Adaptive sliding mode control of a three-mass elastic electromechanical system under parameter uncertainty and external disturbances

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ABSTRAST

This paper enhances the control quality of a three-mass elastic electromechanical system under parameter uncertainties and external disturbances. For the control system design, the paper considers the control object as a three-mass elastic electromechanical system with three interdependent control loops. A complete mathematical model of the object is then established. A robust adaptive sliding controller is proposed for the three-mass elastic system to counteract the influence of unknown external disturbances and model uncertainties caused by parameter uncertainty. The highlight of the adaptive sliding controller is its mechanism for adaptively adjusting the controller gain by estimating the upper bound of the combined external disturbances and system uncertainties. Thus, the controller ensures compensation for the influence of the above-mentioned negative factors. The stability of the proposed adaptive control system is investigated using Lyapunov stability theory. Simulation results on MATLAB/Simulink comparing the performance of the proposed adaptive sliding mode controller with the conventional PID controller demonstrate the effectiveness of the proposed approach.

Keywords: Elastic system; Three-mass system; Adaptive sliding mode control; Parameter uncertainty; External time-varying disturbances.

1. INTRODUCTION

In advanced industrial applications, the control of complex nonlinear electromechanical systems, particularly multi-mass drive systems exhibiting elastic deformation, is pivotal for optimizing the performance of electromechanical assemblies and high-speed motion devices, including ultra-precise metal-cutting machines, industrial robots, unmanned aerial vehicles (UAVs), and mobile platforms [1]. Elastic deformations in mechanical transmission often compromise control quality, inducing undesirable oscillations and elevating the risk of mechanical wear and failure. Traditional control approaches, such as PID control, prove inadequate under conditions of parameter uncertainty, elastic deformation and disturbances, underscoring the need for adaptive and nonlinear robust control strategies.

Many prior studies have established a critical foundation for solving the control challenges of multi-mass elastic systems. Kazakov [1] developed adaptive control using reference models and neural networks for two- and three-mass elastic systems, achieving notable performance under uncertain conditions. Cychowski et al. [2] introduced model predictive control for a three-mass system, though with limited effectiveness. Huang et al. [3] explored adaptive sliding mode control for nonlinear systems with uncertain parameters, providing a theoretical basis but lacking application to three-mass electromechanical systems. Lanh et al. [4] synthesized adaptive control for opto-mechanical multi-mass drive systems, addressing uncertainty and friction, yet omitting complex external disturbances. Meanwhile, domestic researchers have predominantly concentrated on control challenges in two-mass elastic systems. Hien et al. [5] proposed a fractional-order sliding mode controller for underactuated systems with elastic joints. Thanh et al. [6] developed an adaptive-modal control scheme for a two-mass elastic system. Dong et al. [7] proposed an adaptive backstepping controller for a two-mass elastic system taking into account backlash, dead-zone, and Coulomb friction. Thus, it can be seen that developing a structurally simple yet high-performance adaptive sliding mode controller for three-mass elastic systems under parameter uncertainty and external disturbances is both critically necessary and urgently required.

Recent advances have expanded the development of adaptive sliding mode control. Van and Ge [8] developed finite-time adaptive sliding mode control for nonlinear robotic systems, managing disturbances and uncertainties. Zhang et al. [9] investigated adaptive sliding mode control for precision machinery under nonlinear disturbances. Merabet [10] advanced control strategies for electric drive systems in dynamic environments, while Han and Lee [11] proposed adaptive dynamic surface control for high-precision motion systems with uncertainties. These studies highlight the demand for optimized solutions for three-mass elastic systems.

This paper proposes a robust adaptive sliding mode controller for a three-mass elastic electromechanical system, enhancing position tracking under parameter uncertainty and external disturbances. Unlike the adaptive sliding mode controllers with complex structures mentioned previously, the proposed controller features a simplified architecture suitable for practical implementation while ensuring stability and control quality. This distinction manifests in the adaptation mechanism: while typical adaptive controllers employ parameter estimation to compensate for uncertainties and disturbances, the proposed approach adaptively adjusts the controller gain by estimating the upper bound of the uncertainties and disturbances. The robust stability of the proposed adaptive system is theoretically proven using Lyapunov function method. The next section presents comparative simulation results between the proposed sliding mode controller and a PID controller for the three-mass system, demonstrating the superior performance of the proposed approach. The paper concludes with a summary of key findings and conclusions.

