such that:

$$K > 0 \tag{8}$$

Then there will be sliding mode on sliding surface (2).

When system in the sliding mode, from (2) one can obtain:

$$s = x_2 + c x_1^{p/q} = 0,$$

Or:

$$\dot{x}_1 = -c x_1^{p/q} \tag{9}$$

Rewrite (9) and switch side, one can get:

$$\frac{dx_1}{x_1^{p/q}} = -c dt \tag{10}$$

Assume that at the initial time t_0, when s = 0, $x_1(t_0) = x_1(0)$. When x_1 is reaching 0, then according to (9), $\dot{x}_1 = x_2$ is also reaching 0. The time for x_1 to reach to the origin is determined as follows:

$$\int_{x_1(0)}^{0} \frac{dx_1}{x_1^{p/q}} = -c \int_{t_0}^{t_0+t_s} dt \tag{11}$$

Integrating (11), getting:

$$\frac{q}{q-p}x_1(0)^{\frac{q-p}{q}} = ct_s$$

or:
$$t_s = \frac{q}{c(q-p)}x_1(0)^{\frac{q-p}{q}} \tag{12}$$

t_s in (12) is the time required for the system states to return to the origin after the sliding mode occurs.

In fact, when $x_1 \to 0$, $\left| c\frac{p}{q}\frac{\dot{x}_1}{x_1^{(q-p)/q}} \right|$ may approach ∞, and $u_c \to \infty$ then the condition (7) not satisfied, so existence of sliding mode $\dot{V} < 0$ may not be satisfied, i.e. the terminal sliding mode is degraded. To overcome this, non-singular terminal sliding mode control was introduced (NSTSM).

2.2. Non-singular terminal sliding mode control (NSTSM)

To avoid the degradation phenomenon in 2.1 using a new sliding variable for system [7-10] (1):

$$s = x_2^{q/p} + cx_1 \quad \text{with } q>p \tag{13}$$

In system (1) by using sliding variable (13), when sliding mode occurs, the system states will satisfy following equation:

$$x_2^{q/p} = -cx_1 \tag{14}$$

Or after taking the *p/q* exponent on both sides:

$$x_2 = \dot{x}_1 = -c^{p/q}x_1^{p/q} \tag{15}$$

In (15), it can be seen that t_s - Convergence time or the time required for the state of the system to return to the origin after the sliding mode occurs, whose value is:

$$t_s = \frac{q}{c^{p/q}(q-p)}x_1(0)^{\frac{q-p}{q}} \tag{16}$$

To ensure the sliding mode exists on the sliding surface (13), the control input is selected according to (3):

$$\begin{cases} u_c = -g^{-1}f(x) - \dfrac{c}{g}\dfrac{p}{q}x_2^{\frac{2p-q}{p}} \\ u_d = -Ksign(s) \end{cases} \tag{17}$$

Then, by choosing Lyapunov function $V = \frac{1}{2}s^2$, one can obtain:

$$\dot{V} = s\dot{s} = s(\frac{q}{p}x_2^{\frac{q-p}{p}}\dot{x}_2 + cx_2) = s\left[\frac{q}{p}x_2^{\frac{q-p}{p}}(f(x)+gu) + cx_2\right] \tag{18}$$

Apply (3) and (17) to (18), getting:

$$\dot{V} = s\dot{s} = s(-Kg(\frac{q}{p}x_2^{\frac{q-p}{p}})sign(s)) \tag{19}$$

Necessary condition for the existence of sliding mode, or $\dot{V} < 0$, is $K > 0$