2. MATHEMATICAL MODELING AND CONTROL SYSTEM DESIGN

2.1. Mathematical modeling of the three-mass elastic electromechanical system

Consider a three-mass elastic system, as illustrated in figure 1. The design of a standard control system for the rotational motion of a three-mass elastic electromechanical object is typically framed within a tracking system comprising three interdependent control loops.

The mathematical equations for the three-mass elastic electromechanical system, incorporating current I, motor shaft speed ω_1, and load position q control loops, are expressed as [1]:

$$\begin{cases} \dot{q} = \omega_3;\ \dot{\omega}_3 = J_3^{-1} M_{\text{dh2}};\ \dot{M}_{\text{dh2}} = p_{02}\left(\omega_2 - \omega_3\right); \\ \dot{\omega}_2 = J_2^{-1}\left(M_{\text{dh1}} - M_{\text{dh2}}\right);\ \dot{M}_{\text{dh1}} = p_{01}\left(\omega_1 - \omega_2\right); \\ \dot{\omega}_1 = J_1^{-1}\left(M - M_{\text{dh1}}\right);\ L\dot{I} = U - E - RI; \\ M = k_M I;\ E = k_E \omega_1;\ U = k_A u_D; \\ u_D = \beta_D\left(u_T - k_D I\right);\ u_T = \beta_T\left(u_V - k_T \omega_1\right); \\ u_V = \beta_V\left(u_\Sigma - k_V q\right);\ u_\Sigma = u_0 + u_a; u_0 = q_d, \end{cases} \tag{1}$$

Where u_Σ – input signal to the position control loop, comprising the desired load angle q_d and adaptive control signal u_a; q – load position angle; ĐV, ĐT, ĐD – proportional controllers for position, speed, and current loops; KĐ – power amplifier; ĐC – motor; CV, CT, CD – position, speed, and current sensors; u_V, u_T, u_D – control signals for position, speed, and current loops; β_V, β_T, β_D – amplification coefficients of the position, speed, and current controllers; k_V, k_T, k_D – transmission coefficients of position, speed, and current sensors; k_A – transmission coefficient of the power amplifier; L, R – inductance and resistance of the motor armature; k_E, k_M – motor structural constants (with constant flux and negligible inductance); E – back electromotive force; U – armature voltage; I – armature current; ω_1, ω_2 ω_3 – angular velocity of the motor, gearbox, and load; J_1, J_2, J_3 – moments of inertia of the three masses; M – motor electromagnetic torque; M_{dh1}, M_{dh2} – elastic torques between masses 1-2 and 2-3; p_{01}, p_{02} – stiffness coefficients of the drive shaft between motor-gearbox and gearbox-load.

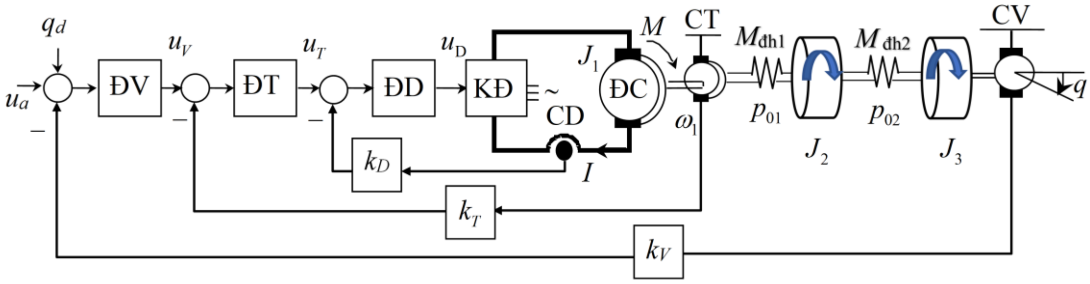

Figure 1. *Schematic diagram of the three-loop interdependent control system for a three-mass elastic structure.*