In (17), to ensure that u_c has a limited value, or that system is not degraded then exponent of x_2, $\frac{2p-q}{q}$ must be positive, or $2p > q$. Combined with the condition in expression (13), getting the condition that the sliding surface (13) ensures the non-singular terminal sliding mode is:

$$2p > q > p \tag{20}$$

2.3. Synthesis of non-singular terminal sliding mode controller in longitudinal stabilization of fixed wing UAV

Dynamic of longitudinal stabilization of fixed wing UAV can be described by the transfer function with the input as the elevator and the output as the altitude of UAV [11]:

$$G(s) = \frac{b}{s^4 + a_4 s^3 + a_3 s^2 + a_2 s + a_1} \tag{21}$$

Rewrite dynamic of system (21) in state space form, where altitude is set to z_1, then its derivatives are also set as: $z_2 = \dot{z}_1, z_3 = \dot{z}_2, z_4 = \dot{z}_3$, set $\mathbf{z} = (z_1 \quad z_2 \quad z_3 \quad z_4)^T$. Then system (23) has following dynamic equations:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 & -a_4 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix} u \tag{22}$$

Or: $$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}u$$

Set $e_1 = z_1 - z_{1d}$ as the altitude tracking error, z_{1d} as desired altitude, and also its derivatives: $e_2 = \dot{e}_1, e_3 = \dot{e}_2, e_4 = \dot{e}_3$. Set:

$$\mathbf{e} = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 \end{bmatrix}^T,$$

$$\mathbf{z_d} = \begin{bmatrix} z_{1d} & z_{2d} & z_{3d} & z_{4d} \end{bmatrix}^T, z_{2d} = \dot{z}_{1d}, z_{3d} = \dot{z}_{2d}, z_{4d} = \dot{z}_{3d}.$$

$$\mathbf{z} = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T, z_2 = \dot{z}_1, z_3 = \dot{z}_2, z_4 = \dot{z}_3.$$

Then, dynamic of system (24) can be written based on tracking error as:

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}u - \dot{\mathbf{z}}_\mathbf{d} + \mathbf{A}\mathbf{z}_\mathbf{d} \tag{23}$$

To apply NSTSM control for system (25), choose sliding surface as follows:

$$\sigma = e_3 + c_2 e_2 + c_1 e_1, \quad c_1 > 0, c_2 > 0 \tag{24}$$

Then: $$\dot{\sigma} = \dot{e}_3 + c_2 \dot{e}_2 + c_1 \dot{e}_1 = e_4 + c_2 e_3 + c_1 e_2$$

The control task is to synthesize the controller for system (25) such that there exists a sliding mode on the sliding surface (26). To accomplish this task, use the 2nd order sliding mode with a non-singular terminal sliding surface (13):

$$s = \dot{\sigma}^{q/p} + c\sigma = 0 \tag{25}$$

To synthesize the control command so that the state of the system lies on the sliding surface (25), use the Lyapunov function $V = \frac{1}{2} s^2$

Then: $$\dot{V} = s\dot{s}$$

$$\dot{V}=s\left(\frac{q}{p}\dot{\sigma}^{\frac{q-p}{p}}\ddot{\sigma}+c\dot{\sigma}\right)=s\left(\frac{q}{p}(e_4+c_2e_3+c_1e_2)^{\frac{q-p}{p}}(\dot{e}_4+c_2e_4+c_1e_3)+c(e_4+c_2e_3+c_1e_2)\right)$$
$$\dot{V}=s\left[\frac{q}{p}\dot{\sigma}^{\frac{q-p}{p}}\left(-\sum_{i=1}^{4}a_ie_i-\sum_{i=1}^{4}a_ix_{id}-\dot{x}_{4d}+bu+c_2e_4+c_1e_3\right)+c\dot{\sigma}\right] \quad (26)$$

Similar to (17), choose :

$$\begin{cases} u_c=-\frac{1}{b}\left(-\sum_{i=1}^{4}a_ie_i-\sum_{i=1}^{4}a_ix_{id}-\dot{x}_{4d}+c_2e_4+c_1e_3\right)-\frac{c}{b}\frac{p}{q}\dot{\sigma}^{\frac{2p-q}{q}} \\ u_d=-\frac{1}{b}Ksign(s) \end{cases} \quad (27)$$

Then: $\dot{V}=s\dot{s}=s(-Kgsign(s))=-K|s|<0$, sliding mode will occur on sliding surface (25).