To simplify the system description (1), the following parameters are defined:

$$\begin{cases} a_1 = J_3^{-1};\ a_2 = p_{02};\ a_3 = J_2^{-1};\ a_4 = p_{01};\ a_5 = -J_1^{-1}; \\ a_6 = k_M J_1^{-1};\ a_7 = -k_A k_V \beta_D \beta_T \beta_V L^{-1};\ a_8 = -\left(k_A k_T \beta_D \beta_T + k_E\right) L^{-1}; \\ a_9 = -\left(k_A k_D \beta_D + R\right) L^{-1};\ b = k_A \beta_D \beta_T \beta_V L^{-1}. \end{cases} \tag{2}$$

Thus, the system equations (1) can be rewritten as:

$$\begin{cases} \dot{q} = \omega_3;\ \dot{\omega}_3 = a_1 M_{dh2};\ \dot{M}_{dh2} = a_2\left(\omega_2 - \omega_3\right); \\ \dot{\omega}_2 = a_3\left(M_{dh1} - M_{dh2}\right);\ \dot{M}_{dh1} = a_4\left(\omega_1 - \omega_2\right); \\ \dot{\omega}_1 = a_5 M_{dh1} + a_6 I;\ \dot{I} = a_7 q + a_8 \omega_1 + a_9 I + b u_\Sigma. \end{cases} \tag{3}$$

The starting point for developing control systems is a three-mass elastic electromechanical system with three interdependent control loops for armature current I, motor shaft speed ω_1, and load position q.

2.2. Synthesis of the adaptive sliding mode control law for position tracking

To synthesize the adaptive sliding mode control law, the system equations (3) are first transformed into canonical form. Let $x_1 = I$, $x_2 = \omega_1$, $x_3 = M_{dh_1}$, $x_4 = \omega_2$, $x_5 = M_{dh2}$, $x_6 = \omega_3$, $x_7 = q$ yielding:

$$\begin{cases} \dot{x}_1 = a_9 x_1 + a_8 x_2 + a_7 x_7 + bu; \\ \dot{x}_2 = a_6 x_1 + a_5 x_3; \\ \dot{x}_3 = a_4 x_2 - a_4 x_4; \\ \dot{x}_4 = a_3 x_3 - a_3 x_5; \\ \dot{x}_5 = a_2 x_4 - a_2 x_6; \\ \dot{x}_6 = a_1 x_5; \\ \dot{x}_7 = x_6. \end{cases} \tag{4}$$

Let $\mathbf{X} = \left[y, \dot{y}, \ddot{y}, ..., y^{(6)}\right]^{\mathrm{T}} \in R^n$, $n = 7$. The output variable is $y = x_7$, so the derivatives of the position output are as follows:

$$\dot{y} = x_6\,;\ \ddot{y} = a_1x_5\,;\ y^{(3)} = a_1a_2x_4 - a_1a_2x_6 = s_1x_4 + s_2x_6\,;$$

$$y^{(4)} = a_1a_2a_3x_3 - \left(a_1a_2a_3 + a_1^2a_2\right)x_5 = s_3x_3 + s_4x_5\,;$$

$$y^{(5)} = a_1a_2a_3a_4x_2 - \left(a_1a_2a_3a_4 + a_1^2a_2^2 + a_1a_2^2a_3\right)x_4 + \left(a_1^2a_2^2 + a_1a_2^2a_3\right)x_6$$
$$= s_5x_2 + s_6x_4 + s_7x_6\,;$$

$$y^{(6)} = a_1a_2a_3a_4a_6x_1 + \left(a_1a_2a_3a_4a_5 - a_1a_2a_3^2a_4 - a_1^2a_2^2a_3 + a_1a_2^2a_3^2\right)x_3 +$$
$$+\left(a_1a_2a_3^2a_4 + a_1^2a_2^2a_3 + a_1a_2^2a_3^2 + a_1^3a_2^2 + a_1^2a_2^2a_3\right)x_5$$
$$= s_8x_1 + s_9x_3 + s_{10}x_5\,;$$

$$y^{(7)} = s_8\left(a_9x_1 + a_8x_2 + a_7x_7 + bu\right) + s_9\left(a_4x_2 - a_4x_4\right) + s_{10}\left(a_2x_4 - a_2x_6\right)$$
$$= s_{11}x_1 + s_{12}x_2 + s_{13}x_4 + s_{14}x_6 + s_{15}x_7 + bs_8u,$$

Where:

$s_1 = a_1a_2$; $s_2 = -a_1a_2$; $s_3 = a_1a_2a_3$; $s_4 = -\left(a_1a_2a_3 + a_1^2a_2\right)$; $s_5 = a_1a_2a_3a_4$;

$s_6 = -\left(a_1a_2a_3a_4 + a_1^2a_2^2 + a_1a_2^2a_3\right)$; $s_7 = a_1^2a_2^2 + a_1a_2^2a_3$; $s_8 = a_1a_2a_3a_4a_6$;

$s_9 = a_1a_2a_3a_4a_5 - a_1a_2a_3^2a_4 - a_1^2a_2^2a_3 + a_1a_2^2a_3^2$; $s_{10} = a_1a_2a_3^2a_4 + a_1^2a_2^2a_3 + a_1a_2^2a_3^2 + a_1^3a_2^2 + a_1^2a_2^2a_3$;

$s_{11} = s_8a_9$; $s_{12} = s_8a_8 + s_9a_4$; $s_{13} = s_{10}a_2 - s_9a_4$; $s_{14} = -s_{10}a_2$; $s_{15} = s_8a_7$.

The mathematical model of the system (3) in canonical form is:

$$\dot{\mathbf{X}} = \mathbf{f}_0\left(\mathbf{X}\right) + \mathbf{b}_0u + \mathbf{d}, \tag{5}$$

Where $\mathbf{f}_0$ and $\mathbf{b}_0$ are the known components of the system with nominal parameter values, calculated as:

$$\mathbf{f}_0 = \begin{bmatrix} x_6 \\ a_1x_5 \\ s_1x_4 + s_2x_6 \\ s_3x_3 + s_4x_5 \\ s_5x_2 + s_6x_4 + s_7x_6 \\ s_8x_1 + s_9x_3 + s_{10}x_5 \\ s_{11}x_1 + s_{12}x_2 + s_{13}x_4 + s_{14}x_6 + s_{15}x_7 \end{bmatrix} \in R^n\,;\ \mathbf{b}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ bs_8 \end{bmatrix} \in R^n;$$

$\mathbf{d} = \left[\Delta\mathbf{f}\left(\mathbf{X}\right) + \Delta\mathbf{b}u + \mathbf{w}\left(t\right)\right] \in R^n$ is the total disturbance caused by unknown external disturbances $\mathbf{w}\left(t\right)$ and unknown parameter variations of the system $\Delta\mathbf{f}\left(\mathbf{X}\right) + \Delta\mathbf{b}u$.

Let $\mathbf{X}_d$ – desired state vector and $\mathbf{e} = \mathbf{X} - \mathbf{X}_d$ – tracking error. The objective is to design a sliding mode controller such that $\mathbf{e}$ converges to 0 or to its neighborhood as $t \to \infty$.

The sliding surface is chosen as [3, 4]:

$$\sigma = \mathbf{c}^{\mathrm{T}}\mathbf{e}, \tag{6}$$

Where: $\mathbf{c} = \left[c_1, c_2, ..., c_7\right]^{\mathrm{T}}$; $c_1, c_2, ..., c_7$ are positive constants such that the polynomial $\varphi\left(\lambda\right) = c_7\lambda^6 + c_6\lambda^5 + ... + c_1$ is Hurwitz.

In practice, the disturbance **d** is bounded, so there exists an unknown constant $D > 0$ such that

$\left|\mathbf{c}^{\mathrm{T}}\mathbf{d}\right| < D$. Let $\hat{D}$ be the estimate of *D*. The adaptive control law is:

$$u = -\left(\mathbf{c}^{\mathrm{T}}\mathbf{b}_0\right)^{-1}\left(\mathbf{c}^{\mathrm{T}}\mathbf{f}_0\left(\mathbf{X}\right) - \mathbf{c}^{\mathrm{T}}\dot{\mathbf{X}}_d + K\sigma + \hat{D}\mathrm{sign}\left(\sigma\right)\right); \tag{7}$$

$$\dot{\hat{D}} = \alpha\left|\sigma\right|, \tag{8}$$

with $K > 0$, $\alpha > 0$ are positive constants selected during simulation.