When (25) is satisfied, $\sigma,\dot{\sigma}$ will approach 0 in finite time, determined by formula (16):

$$t_s=\frac{q}{c^{\frac{p}{q}}(q-p)}\sigma(0)^{\frac{q-p}{q}} \quad (28)$$

After the sliding mode on the sliding surface (24) occurs, the system (23) will stabilize asymptotically. The coefficients are selected by the pole placement method. The coefficients *q*, *p* and *c* are design parameters, their influence on control quality will be evaluated through the simulation results below.

3. SIMULATION AND RESULTS

The presentation algorithm will be verified through simulation. The parameters of the UAV are taken according to [11, 12]:

$$G(s)=\frac{61.68}{s^4+0.8699s^3+2.112s^2+1.442s+0.2138} \quad (29)$$

The simulation conditions are two cases, the UAV climbs 200 m and descends 200 m along the transition line of the fourth step.

Choose sliding surface:

$$s=\dot{\sigma}^{9/5}+50\sigma \quad (30)$$

The simulation results are presented below.

Figures 1, 2, 3, 4, 5 show the graph in descending mode.

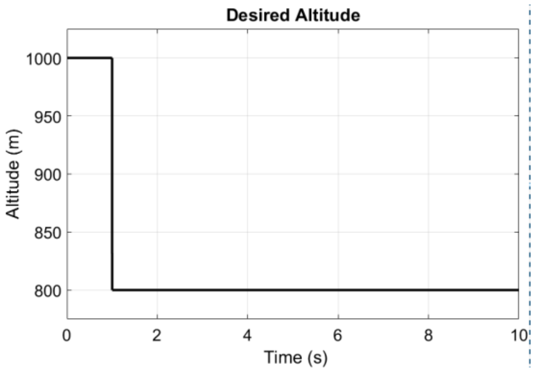

Figure 1. *Desired altitude.*

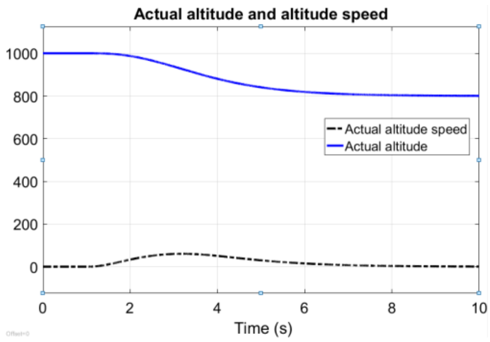

Figure 2. *Actual altitude and altitude change speed.*

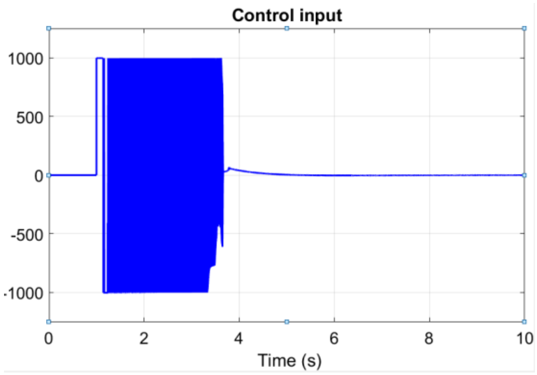

Figure 3. *Control input.*

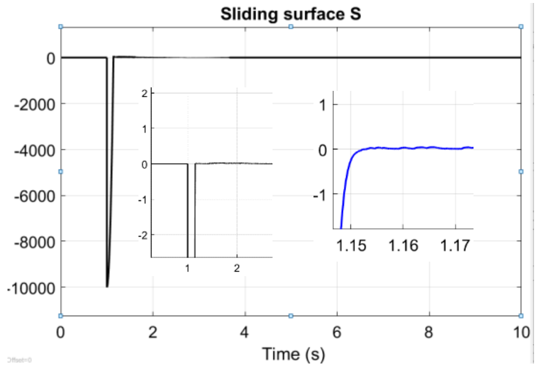

Figure 4. *Sliding surface s.*

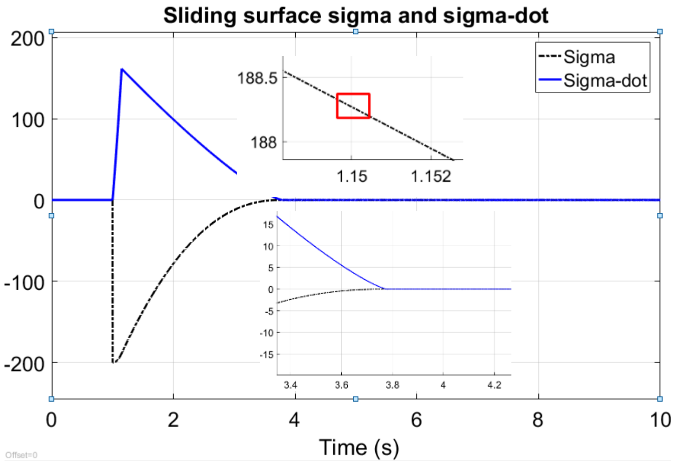

Figure 5. *Sliding surfaces* $\sigma,\dot{\sigma}$.

Figure 4 shows that $s \to 0$ at time $t_0 = 1.15$ s. At that time $\sigma(0) = 188.25$ (figure 5). According to (28) convergence time t_s will be:

$$t_s = \frac{9}{50^{\frac{5}{9}}(9-5)} 188.25^{\frac{9-5}{9}} = 2.63s \tag{31}$$

Time when $\sigma,\dot{\sigma}$ converge to 0 is $t = t_0 + t_s = 1.15 + 2.63 = 3.78s$

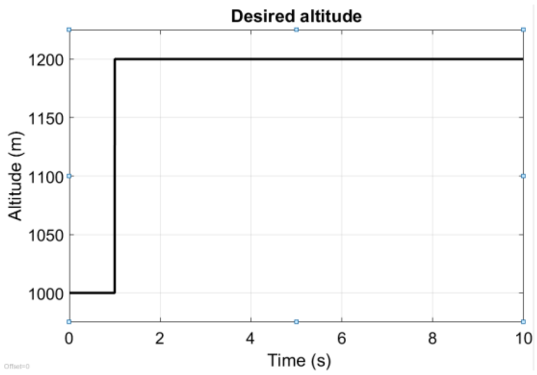

Figure 6. *Desired altitude.*

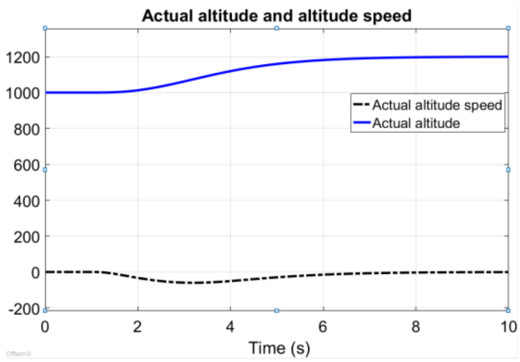

Figure 7. *Actual altitude and altitude change speed.*

Figure 5 shows that $\sigma \to 0, \dot{\sigma} \to 0$ at time t=3.78 s as calculated above. So the simulation results verify the correctness of the method.

Figures 6, 7, 8, 9, 10 show the graph in increasing mode.