The control law (7) with the parameter adjustment law (8) ensures that the system (3) and (5) is stable with the tracking error approaching 0. To prove the stability of the system, the Lyapunov method is used, and the Lyapunov function is chosen as:

$$V = \frac{1}{2}\sigma^2 + \frac{1}{2\alpha}\tilde{D}^2, \tag{9}$$

Where $\tilde{D} = \hat{D} - D$ – the estimation error of *D*.

Taking the derivative of both sides of equation (9), we obtain the following equation:

$$\dot{V} = \sigma\dot{\sigma} + \frac{1}{\alpha}\tilde{D}\dot{\hat{D}}, \tag{10}$$

Since $\dot{\sigma} = \mathbf{c}^{\mathrm{T}}\dot{\mathbf{e}} = \mathbf{c}^{\mathrm{T}}\left(\dot{\mathbf{X}} - \dot{\mathbf{X}}_d\right)$ we have:

$$\dot{\sigma} = \mathbf{c}^{\mathrm{T}}\left[\mathbf{f}_0\left(\mathbf{X}\right) + \mathbf{b}_0 u + \mathbf{d} - \dot{\mathbf{X}}_d\right]. \tag{11}$$

Substituting (7) into (11), we derive the stability condition:

$$\dot{\sigma} = \mathbf{c}^{\mathrm{T}}\mathbf{d} - \hat{D}\mathrm{sign}\left(\sigma\right) - K\sigma. \tag{12}$$

Combining (8), (10) and (12) we obtain the following equation:

$$\dot{V} = \sigma\left[\mathbf{c}^{\mathrm{T}}\mathbf{d} - \hat{D}\mathrm{sign}\left(\sigma\right)\right] + \tilde{D}\left|\sigma\right| - K\sigma^2 = \left(\sigma\mathbf{c}^{\mathrm{T}}\mathbf{d} - D\left|\sigma\right|\right) - K\sigma^2. \tag{13}$$

Since $\left|\mathbf{c}^{\mathrm{T}}\mathbf{d}\right| < D$, $\sigma\mathbf{c}^{\mathrm{T}}\mathbf{d} \le \left|\sigma\right|\left|\mathbf{c}^{\mathrm{T}}\mathbf{d}\right| < D\left|\sigma\right|$ we have:

$$\dot{V} < -K\sigma^2 \le 0. \tag{14}$$

Thus, according to Barbalat's lemma, the system is stable and the sliding surface approaches 0 ($\sigma \to 0$), meaning the tracking error approaches 0 as designed ($e \to 0$).

In practical applications, high-frequency oscillations phenomenon in sliding mode control (chattering phenomenon) can significantly reduce the lifespan of mechanical components. To overcome this limitation while preserving control performance, this paper implements a smooth hyperbolic tangent (tanh) function as a continuous approximation of the discontinuous sign function in the control law (7). Furthermore, the presence of external disturbances and parameter variations can lead to uncontrolled growth of the adjustment parameter, potentially causing system instability. To prevent drift and parameter escalation, the paper proposes adding a negative feedback term to the parameter adjustment law (8), resulting in:

$$u = -\left(\mathbf{c}^{\mathrm{T}}\mathbf{b}_0\right)^{-1}\left(\mathbf{c}^{\mathrm{T}}\mathbf{f}_0\left(\mathbf{X}\right) - \mathbf{c}^{\mathrm{T}}\dot{\mathbf{X}}_d + K\sigma + \hat{D}\tanh\left(\sigma/\varepsilon\right)\right); \tag{15}$$

$$\dot{\hat{D}} = \alpha\left(\left|\sigma\right| - \gamma\hat{D}\right). \tag{16}$$

Where: $\varepsilon, \gamma > 0$ – small positive constants. The value of ε is inversely proportional to the steepness (slope) of the tanh function.

As demonstrated in [12], the parameter adjustment law (16) with negative feedback term ensures boundedness of all closed-loop system signals and guarantees their exponential convergence to an invariant set. The size of this set can be made arbitrarily small through the proper selection of control parameters. By increasing control gains K and α while decreasing design parameters ε and γ, we achieve: reducing the size of this invariant set and increasing the convergence rate of all signals. Consequently, the proposed adaptive sliding mode control (15), (16) ensures system stability while driving the position tracking error to a neighborhood of 0.

3. SIMULATION OF THE ADAPTIVE SLIDING MODE CONTROL SYSTEM

In the system, the drive motor is a 5MT-type DC motor with a power rating of 7,4 kW. The initial parameters of the system are as follows [1]: $J_1 = 0.05$ $kg.m^2$; $J_2 = 0.03$ $kg.m^2$; $J_3 = 0.1$ $kg.m^2$; $p_{01} = 30$ Nm/rad; $p_{02} = 25$ Nm/rad; $L = 0.01$ H; $R = 2$ Ω; $k_E = 1$ $V.s$; $k_M = 1$ $V.s$; $k_A = 22$; $k_D = 0.0095$ V/rad; $k_T = 0.0095$ Vs/rad; $k_V = 0.0095$ V/rad. The external disturbance is of the form: $w(t) = 1.5\sin(2\pi t)$. The parameters of the adaptive sliding mode controller are selected as follows:

$$K = 50; \alpha = 10^3; \varepsilon = \gamma = 0.01; \mathbf{c} = \left[1,\ 0.024,\ 0.00024,\ 1.28\cdot10^{-6},\ 3.84\cdot10^{-9},\ 6.144\cdot10^{-12},\ 4.096\cdot10^{-15}\right]^T.$$

The paper simulates the system response to position tracking and angular trajectory tracking under conditions of parameter uncertainty and external disturbances. To demonstrate the stability and effectiveness of the proposed adaptive sliding mode controller, we choose the following PID control law for comparison: $u_{PID} = 3.15e(t) + 0.3\int_0^t e(t_1)dt_1 + 4.8\dot{e}(t)$.

The simulation results for position tracking are presented in figures 2-7, where "ASMC" – Adaptive Sliding Mode Control; "PID" – PID control. In figures 6 and 7, the subscript n denotes the nominal value of the system parameters.

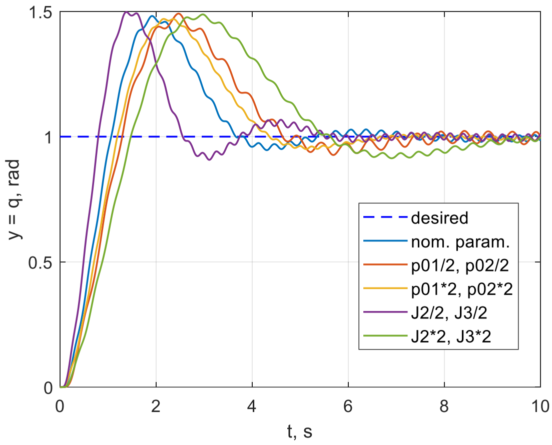

Figure 2. *System response when parameters change without external disturbances (PID).*

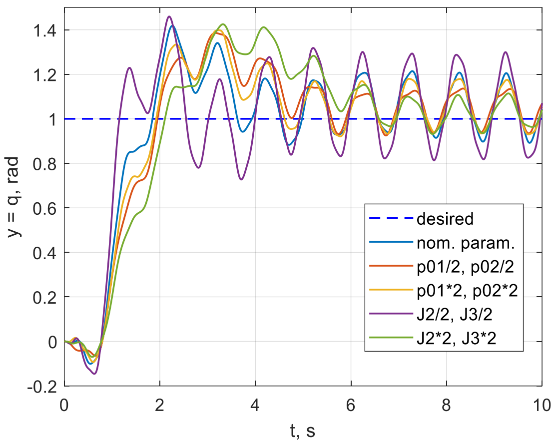

Figure 3. *System response when parameters change with external disturbances (PID).*

Observations:

- In the case of parameter changes without external disturbances, the system with a PID controller performs with poor control quality: the settling time exceeds 3s, the overshoot reaches 50%, and there are persistent oscillations due to the system's elastic properties (figure 2). Meanwhile, the adaptive sliding mode control system remains fully stable. The system response meets the design quality criteria: the settling time does not exceed 0.25s with no overshoot. As observed, the transient response of the proposed adaptive system does not change significantly when parameters vary (figure 4).