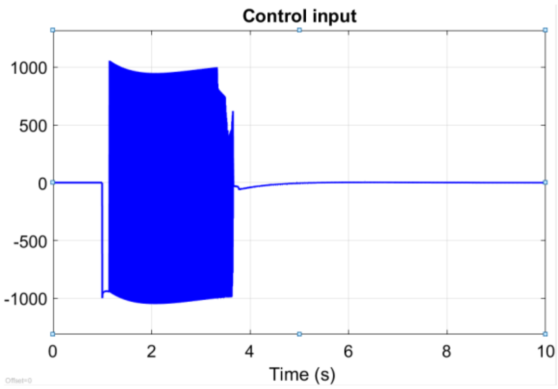

Figure 8. *Control input.*

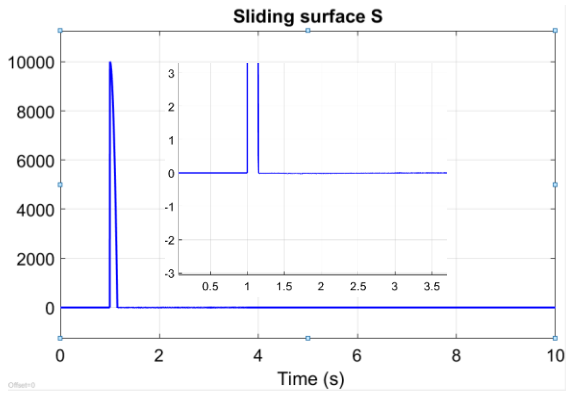

Figure 9. *Sliding surface s.*

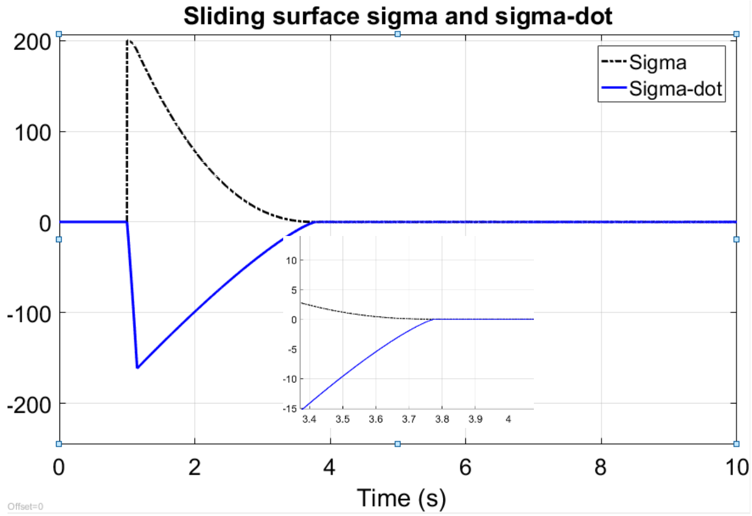

Figure 10. *Sliding surfaces* $\sigma, \dot{\sigma}$.

The figures above show that, with the proposed algorithm, the UAV actual altitude aproach the desired altitude with the static error after 8s is zero, the UAV keeps a stable altitude.

As shown on the simulation results, the sliding surface s goes to 0 first, then both sliding surface $\sigma, \dot{\sigma}$ goes to 0, and finally the state variables of the system. Thus, the system operates in sliding mode.

4. CONCLUSIONS

Non-singular terminal sliding mode control is an effective tool in control system synthesis. When the variables in the sliding surface are not state variables but a sliding surface and its derivative, the non-singular terminal sliding control will also ensure the 2nd order sliding mode, i.e. the sliding mode occurs both on the sliding surface and its derivative. This is evident in the example of a longitudinal control system for fixed wing UAV. The results presented in the paper can be used to synthesize different control systems effectively when there is a requirement for the system's stability time.

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TÓM TẮT

Điều khiển chế độ trượt Terminal trong ổn định kênh dọc của UAV cánh bằng

Bài báo trình bày phương pháp ứng dụng điều khiển chế độ trượt Terminal không suy biến bậc hai để điều khiển ổn định kênh dọc cho UAV cánh bằng. Các chế độ trượt xảy ra trên cả mặt trượt và đạo hàm của nó, đồng thời, thời gian hội tụ của các biến trượt cũng được tính toán tường minh đảm bảo phương pháp có thể được áp dụng để tổng hợp bộ điều khiển trong các hệ thống yêu cầu thời gian ổn định. Quá trình tổng hợp luật điều khiển được bảo đảm toán học chặt chẽ. Mô phỏng trong Matlab thể hiện trực quan kết quả nghiên cứu.

Từ khoá: Điều khiển chế độ trượt; Điều khiển chế độ trượt Terminal; TSM không suy biến; UAV; NSTSM.