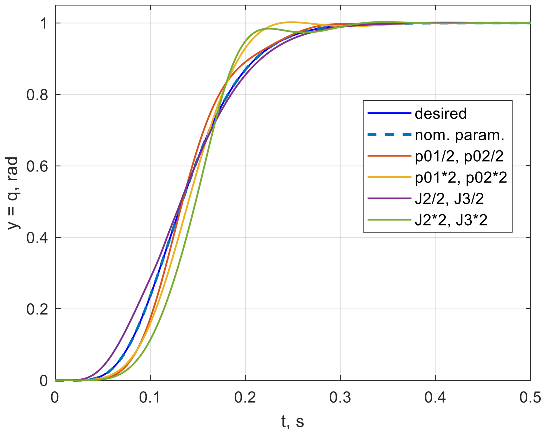

Figure 4. *System response when parameters change without external disturbances (ASMC).*

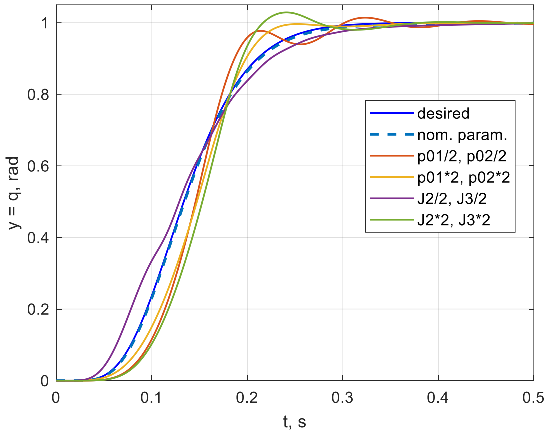

Figure 5. *System response when parameters change with external disturbances (ASMC).*

- In the case of parameter changes with external disturbances, the system with a PID controller becomes completely unstable, exhibiting high-amplitude oscillations that could potentially damage mechanical components (figure 3). In contrast, the adaptive sliding mode control system remains highly stable. The system response aligns with the design requirements: the maximum settling time remains around 0.25s; overshoot is negligible, staying below 2%. Even in the worst-case scenario, the proposed adaptive system exhibits very slight oscillations before quickly returning to the setpoint (figure 5).

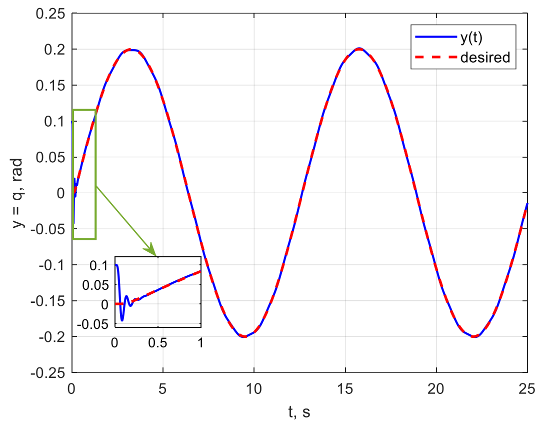

Figure 6. *Position tracking of the desired trajectory when $p_{01}=p_{01n}/2$; $p_{02}=p_{02n}/2$; $J_2=2J_{2n}$; $J_3=2J_{3n}$ with external disturbances (ASMC).*

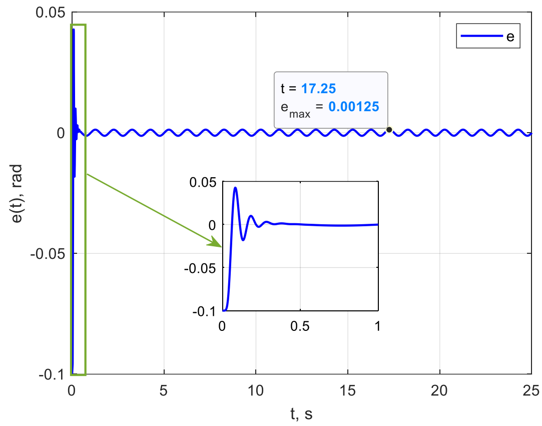

Figure 7. *Position tracking error when $p_{01}=p_{01n}/2$; $p_{02}=p_{02n}/2$; $J_2=2J_{2n}$; $J_3=2J_{3n}$ with external disturbances (ASMC)).*

- The simulation results for tracking the desired trajectory with adaptive sliding mode controller in the worst-case scenario (with external disturbances, moments of inertia of masses 2 and 3 doubled, and stiffness coefficients between masses 1-2 and 2-3 halved) indicate that the system operates very stably, closely following the reference signal with high accuracy (figure 6).

- In this case, the position tracking error at steady state is extremely small e_{max} = 0.00125 rad ≈ 0.072^0, indicating very high tracking accuracy. As analyzed theoretically in the previous section, the steady-state error does not approach 0 but remains in a neighborhood of 0 due to the use of the parameter adjustment law (16) to prevent uncontrolled growth of the adjustment parameter under unknown external disturbances, while still ensuring tracking accuracy (figure 7).

- The simulation results confirm the correctness of the theoretical analysis and the effectiveness of the designed adaptive sliding mode controller for the three-mass elastic electromechanical system under simultaneous effects of parameter uncertainty and unknown external disturbances.

4. CONCLUSIONS

Based on the adaptive sliding mode control method, the authors propose the synthesis of an adaptive sliding mode position tracking controller for a three-mass elastic electromechanical system under conditions of parameter uncertainty and unknown varying external disturbances. The proposed adaptive sliding mode controller has a simple yet effective structure, suitable for practical implementation. Simulation results comparing the proposed controller with a conventional PID controller demonstrate the effectiveness of the adaptive sliding mode controller through evaluation criteria: robustness to parameter changes and external disturbances, assurance of design quality, stability, and high-precision position tracking.

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TÓM TẮT

Điều khiển trượt thích nghi hệ cơ điện đàn hồi ba khối lượng
với các tham số bất định và ảnh hưởng của nhiễu ngoài

Bài báo nghiên cứu nâng cao chất lượng điều khiển hệ cơ điện đàn hồi ba khối lượng trong điều kiện bất định tham số và ảnh hưởng của nhiễu ngoài. Để phát triển hệ thống điều khiển bài cáo coi đối tượng điều khiển là một hệ cơ điện đàn hồi ba khối lượng với ba vòng điều khiển lệ thuộc. Từ đó, mô hình toán học đầy đủ của đối tượng được xây dựng. Một bộ điều khiển trượt thích nghi bền vững được đề xuất áp dụng cho hệ đàn hồi ba khối lượng nhằm chống lại ảnh hưởng của nhiễu loạn bên ngoài chưa biết và phần bất định của mô hình đối tượng gây ra bởi sự bất định tham số. Điểm nổi bật của bộ điều khiển trượt thích nghi trên là nó có cơ chế điều chỉnh thích nghi độ lợi của bộ điều khiển bằng cách ước lượng giới hạn trên của tổng nhiễu ngoài và sự không chắc chắn của mô hình hệ thống. Do đó, bộ điều khiển đảm bảo bù các ảnh hưởng của các yếu tố tiêu cực trên. Tính ổn định của hệ thống điều khiển thích nghi đề xuất được khảo sát bằng lý thuyết ổn định Lyapunov. Kết quả mô phỏng trên MATLAB/Simulink so sánh hiệu suất của bộ điều khiển trượt thích nghi đề xuất với bộ điều khiển PID thông thường chứng minh tính hiệu quả của bộ điều khiển đề xuất.

Từ khóa: Hệ đàn hồi; Ba khối lượng; Điều khiển trượt thích nghi; Tham số không chắc chắn; Nhiễu ngoài thay đổi.